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Investigating The Repeatability Of The Extracted Factors In Relation To The Type Of Rotation Used, And The Level Of Random Error: A Simulation Study

Dimitris Panaretos[1], George Tzavelas[2], Malvina Vamvakari[3], Demosthenes Panagiotakos[1]

[1]*School of Health Science and Education, Harokopio University, Athens, Greece*
[2]*Department of Statistics and Insurance Science, University of Piraeus, Piraeus, Greece*
[3] *Department of Informatics & Telematics, School of Digital Technology, Harokopio University, Athens, Greece*

ABSTRACT

Factor analysis (FA) is the most commonly used pattern recognition methodology in social and health research. A technique that may help to better retrieve true information from FA is the rotation of the information axes. The purpose of this study was to evaluate whether the selection of rotation type affects the repeatability of the patterns derived from FA, under various scenarios of random error introduced, based on simulated data from the Standard Normal distribution. It was observed that when applying promax non - orthogonal rotation, the results were more repeatable as compared to the orthogonal rotation, irrespective of the level of random error introduced in the model.

Keywords: factor analysis; multivariate analysis; recognition pattern analysis; rotation; repeatability

Shortened title: Investigating the repeatability of rotations

1. INTRODUCTION

Accuracy of the results in both research and clinical practice is a cornerstone in research methodology in order to make robust conclusions about the tested hypothesis. An accurate medical assessment is of major importance in research in order to make robust conclusions (Golafshani, 2003; Hammersley, 1987; Rose and Barker, 1978). However, all experiments have some degree of random errors that are caused inevitably in an experimental process due to imponderable and uncontrollable factors and by extension influence the accuracy of a research. For the past twenty years, research has incorporated pattern recognition analysis into analytical methodologies. Pattern analysis is a classical multivariate statistical approach that aims to identify patterns in data in order to show certain attributes. The patterns are usually extracted through Factor Analysis (FA) and Principal Components Analysis (PCA) (Panaretos et al., 2016; Parmet, Edna and Sherman, 2010). Particularly, FA is a statistical procedure, which aims in finding patterns among a set of variables. In other words, that method was created in order to reduce the difficulty of the problem (the number of variables) without missing the initial information. A technique that may help to better retrieve true information from the FA is the rotation of the information axes (factors and components, respectively). It has been suggested that rotation of the axes is required so that the extracting factors can be more interpretable. The rotation maximizes the variance explained of the extracted components and makes the pattern of loadings more well-defined (Thurstone, 1947; Cattel, 1978). The rotation can be orthogonal (the factors are uncorrelated) or non-orthogonal (the factors are correlated). The most common methods of orthogonal rotation are Varimax (Kaiser, 1958) and Quartimax (Carroll, 1953; Neuhaus and Wrigley, 1954; Saunders, 1960) while the most common methods of non-orthogonal rotation are Promax (Hendrickson and White, 1964) and direct Oblimin (Jennrich and Sampson, 1966).

In a previously published paper (Panaretos et. al, 2018) it was evaluated whether rotation type, in the presence of constant random error, influences the repeatability on simulated data derived from Normal and Uniform distributions. However, adding random error into all variables is not realistic for real data application. To the best of our knowledge, the role of the introduction of various levels of random error to different set of variables in relation to the repeatability of the extracted factors through FA, with or without rotation, has not been studied in the literature. Thus, the purpose of this study was to evaluate whether the selection of certain rotation type, under various scenarios of random error introduced in the initial variables, affects the repeatability of the patterns derived from the application of FA, based on simulated data obtained from the Standard Normal distribution.

2. METHODS

2.1 Theoretical background

Let's assume a set of observed variables X = [x1, x2, …, xp], supposed to be linked to a smaller number of common factors F = [f1, f2, …, fm], where $m \leq p$. We may present X and F through a regression model of the form:

$$X_1 = L_{11}F_1 + L_{12}F_2 + \cdots + L_{1m}F_m + u_1$$

$$X_2 = L_{21}F_1 + L_{22}F_2 + \cdots + L_{2m}F_m + u_2$$

$$\vdots$$

$$X_p = L_{p1}F_1 + L_{p2}F_2 + \cdots + L_{pm}F_m + u_p$$

（2.1）

where U = [u1, u2, …up] represents the error term

The aforementioned may be written in matrix notation as

$$X(px1) = L(pxm)\ F(mx1) + U(px1) \qquad (2.2)$$

where X, U are column vectors of p components, F is a column vector of m (≤ p) components and L is a pxm matrix. The Lij are called factor loadings, and express the relationship (i.e., in a form of a correlation) of each variable to the common factor F, when the data are standardized.

