

Identifying the Unique Projection and Follow-up Runs for $k = 4$ or 5 Important Factors from the $n = 12, 20$ or 24-run Plackett Burman Designs

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Abstract: Complexities involved with identifying the projection for a specific set of k factors ($k = 2, \dots, 11$) from an n -run ($n = 12, 20$ or 24) Plackett Burman design are described. Once the correct projection is determined, difficulties with selecting the necessary additional runs to complete either the full or half fraction factorial for the respective projection are noted, especially for $n = 12, 20$ or 24 and $k = 4$ or 5. Because of these difficulties, a user-friendly computational approach that identifies the projection and corresponding necessary follow-up runs to complete the full or half fraction factorial is given. The method is illustrated with a real data example.

Key words: Foldover; follow-up, fractional factorial, main effect, Plackett Burman design, projection, 2-factor interaction.

1. Introduction

Plackett and Burman (1946) presented a set of two-level fractional factorial designs (PB designs) for estimating $(n - 1)$ factors in n runs. If all 2-factor (and above) interactions are “0”, each of the estimated $(n - 1)$ main effects are unbiased. The plans set forth by Plackett and Burman cover the situations where n is a multiple of four and $n \leq 100$. Baumert, Golomb, and Hall (1962) provided the plans when $n = 92$. PB designs are especially attractive in screening situations, i.e., when many factors $(n - 1)$ may affect an outcome of interest but insufficient time, effort and money are available to perform a “full” 2^{n-1} factorial needed to determine the few (k) important factors that significantly influence the outcome.

If 2-factor interactions exist when using a PB design, biased estimation of the $(n - 1)$ main effects occurs, i.e., the estimated main effects actually contain a contribution from existing 2-factor interactions. Lin and Draper (1993) showed how each estimated main effect contains a contribution from a long list of 2-factor interactions. Box, Hunter, and Hunter (1978) suggested one approach to

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eliminate this “blurring”. Their plan is typically referred to as a “fold over”. For each of the original n treatment combinations, the “lo” level of a given factor is changed to the “hi” level and the “hi” level of a factor is changed to a “lo” level, i.e., if a “-” represents a “lo” level and a “+” represents a “hi” level, the “+s” are changed to “-s” and vice-versa. Running the resulting new set of n treatment combinations constitutes a follow-up of size n . Hence, the set of $2n$ runs guarantee that each of the estimated $(n - 1)$ main effects are free of any 2-factor interactions (higher order interactions may be included in these estimates but they are usually assumed to be minimal and are ignored). Box, Hunter and Hunter (1978) showed that following up a PB design of size n with this foldover approach always converts a resolution III design into a resolution IV design but the total run size = $2n$ is doubled. (A PB design of resolution III is such that all estimated main effects estimate both main effects and a long list of 2-factor interactions. All estimated main effects from a resolution IV design are free of any 2-factor interactions but may be distorted by contributions from some 3-factor (and higher) interactions.)

A downside to the foldover approach is the required cost, time and effort. For the projection of $k = 2$ or 3 important factors from $n \leq 84$ PB designs, Box and Tyssedal (1996) listed full factorial properties, i.e., when no follow up runs are needed to complete a full 2^k factorial. Lin and Draper (1995) discussed alternatives to a fold over. Their work showed that after the initial analysis of a PB design, projection of the whole design into a lower dimensional space corresponding to the k important factors leads to identification of helpful additional runs (for some values of k). Specifically, projection properties of the two-level designs for the k factors give insight as to which additional runs should be performed that either complete the full 2^k factorial or complete a 2^{k-1} fractional factorial. The number of these additional runs being significantly less (for $k = 4$ or 5) than a foldover would require. Further, Draper and Linn (1995) provided a characterization of the different possible projections of an n -run PB design into k factors. Characterizations were made for $n = 12, 20$ and 24 where $k = 2, 3, 4$ and 5 respectively. When $n = 12$ and $k = 2, 3, 4$, only one “type” of projection exists for each k . For $n = 12$ and $k = 5$, two “types” of projections were identified. When $n = 20$ and 24 , two “types” of projections exist for $k = 3$. When $n = 20$ and $k = 4$, three “types” of projections occur. When $n = 24$ and $k = 4$, four “types” of projections occur. When $n = 20$ or 24 and $k = 5$, nine “types” of projections were identified. When $n = 20$ or 24 and $k > 5$, there are a multitude of projection types for each combination of n and k . Hence, after analyzing the initial PB design and identifying the potentially important k factors, determining the correct projection type and necessary additional runs to complete the full or half fraction factorial can be quite tedious and difficult for the user.