We assume that E(εi) = 0, Var(εi) = ψi , Cov(εi, εκ) = 0 , i≠κ and Cov(εi, fj) = 0 for all i and j. These assumptions are natural consequences of the basic model and the goals of factor analysis. (Rencher and Christensen, 2012) The assumptions Var(fj) = 1 and those that are described above, imply that the variance of the variable is given by

$$\sigma_{ii} = Var(X_i) \ = \ Var(L_{i1}F_1 + L_{i2}F_2 + \ldots + L_{im}F_m + u_i)$$

$$= L_{i1}^2 + L_{i2}^2 + \ldots + L_{im}^2 + u_i \ = \sum_{j=1}^{m} L_{ij}^2 + u_i$$

So, FA model implies that the variance of each observed variable can be split into two parts. The first given by

$$h_i^2 = \ \sum_{j=1}^{m} L_{ij}^2$$

called communality of the variable and represent the variance shared with other variables via the common factors. The second part, ui, called specificity and represent the variance not shared with other variables.

So that

$$Cov(X) = Cov(LF + u) = Cov(LF) + Cov(u) \ = L\ Cov(F)\ L' + U = L\ I\ L' + \Psi \qquad = L\ L' + \Psi$$

where LL΄ is the common factor covariance matrix.

A technique that may help to better retrieve true information from the FA is the rotation of the information axes (factors). It has been suggested that rotation of the axes is required so that the extracting factors can be more interpretable. The objective of factor rotation is to achieve the most parsimonious and simple structure possible through the manipulation of the factor pattern matrix. There are two broad classes of rotation, orthogonal and non-orthogonal, which have different underlying assumptions, but which share the common goal of simple structure (Pett, Lackey and Sullivan, 2003). Orthogonal rotation shifts the factors in the factor space maintaining 900 angles of the factors to one another to achieve the best simple structure. In contrast, a non-orthogonal rotation follows the same rotation principles as an orthogonal rotation, but due to the factors not being independent, a 900 angle of rotation is not fixed between the axes (Kieffer, 1998; Chris Chatfield, 2018). The most commonly used methods of orthogonal rotation are Varimax and Quartimax, while the methods of non-orthogonal rotation are direct Oblimin and Promax.

The Varimax procedure chooses a rotation that maximizes the variation in the squares of the column entries of the estimated factor loading matrix, namely look for those Lij where maximize the function

$$V = \frac{1}{p}\sum_{j=1}^{m}\left[\sum_{i=1}^{p} L^{*4}_{ij} - \frac{\left(\sum_{i=1}^{p} L^{*2}_{ij}\right)^2}{p}\right] \tag{2.3}$$

On the other hand, the Quartimax procedure look for those Lij where maximize the function

$$Q = \sum_{j=1}^{m}\sum_{i=1}^{p} L^{*4}_{ij} \tag{2.4}$$

The Oblimin rotation (Jennrich & Sampson, 1966) is a non-orthogonal rotation, which carry out by nminimizing the expression $OBMIN = \sum_{s<j=1}^{m}\left[\sum_{i=1}^{p} L^{*2}_{is}L^{*2}_{ij} - \frac{\delta}{p}\sum_{i=1}^{p} L^{*2}_{is}\sum_{i=1}^{p} L^{*2}_{ij}\right]$ where δ is a parameter which controls the degree of correlation between the factors. On the other hand, Promax (Hendrickson & White, 1964), of which name derives from Procrustean rotation, is similar to direct Oblimin but it is mostly used for very large datasets. Promax is often the oblique rotation strategy of choice, as it is relatively easy to use, typically provides good solutions, and tends to generate more replicable results than the direct Oblimin rotations (Kieffer &Kevin M, 1998).

2.2 Simulation Study's setting

To test the research hypothesis of the present study, i.e., the repeatability of factors derived through factor analysis under various rotation methods, a data file with 10 variables of 1000 observations each, was created. This was conducted by simulating 1000 observations from the Standard Normal distribution. Random error from a Normal distribution on [-0.1, 0.1], [-0.3, 0.3] and [-0.5, 0.5] was added respectively to each element of the constructed matrix. Therefore, it was assumed that the same research takes place under the same conditions at a later time. Consequently, the random error has been distributed on 20%, 40%, 60%, 80% and 100% of variables into the original data.

2.2 Factor analysis with and without rotation

Then, FA was applied in order to identify common factors between the 10 variables group in each case separately. For each case, 10 new variables were created, the factors, which can be generally identified in a subjective way as some non-measurable variables. The loadings matrix was saved separately, to take account of the three cases: no rotation, orthogonal rotation and non-orthogonal rotation. The exact same procedure was applied to the matrices with added error.