The *major contributions of this paper are user-friendly* computational methods for various (n, k) combinations from a *user-defined* n -run PB design that determine: 1. the corresponding projection type and 2. additional runs, in terms of the user's k important factors (columns), necessary to complete a 2^k and 2^{k-1} factorial design for $k = 4$ and 5 and $n = 12, 20$ or 24 . Section 2 highlights the excellent work by Lin and Draper (1995), Draper and Lin (1995) and Lin and Draper (1992) pertaining to projection types and follow-up runs to a PB designed experiment. Practical difficulties associated with implementation of their important work are noted, providing motivation for the above-mentioned computational approach. Section 3 presents an application of our two computer programs in the context of a real-life data example. The Appendix includes "layouts" of the PB designs used by JMP (2000), tables of corresponding projection types (determined by our computer program), and "values" of the criteria used to identify corresponding projection types based on the JMP (2000) PB design layouts.

2. Plackett Burman Designs

In the Appendix, Tables 1, 2 and 3 show the 12-run, 20-run and 24-run Plackett Burman designs used by the statistical software JMP (2000), respectively. Consider the 12-run design. The first row (run) is all plus signs. The second row (run) is

$$- + - + + + - - - + -.$$

Permute the last sign on the right to become the first sign on the left gives the third row (run)

$$- - + - + + + - - - +.$$

Continuing with this permutation approach produces the remaining 9 runs of the 12-run Plackett Burman design in Table 1. Consider the 20-run PB design. The first run is all plus signs. The second run is $- + - - + + + + - + - + - - - + + -$. Permuting as described above produces the remaining 18 runs of the 20-run PB design in Table 2. Consider the 24-run PB design. The first run is all plus signs. The second run is

$$- + + + + - + - + + - - + + - - + - + - - - -.$$

Permuting as described above produces the remaining 22 runs of the 24-run PB design in Table 3.

Draper and Lin (1995) and Lin and Draper (1995) presented two other schemes for generating PB designs. Both of their approaches outlined the following "first run" for $n = 12, 20$ and 24 :

$$\begin{aligned}
 n = 12 & : + + - + + + - - - + - \\
 n = 20 & : + + - - + + + + - + - + - - - - + + - \\
 n = 24 & : + + + + + - + - + + - - + + - - + - + - - - - .
 \end{aligned}$$

However, generation of the remaining $(n - 1)$ columns differed. Draper and Lin (1995) recommended removing the leftmost sign, place it on the extreme right and move all signs one place back to the left — the n th row being all $-$'s. In their other article, Lin and Draper (1995) recommended removing the rightmost sign, placing it on the extreme left and moving all signs one place to the right — the n th row again being all $-$'s. The result is that the “generating” columns identified by Draper and Lin (1995) for the different projections are only appropriate for the particular PB design espoused in their article. Further, the follow-up runs Lin and Draper (1995) identified as those needed to complete the full or fractional factorials are only appropriate for their PB design (based on their column labels). A significant improvement for the user would be a method that could easily identify *the actual runs (in terms of the user's column labels)* necessary to complete the full and half fraction factorial as well as the projection type for the *user's particular PB design*.