2.3 Testing repeatability of Factor Analysis under different scenarios

After applying the previous steps, 4 matrices were created for each case: a matrix with loadings of the extracted factors before and 3 after the random error has been distributed (0.1, 0.3 and 0.5). Then, for each case, 3 new matrices were created, which were the matrices of the difference between the loadings resulting from the factorial analysis before the error was introduced and the loadings after the error was introduced. The purpose in this step was to find the empirical expectation of the Frobenius matrix norm, $\|L - L'\|_F = \left[\sum_{i,j}\left(l_{ij} - l'_{ij}\right)^2\right]^{1/2}$ that calculated as a more global measure of showing how the input of random errors affect the repeatability of the results, under the Normal distributions (Tables 1, 2, 3, 4 & 5). Thus, the lowest the number of the metric, the higher the repeatability. The procedure was repeated, separately in case of not selecting any type of rotation and in the case of orthogonal and non-orthogonal rotation.

2.4 Simulation Process

The above procedure was repeated 1000 times. Each time the percentage of differences in the loadings and the 95% Confidence Interval for the percentage was calculated, in order to measure to what extent, the introduction of random error between multiple and independent measurements, with and without rotation, affects the repeatability of the procedure.

3. RESULTS

Tables 1 and 5 presents the results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error adding into the data, based on 1000 simulations from Standard Normal distribution, respectively. In all cases it is noticed that as the error and percentage of variables with error increased, the deviations also increased in almost with a linear way (Figures 1-5).

The orthogonal rotation, which was then applied as the most commonly used technique in order to improve the interpretation of the common factors, resulted in worse results compared to the absence of rotation of axes. In particular, in the case that the error was ± 0.1 and varimax rotation was applied, it was found that, deviations of the loadings of the factorial analysis matrices, before and after the error input the 20% of variables, was 3.54 (95%CI: 3.52 - 3.57) while the deviation was reduced to 3.61 in the case that the error was ±0.5 (95%CI: 3.58 - 3.64). Respectively, in the case that the error (± 0.1) input the 100% of variables the deviation was 4.15 (95%CI: (4.09, 4.21). Similar behavior has occurred in the case of Quartimax rotation with slightly better results.

Concerning the non-orthogonal rotation, the results were much better than the previous procedures. When selecting promax rotation with a random error input of ± 0.1, deviations of the loadings of the factorial analysis matrices, before and after the error input the 20% of variables, was 1.74 (95%CI: 1.68 - 1.80). Respectively, in the case that the error (± 0.1) input the 100% of variables the deviation was 2.55 (95%CI: 2.49 - 2.61). Similar behavior was also observed in the case of Oblimin rotation with worse results. It is also important to note the fact that, when we did not apply any type of rotation, the results were much better from orthogonal or non – orthogonal rotation results.

4. DISCUSSION

In this work the influence of the factor's rotation in factor analysis on the repeatability (robustness) of the extracted patterns was examined based on simulated data; the main goal was to test the repeatability of the results derived through a commonly used pattern recognition methodology, i.e., factor analysis, and to reveal the best rotation method under various scenarios. The simulation studies showed that when the Promax method is used, the results are more robust (i.e., repeatable) as compared to the orthogonal rotation. It is also important to note the fact that, any type of rotation is not applied, the results were much less repeatable as it can be seen from Tables 1, 2, 3 and 4.

An issue that has rarely been examined in the literature is the repeatability of the extracted patterns derived through the factor analysis. The importance of achieving repeatable and reliable information in both research and clinical practice is a fundamental issue in research methodology.

Generally, it is argued that employing a method of orthogonal rotation may be preferred over oblique rotation, due to better understanding the results in terms of interpretation. However, orthogonal rotations often do not honor a given researcher's view of reality as the researcher may believe that two or more of the extracted and retained factors are correlated. Secondly, orthogonal rotation of factor solutions may oversimplify the relationships between the variables and the factors and may not always accurately represent these relationships. In contrast, an oblique rotation offers a better chance of finding simple structure, but at the price of complicating the interpretation (Marley W. Watkins, 2018). In practical terms there is no obvious reason why factors of substantive interest should be uncorrelated as orthogonality implies (Bartholomew, Knott and Moustaki, 2011).

In our study, of course it was found that in case of Quartimax rotation the results were worse than Promax rotation. The inconsistency between the methods of rotation is inevitable due to the different mathematical structure, but what really matters is how different they are. Direct Oblimin (Jennrich & Sampson, 1966) results in higher eigenvalues but diminished interpretability of the factors.