Consider the $n = 12$ run PB design in Table 1. The first run in Table 1 is the negative of the last run of the 12-run PB design presented in the articles Draper and Lin (1995) and Lin and Draper (1995). Multiplying every entry in the JMP version by -1 and re-labeling columns 11 as 1, 10 as 2, \dots and 1 as 11, row 1 becomes row 12 in both versions outlined by Draper and Lin. Rows 2 - 11 are now rows 2 - 11 in the “remove the leftmost sign approach”. Rows 2 - 11 are now rows 11- 2 in the “remove the rightmost sign approach”. Similar column/row relationships occur amongst the 3 methods for generating a PB design for the $n = 20$ and 24 scenarios.

The relationships are described above because as mentioned, identifying a projection type and which runs are needed to complete a full or half-fraction factorial are dependent on column labels and the original version of the selected n -run PB design. When discussing the $k = 4$ and $n = 20$ PB design, Lin and Draper (1992) noted this problem stating “ \dots For $n = 20$, only three types of projections exist (*apart from sign changes in the columns, permutations of the columns and rearrangement of the rows*)” (our italics added).

Draper and Lin (1995) and Lin and Draper (1995) established a characterization scheme for PB designs. For a particular set of k columns from an n -run PB design, their scheme involved identifying triples and pairs of repeat runs, mirror image runs and single runs to distinguish unique projections. This criteria is independent of the different PB design layouts described above. Manually counting

the number of “repeat pairs” and “mirror image pairs” (and in some cases the number of triples or number of unique runs) can be tedious, time consuming, confusing and prone to error, especially when $n = 20$ or 24 and $k > 3$. Lin and Draper (1995) established the columns needed to generate the different projection types for $k = 4$ and 5 when $n = 12, 20$ and 24 using their PB design layout described above. Further, the number of different ways that a particular projection may be determined was also given by Draper and Lin (1995). In two technical reports, Lin and Draper (1991) and Draper and Lin (1988) give a more complete list of the number of ways a projection can be determined. Since there are literally 100s of configurations that produce the same projection and each particular n -run PB design can be constructed differently (with correspondingly different column labels), *manually identifying the projection type and necessary follow-up runs to complete a full factorial or half-fraction factorial for a selected example can be prohibitive, even if one uses the exact PB design given by Lin and Draper (1995) and $k = 4$ or $k = 5$* . As mentioned earlier, one of the major contributions of this paper is a user-friendly computer program that quickly identifies the projection type for a $n = 12, 20$ or 24 PB design and a selected k ($1 < k < n$). The program works for the *user’s version* of the n -run PB design and the k important factors (in terms of the *user’s columns*). (Recall the 3 different approaches to setting up a PB design outlined in Section 2). Criteria established by Draper and Lin (1995) and Lin and Draper (1995) are implemented by our program. The other major contribution of this paper is a user-friendly computer program that identifies the follow-up runs (in terms of the *user’s identified k important column #s*) needed to complete the positive or negative half fraction for any $n = 12, 20$ or 24 run PB design and any selected $k = 4$ or 5 columns. (Thus, the runs needed for completion of the full factorial are known). (If a user has, say, 12 runs but only 9 factors, he may select a 12-run PB design and enter two columns of 0’s. Both programs will work with this type of scenario.) The other major contribution or our paper are Tables 4, 5 and 6 in the Appendix which summarize the projections corresponding to the PB design layout used by JMP(2000). The following section presents an example. How the programs assist the analysis by identifying both the projection type and necessary follow-up runs (those that either complete the full factorial or the positive/negative half fraction) is illustrated.

3. Example

Srinivas, Chand and Lonsane (1994) used a 20-run PB design to screen important factors that affect production of alpha-galactosidase by *Aspergillus niger* MRSS 234 in solid state fermentation system. Their 20-run PB design had the

same first run as Plackett and Burman (1946). It is given below

+ + - - + + + + - + - + - - - - + + -.

However, they cycled to the left; that is, the leftmost sign (+) was moved to the far right and the whole row was shifted left. Thus, the second row (run) became

+ - - + + + + - + - + - - - - + + - +.

And the third row (run) became

- - + + + + - + - + - - - - + + - + +.

The last row being all -'s.