In a previously published paper (Panaretos et. al, 2018) it was evaluated whether rotation type, in the presence of constant random error, influences the repeatability on simulated data derived from Normal and Uniform distributions. From the results of the simulation studies performed there it was observed that when applying non-orthogonal rotation, and specifically the Promax method, the results were more robust (i.e., repeatable) as compared to the orthogonal rotation, while when we did not apply any type of rotation, the results were much less repeatable and thus, it was not possible to generalize.

As shown in the present study, when a random error occurs in measurements, except for promax non – orthogonal rotation, the results are not repeatable. This was confirmed by using the Frobenius matrix norms. The result from the aforementioned findings should be made with conscious and further research is needed in order to prove them. To the best of our knowledge, this kind of study has not been carried out before, making this study unique in exploring the inherent properties of factor analysis as a robust pattern recognition tool. According to the findings of this simulation study, it is strongly concluded that when rotation is needed to improve the interpretation of patterns derived through factor analysis, promax non-orthogonal rotation seems to produce more robust results.

Conflict of interest

None to declare

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REFERENCES

[1] Basilevsky A. (2008). Statistical Factor Analysis and Related Methods: Theory and Applications, Wiley Series in Probability and Statistics.

[2] Andrew, L. Comrey, and Howard, B. Lee, (1992). A first course in factor analysis, Lawrence Erlbaum Associates, Inc.

[3] Carroll, J. B. (1953) An analytical solution for approximating simple structure in factor analysis, Psychometrika 18 (1) 23-38.

[4] Cattel, R.B. (1978). The scientific use of factor analysis. New York: Plenum.

[5] Chatfield Chris. (2018) Introduction to Multivariate Analysis, Taylor and Francis Group

[6] David J. Bartholomew, Martin Knott, and Irini Moustaki. (2011). Latent Variable Models and Factor Analysis. A Unified Approach, 3rd Edition, Wiley Series in Probability and Statistics.

[7] Golafshani N. (2003). Understanding Reliability and Validity in Qualitative Research. The Qualitative Report; 8 (4) 597-607.

[8] Gorsuch and Richard L., (1983). Factor Analysis, second edition, Hillsdale: Lawrence Erlbaum Associates

[9] Hair et al. (1995). Multivariate data analysis. 4th ed. New Jersey: Prentice-Hall Inc.

[10] Hammersley M. (1987). Some notes on the terms «validity» and «reliability». Br. Educ. Res. J. (13) 73-81.

[11] Hendrickson and White. (1964). Promax: A Quick Method for Rotation to Oblique Simple Structure, British Journal of Mathematical and Statistical Psychology 17 (1): 65-70

[12] Jennrich RI and Sampson PF. (1966). Rotation for simple loadings, Psychometrika 31(3): 313-23.

[13] Kaiser, H.F. (1958). The varimax criterion for analytic rotation in factor analysis. Psychometrika 23, 187–200.

[14] Kieffer, KM. (1998). Orthogonal Versus Oblique Factor Rotation: A Review of the Literature Regarding the Pros and Cons, Annual Meeting of the Mid-South Educational Research Association 4-6.

[15] Marley W. Watkins (2018). Exploratory Factor Analysis: A Guide to Best Practice, Journal of Black Psychology, 44 (3) 219–246

[16] Neuhaus, J. O. and Wrigley, C. (1954). The quartimax method: an analytical approach to orthogonal simple structure. British Journal of Statistical Psychology 7, 187-191.

[17] Panaretos, D. Tzavelas, G. Vamvakari M. and Panagiotakos D. (2016). Repeatability of dietary patterns extracted through multivariate statistical methods: a literature review in methodological issues. Int J Food Sci Nutr.68 (4) 385-391.

[18] Panaretos D, Tzavelas G, Vamvakari M, Panagiotakos D (2018). Investigating the role of orthogonal and non – orthogonal rotation in multivariate factor analysis, in regard to the repeatability of the extracted factors: a simulation study. Communications in Statistics – Simulation and Computation, 1-12

[19] Pett MA, Lackey NR and Sullivan JJ. (2003). Making Sense of Factor Analysis: The use of factor analysis for instrument development in health care research. California: Sage Publications Inc.

[20] Alvin C. Rencher and William F. Christensen. (2012). Methods of Multivariate Analysis. Wiley-Interscience

[21] Rose G, and Barker DJP. (1978). Repeatability and validity. Br. Med. J. (2): 1070-1071.

[22] Saunders, D. R. (1962). Trans Varimax: Some properties of the ratiomax and Equimax criteria for blind orthogonal rotation. Paper presented at the meeting of the American Psychological Association, St. Louis.

[23] Saunders D.R. (1960). A computer program to find the best – fitting orthogonal factors for a given hypothesis, Psychometrica 25, 199-205

[24] Tabachnick, B. G., and Fidell, L. S. (2001). Using Multivariate Statistics. Boston: Allyn and Bacon.

[25] Thurstone, L.L. (1947). Multiple factor analysis: A development and expansion of vectors of the mind. University of Chicago Press Chicago, xix 535.

[26] Yisrael Parmet, Edna Schechtman and Michael Sherman. (2010). Factor Analysis Revisited – How Many Factors are There? Communications in Statistics: Simulation and Computation. 39 (10):1893-1908. DOI: 10.1080/03610918.2010.524332

Table 1: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 20% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non – orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error = ±0.1	0.49 (0.45, 0.54)	3.54 (3.52, 3.57)	2.24 (2.18, 2.30)	1.74 (1.68, 1.80)	4.18 (4.16, 4.20)
Error = ±0.3	1.49 (1.44, 1.54)	3.53 (3.50, 3.56)	3.05 (3.01, 3.09)	2.61 (2.55, 2.66)	4.22 (4.20, 4.23)
Error = ±0.5	2.22 (2.17, 2.27)	3.61 (3.58, 3.64)	3.41 (3.38, 3.45)	3.07 (3.03, 3.12)	4.23 (4.21, 4.24)

Table 2: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 40% of the variables from the Normal distribution, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non – orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error = ±0.1	0.70 (0.66, 0.75)	3.59 (3.57, 3.62)	2.78 (2.73, 2.83)	1.85 (1.79, 1.92)	4.18 (4.16, 4.20)
Error = ±0.3	2.02 (1.97, 2.07)	3.62 (3.60, 3.65)	3.40 (3.36, 3.43)	2.85 (2.80, 2.90)	4.23 (4.22, 4.25)
Error = ±0.5	2.88 (2.83, 2.92)	3.77 (3.74, 3.79)	3.69 (3.65, 3.72)	3.30 (3.34, 3.34)	4.24 (4.22, 4.25)

Table 3: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 60% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non – orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error = ±0.1	0.89 (0.84, 0.94)	3.61 (3.58, 3.63)	2.90 (2.86, 2.95)	2.18 (2.12, 2.24)	4.19 (4.18, 4.21)
Error = ±0.3	2.36 (2.31, 2.41)	3.71 (3.68, 3.74)	3.55 (3.52, 3.59)	3.28 (3.24, 3.32)	4.25 (4.23, 4.26)
Error = ±0.5	3.16 (3.11, 3.20)	3.88 (3.86, 3.91)	3.82 (3.80, 3.85)	3.67 (3.64, 3.70)	4.25 (4.24, 4.27)

Table 4: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 80% of the variables from the Normal distribution, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non – orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error = ±0.1	0.97 (0.92, 1.01)	3.62 (3.59, 3.64)	3.03 (2.99, 3.07)	2.41 (2.35, 2.46)	4.21 (4.19, 4.22)
Error = ±0.3	2.55 (2.50, 2.60)	3.75 (3.73, 3.78)	3.68 (3.65, 3.71)	3.45 (3.41, 3.49)	4.25 (4.23, 4.26)
Error = ±0.5	3.35 (3.31, 3.40)	3.94 (3.92, 3.96)	4.22 (4.17, 4.27)	3.81 (3.78, 3.84)	4.25 (4.23, 4.26)

Table 5: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 100% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non – orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error = ±0.1	1.05 (1.00, 1.10)	3.66 (3.63, 3.68)	3.05 (3.01, 3.09)	2.55 (2.49, 2.61)	4.22 (4.20, 4.23)
Error = ±0.3	2.77 (2.72, 2.82)	3.80 (3.77, 3.83)	3.72 (3.70, 3.75)	3.53 (3.49, 3.57)	4.26 (4.24, 4.27)
Error = ±0.5	3.50 (3.46, 3.54)	4.15 (4.09, 4.21)	3.72 (4.10, 4.21)	3.87 (3.84, 3.90)	4.25 (4.23, 4.27)