Srinivas, Chand and Lonsane (1994) identified factors (columns) 4, 6, 7, 8 and 13 as deserving further attention, i.e., $k = 5$. Using the form of their 20×19 PB design, a value $k = 5$ and the identified important factors (columns 4, 6, 7, 8 and 13), our first program determined the projection to be 5.1 as well as the number of "mirror image pairs" (5) and the number of "repeat pairs" (0). Further, the runs that make up the mirror image pairs and repeats were produced. Based on the output from the program and the work by Lin and Draper (1995), 6 follow-up runs were needed to complete the positive half fraction and another 6 follow-up runs were needed to complete the negative half fraction (thus 12 total runs needed to complete the full factorial). Their article, however, does not tell the user which runs were needed in terms of the user's columns (factors) 4, 6, 7, 8 and 13. Our second program provided that answer.

Based on the same input information as the first program, our second program specified which treatment combinations (from the 2^k possible combinations) were needed to complete the respective positive and negative half fractions, i.e., the necessary follow-up runs (in terms the user understands, i.e., the user's column #s). Thus, to complete a positive 2^{5-1} fractional factorial, where all estimated main effects are free of any 2-factor interactions and all estimated 2-factor interactions are free of all main effects and all other 2-factor interactions, the 6 follow-up runs needed in addition to the original 20 runs for the user's columns 4, 6, 7, 8 and 13 were:

| Columns | | | | |
|---------|----|----|----|----|
| 4 | 6 | 7 | 8 | 13 |
| 1 | -1 | -1 | -1 | -1 |
| -1 | -1 | 1 | -1 | -1 |
| 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | 1 |
| -1 | -1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Similarly, the 6 follow-up runs needed to complete the negative half fraction (in addition to the original 20 runs for the user's columns 4, 6, 7, 8 and 13) were identified as:

| Columns | | | | |
|---------|----|----|----|----|
| 4 | 6 | 7 | 8 | 13 |
| -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | -1 | 1 | -1 |
| 1 | 1 | 1 | 1 | -1 |
| -1 | 1 | -1 | -1 | 1 |
| -1 | -1 | -1 | 1 | 1 |

To complete the full 2^5 , the combined set of follow-up runs (positive half and negative half) would be needed in addition to the original 20 runs for the user's columns 4, 6, 7, 8 and 13.

When $k > 5$ and $n = 12, 20$ or 24 , sometimes it is advantageous to do a fold over and sometimes it is advantageous (less runs with same "accuracy", i.e., both follow-up procedures result in estimated main effects being free from 2-factor interactions but one follow-up procedure may have less runs) to do a follow-up with the runs that complete the positive or negative half fraction. This problem is currently being investigated.

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Appendix

Table 1: JMP 12-run Plackett Burman design

| | | Columns | | | | | | | | | |
|-----|----|---------|----|----|----|----|----|----|----|----|----|
| Run | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 |
| 3 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 |
| 4 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 |
| 5 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 |
| 6 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 |
| 7 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 |
| 8 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 |
| 9 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 10 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 |
| 11 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 |
| 12 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 |

Table 2: JMP 20-run Plackett Burman design

| | | Columns | | | | | | | | | | | | | | | | | |
|-----|----|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Run | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 |
| 3 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 4 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 |
| 5 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 |
| 6 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 |
| 7 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 |
| 8 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 9 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 |
| 10 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 |
| 11 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 |
| 12 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 |
| 13 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 |
| 14 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 |
| 15 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 |
| 16 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 |
| 17 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 |
| 18 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 |
| 19 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 20 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 |

Table 3: JMP 24-run Plackett Burman design

| Columns | | | | | | | | | | | | | | | | | | | | | | | |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Run | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 |
| 3 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 |
| 4 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 |
| 5 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 |
| 6 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 7 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 |
| 8 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 9 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 10 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| 11 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |
| 12 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 13 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 14 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 15 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 |
| 16 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 |
| 17 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 |
| 18 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 |
| 19 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 |
| 20 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| 21 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| 22 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |
| 23 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 |
| 24 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 |