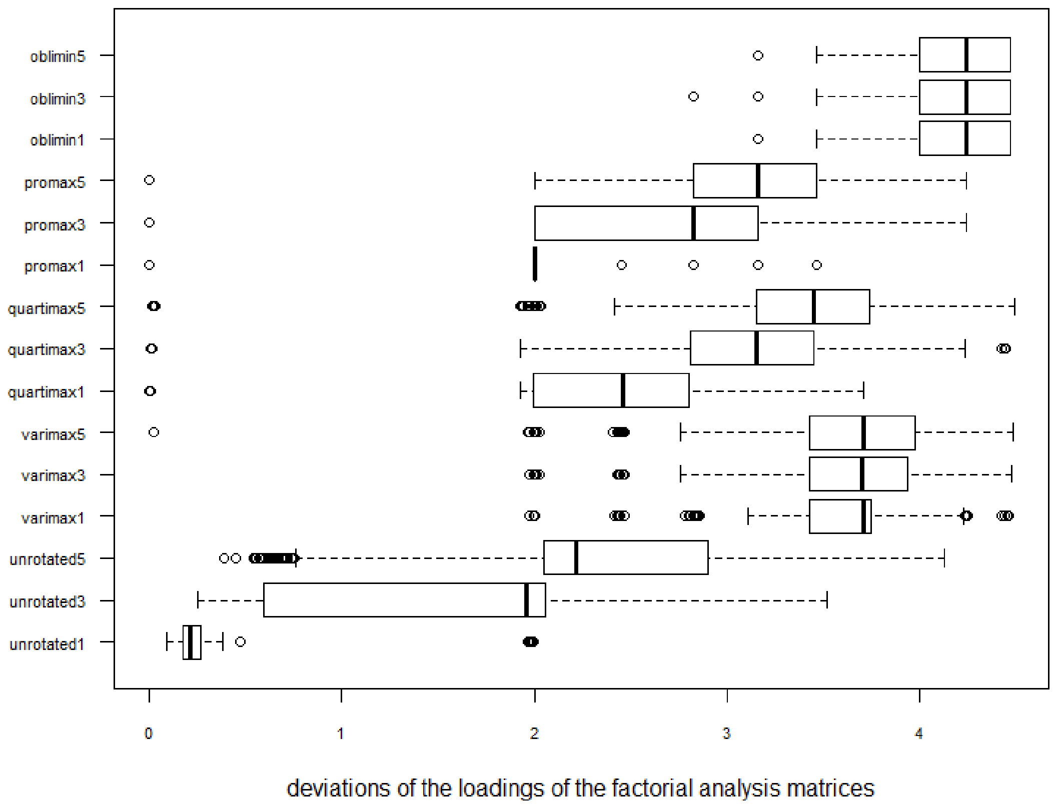

Figure 1: Illustration of the results derived from the simulation study, showing the Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and

after the error input into the 20% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

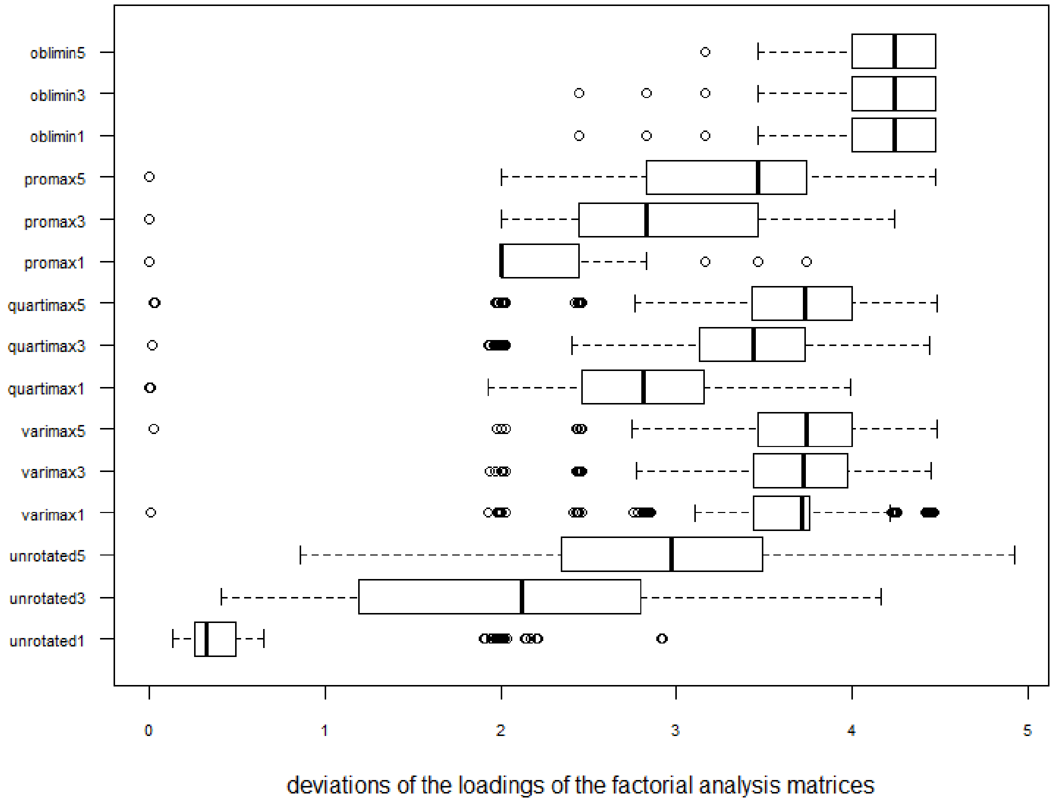

Figure 2: Illustration of the results derived from the simulation study, showing the Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 40% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

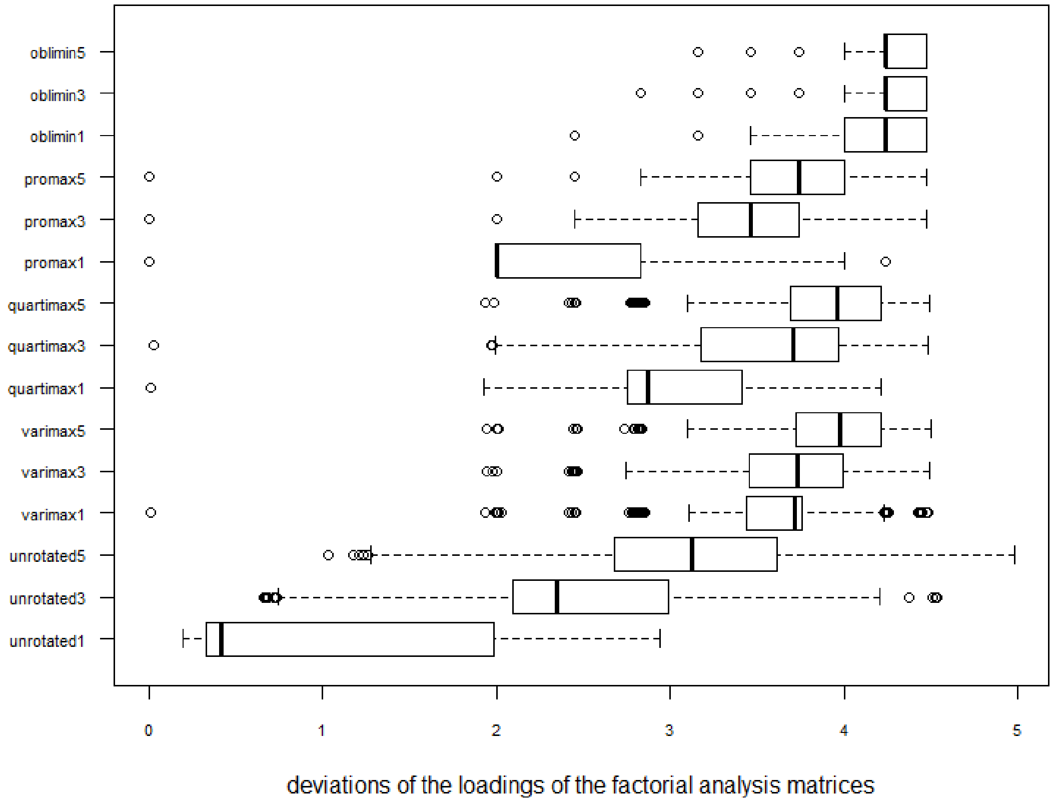

Figure 3: Illustration of the results derived from the simulation study, showing the Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 60% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