Table 4: 12-run Plackett Burman projection characterization

| JMP Columns | k | Projection Type | # repeat pairs | # mirror image pairs |
|-------------------|-----|-----------------|----------------|----------------------|
| (10, 11) | 2 | 2.1 | 12 | 12 |
| (9, 10, 11) | 3 | 3.1 | 4 | 8 |
| (8, 9, 10, 11) | 4 | 4.1 | 1 | 3 |
| (7, 8, 9, 10, 11) | 5 | 5.1 | 0 | 1 |
| (4, 7, 9, 10, 11) | 5 | 5.2 | 1 | 0 |

Table 5: 20-run Plackett Burman projection characterization

| JMP Columns | k | Projection Type | # repeat pairs | # mirror image pairs |
|------------------------------|-----|-----------------|----------------|----------------------|
| (18, 19) | 2 | 2.1 | 40 | 50 |
| (17, 18, 19) | 3 | 3.1 | 16 | 24 |
| (14, 17, 19) | 3 | 3.2 | 24 | 16 |
| (16, 17, 18, 19) | 4 | 4.1 | 5 | 11 |
| (14, 17, 18, 19) | 4 | 4.2 | 9 | 7 |
| (4, 17, 18, 19) | 4 | 4.3 | 9 | 15 |
| (15, 16, 17, 18, 19) | 5 | 5.1 | 0 | 5 |
| (14, 16, 17, 18, 19) | 5 | 5.2 | 3 | 2 |
| (11, 16, 17, 18, 19) | 5 | 5.3 | 2 | 3 |
| (4, 14, 17, 18, 19) | 5 | 5.4 | 4 | 5 |
| (6, 16, 17, 18, 19) | 5 | 5.5 | 1 | 4 |
| (5, 16, 17, 18, 19) | 5 | 5.6 | 3 | 6 |
| (8, 15, 17, 18, 19) | 5 | 5.7 | 2 | 7 |
| (11, 14, 17, 18, 19) | 5 | 5.8 | 6 | 3 |
| (10, 14, 17, 18, 19) | 5 | 5.9 | 4 | 1 |
| (14, 15, 16, 17, 18, 19) | 6 | 6.1 | 0 | 1 |
| (13, 15, 16, 17, 18, 19) | 6 | 6.2 | 0 | 0 |
| (13, 14, 16, 17, 18, 19) | 6 | 6.3 | 1 | 0 |
| (13, 14, 15, 17, 18, 19) | 6 | 6.4 | 0 | 2 |
| (13, 14, 15, 16, 18, 19) | 6 | 6.5 | 1 | 1 |
| (12, 13, 15, 16, 18, 19) | 6 | 6.6 | 1 | 2 |
| (12, 13, 14, 15, 18, 19) | 6 | 6.7 | 1 | 5 |
| (11, 14, 16, 17, 18, 19) | 6 | 6.8 | 2 | 0 |
| (11, 13, 15, 16, 17, 19) | 6 | 6.9 | 0 | 3 |
| (11, 13, 14, 15, 16, 19) | 6 | 6.10 | 2 | 2 |
| (11, 12, 15, 17, 18, 19) | 6 | 6.11 | 2 | 1 |
| (11, 12, 15, 16, 18, 19) | 6 | 6.12 | 1 | 3 |
| (11, 12, 13, 14, 15, 19) | 6 | 6.13 | 0 | 4 |
| (10, 11, 12, 14, 16, 19) | 6 | 6.14 | 2 | 4 |
| (9, 13, 16, 17, 18, 19) | 6 | 6.15 | 3 | 0 |
| (8, 11, 14, 17, 18, 19) | 6 | 6.16 | 3 | 1 |
| (7, 12, 15, 17, 18, 19) | 6 | 6.17 | 3 | 3 |
| (13, 14, 15, 16, 17, 18, 19) | 7 | 7.1 | 0 | 0 |
| (12, 13, 14, 15, 17, 18, 19) | 7 | 7.2 | 0 | 1 |
| (11, 13, 14, 15, 16, 18, 19) | 7 | 7.3 | 1 | 0 |
| (11, 12, 14, 15, 17, 18, 19) | 7 | 7.4 | 1 | 1 |
| (11, 12, 13, 14, 17, 18, 19) | 7 | 7.5 | 1 | 2 |
| (11, 12, 13, 14, 15, 18, 19) | 7 | 7.6 | 0 | 2 |
| (9, 10, 11, 12, 15, 16, 19) | 7 | 7.7 | 0 | 3 |
| (8, 11, 12, 14, 17, 18, 19) | 7 | 7.8 | 2 | 1 |