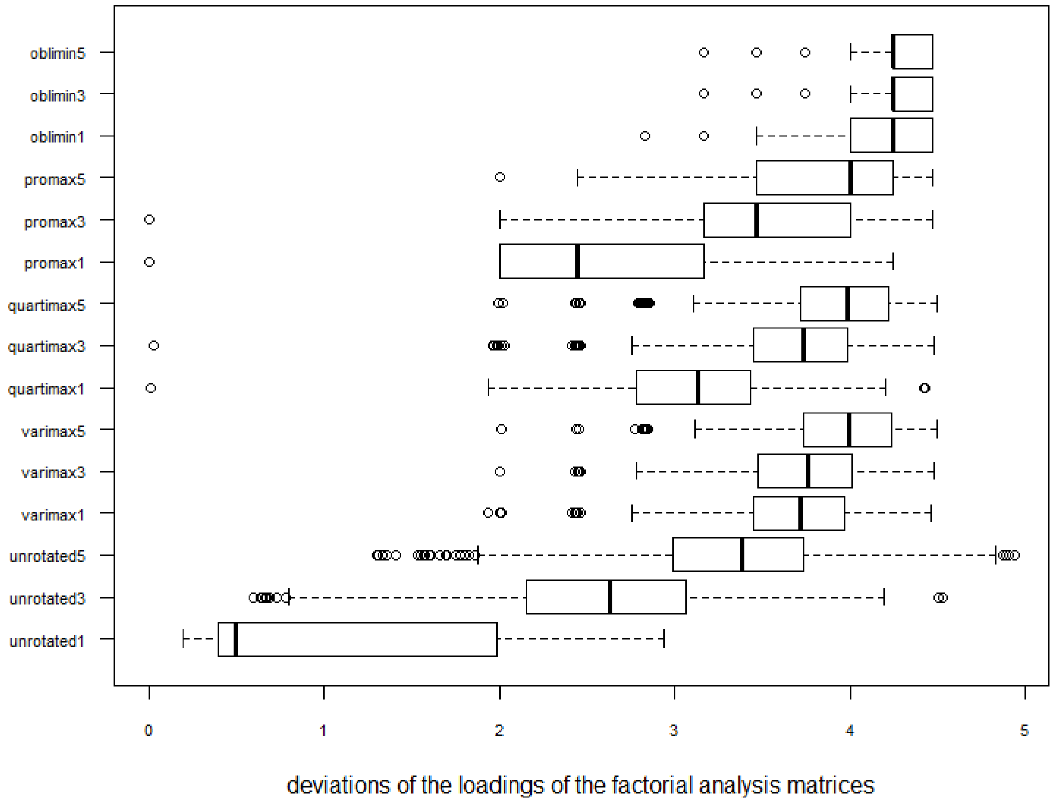

Figure 4: Illustration of the results derived from the simulation study, showing the Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 80% of variables from the Normal distribution, under various rotation methods used (1000 simulations).

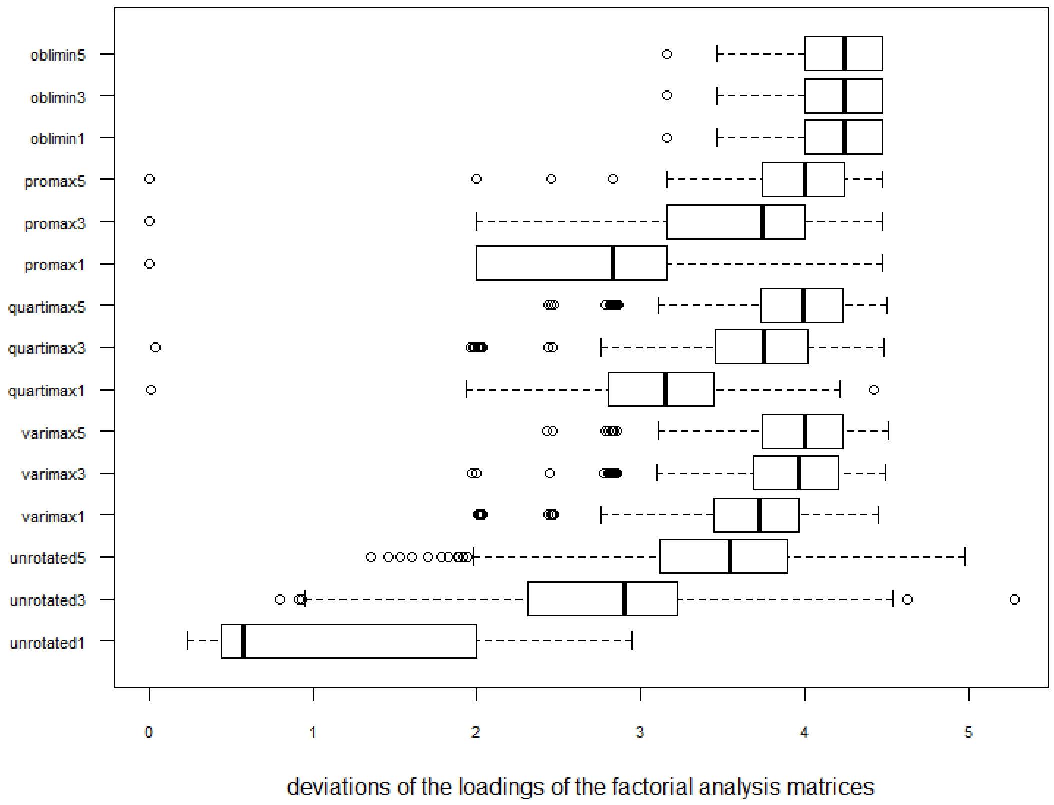

Figure 5: Illustration of the results derived from the simulation study, showing the Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the 100% of variables from the Normal distribution, under various rotation methods used (1000 simulations).