Table 5 (continued): 20-run Plackett Burman projection characterization

| JMP Columns | k | Projection Type | # repeat pairs | # mirror image pairs |
|------------------------------|-----|-----------------|----------------|----------------------|
| (8, 9, 11, 13, 16, 18, 19) | 7 | 7.9 | 2 | 0 |
| (12, 13, 14,15,16,17,18,19) | 8 | 8.1 | 0 | 0 |
| (11,12,13,14,15,17,18,19) | 8 | 8.2 | 0 | 1 |
| (10,11,13,14,15,16,18,19) | 8 | 8.3 | 1 | 0 |
| (8,11,12,14,15,17,18,19) | 8 | 8.4 | 1 | 1 |
| (8,9,10,11,14,15,16,19) | 8 | 8.5 | 0 | 2 |
| (11,12,13,14,15,16,17,18,19) | 9 | 9.1 | 0 | 0 |
| (8,11,12,13,14,15,17,18,19) | 9 | 9.2 | 0 | 1 |
| (7,8,11,12,14,15,17,18,19) | 9 | 9.3 | 1 | 0 |

Table 6: 24-run Plackett Burman projection characterization

| JMP Columns | k | Projection Type | # repeat pairs | # mirror image pairs |
|--------------------------|-----|-----------------|----------------|----------------------|
| (22, 23) | 2 | 2.1 | 60 | 72 |
| (21, 22, 23) | 3 | 3.1 | 24 | 36 |
| (18, 21, 23) | 3 | 3.2 | 28 | 32 |
| (20, 21, 22, 23) | 4 | 4.1 | 8 | 20 |
| (19, 21, 22, 23) | 4 | 4.2 | 8 | 16 |
| (18, 21, 22, 23) | 4 | 4.3 | 12 | 16 |
| (16, 21, 22, 23) | 4 | 4.4 | 12 | 12 |
| (13, 20, 21, 22, 23) | 5 | 5.1 | 2 | 10 |
| (17, 20, 21, 22, 23) | 5 | 5.2 | 4 | 8 |
| (16, 20, 21, 22, 23) | 5 | 5.3 | 2 | 6 |
| (15, 20, 21, 22, 23) | 5 | 5.4 | 6 | 6 * no 3's |
| (10, 20, 21, 22, 23) | 5 | 5.5 | 0 | 8 |
| (9, 19, 21, 22, 23) | 5 | 5.6 | 4 | 4 |
| (15, 18, 21, 22, 23) | 5 | 5.7 | 6 | 6 * two 3's |
| (14, 19, 20, 22, 23) | 5 | 5.8 | 6 | 6 * one 3 |
| (15, 17, 18, 22, 23) | 5 | 5.9 | 6 | 2 |
| (18, 19, 20, 21, 22, 23) | 6 | 6.1 | 1 | 5 *15 clean |
| (17, 19, 20, 21, 22, 23) | 6 | 6.2 | 0 | 6 |
| (17, 18, 20, 21, 22, 23) | 6 | 6.3 | 1 | 3 |
| (17, 18, 19, 21, 22, 23) | 6 | 6.4 | 2 | 4 * 12 clean |
| (17, 18, 19, 20 21, 23) | 6 | 6.5 | 0 | 4 |
| (16, 19, 20, 21, 22, 23) | 6 | 6.6 | 1 | 1 |

Table 6 (continued): 24-run Plackett Burman projection characterization

| JMP Columns | k | Projection Type | # repeat pairs | # mirror image pairs |
|------------------------------------|-----|-----------------|----------------|----------------------|
| (16, 18, 19, 21, 22, 23) | 6 | 6.7 | 2 | 2 |
| (16, 18, 19, 20, 22, 23) | 6 | 6.8 | 0 | 2 |
| (15, 18, 19, 21, 22, 23) | 6 | 6.9 | 2 | 0 |
| (15, 17, 20, 21, 22, 23) | 6 | 6.10 | 4 | 2 |
| (15, 17, 18, 20, 22, 23) | 6 | 6.11 | 3 | 1 |
| (15, 16, 19, 21, 22, 23) | 6 | 6.12 | 0 | 0 |
| (15, 16, 18, 20, 22, 23) | 6 | 6.13 | 2 | 4 *15 clean |
| (14, 15, 19, 20, 22, 23) | 6 | 6.14 | 3 | 3 *12 clean |
| (14, 15, 17, 19, 21, 23) | 6 | 6.15 | 1 | 5 *12 clean |
| (14, 15, 17, 19, 20, 23) | 6 | 6.16 | 2 | 4 *18 clean |
| (13, 18, 19, 21, 22, 23) | 6 | 6.17 | 3 | 3 *15 clean |
| (12, 14, 15, 19, 20, 23) | 6 | 6.18 | 4 | 0 |
| (16, 18, 19, 20, 21, 22, 23) | 7 | 7.1 | 0 | 1 |
| (15, 18, 19, 20, 21, 22, 23) | 7 | 7.2 | 1 | 0 |
| (15, 17, 18, 19, 21, 22, 23) | 7 | 7.3 | 0 | 0 |
| (15, 16,17, 18, 20, 22, 23) | 7 | 7.4 | 1 | 1 |
| (14, 15, 17, 18, 19, 22, 23) | 7 | 7.5 | 2 | 0 |
| (17, 18, 19, 20, 21, 22, 23) | 7 | 7.6 | 0 | 3 |
| (16, 17, 18, 19, 21, 22, 23) | 7 | 7.7 | 1 | 2 |
| (15, 17, 19, 20, 21, 22, 23) | 7 | 7.8 | 0 | 2 |
| (14, 15, 17, 20, 21, 22, 23) | 7 | 7.9 | 2 | 1 |
| (13, 14, 16, 17, 18, 21, 23) | 7 | 7.10 | 2 | 2 |
| (12, 17, 19, 20, 21, 22, 23) | 7 | 7.11 | 0 | 4 |
| (12,13,14,16,17,18,23) | 7 | 7.12 | 1 | 3 |
| (16,17,18,19,20,21,22,23) | 8 | 8.1 | 0 | 1 |
| (15,17,18,19,20,21,22,23) | 8 | 8.2 | 0 | 0 |
| (14,15,17,18,19,20,22,23) | 8 | 8.3 | 1 | 0 |
| (13,14,15,16,17,18,21,23) | 8 | 8.4 | 1 | 1 |
| (12,17,18,19,20,21,22,23) | 8 | 8.5 | 0 | 2 |
| (15,16,17,18,19,20,21,22,23) | 9 | 9.1 | 0 | 0 |
| (13,14,15,16,18,19,20,22,23) | 9 | 9.2 | 0 | 1 |
| (12,14,15,17,18,19,20,22,23) | 9 | 9.3 | 1 | 0 |
| (14,15,16,17,18,19,20,21,22,23) | 10 | 10.1 | 0 | 0 |
| (11,12,13,14,16,17,18,20,21,23) | 10 | 10.2 | 0 | 1 |
| (10,11,13,14,15,18,19,21,22,23) | 10 | 10.3 | 1 | 0 |
| (13,14,15,16,17,18,19,20,21,22,23) | 11 | 11.1 | 0 | 0 |
| (9,10,11,13,14,15,18,19,21,22,23) | 11 | 11.2 | 1 | 0 |
| (9,10,11,12,14,16,18,20,21,22,23) | 11 | 11.3 | 0 | 1 |

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