Web-based Supplementary Information for (Time Series Regression Models for COVID-19 Deaths) by (Marinho G. Andrade, Jorge A. Achcar, Katiane S. Conceição, Nalini Ravishanker)

1 Rational functions and Nonlinear Rational Polynomial Model

A Rational function is a function which is the ratio of polynomial functions. That is, $r(x)$ is a Rational function if it is of the form

$$r(x) = p(x)/q(x),$$

where $p(x)$ and $q(x)$ are polynomial functions. Clearly, $Dom(r) = \{x \in \mathbb{R} : q(x) \neq 0\}$.

In general, a nonlinear Rational function model is a generalization of a polynomial model. Rational function models contain polynomial models as a subset. In the literature, fitting Rational function models is also referred to as the Pade approximation. The nonlinear model can be written as,

$$y = \frac{a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0}{b_m x^m + \cdots + b_2 x^2 + b_1 x + b_0},$$

where n denotes a non-negative integer that defines the degree of the numerator and m denotes a non-negative integer that defines the degree of the denominator. One common difficulty in fitting nonlinear models is finding suitable starting values. A major advantage of Rational function models is the ability to compute starting values using a linear least squares fit. For example, given the linear/quadratic model,

$$y = \frac{\theta_0 + \theta_1 x}{1 + \theta_2 x + \theta_3 x^2},$$

we could perform a linear fit on the model as

$$y = \theta_0 + \theta_1 x - \theta_2 xy - \theta_3 x^2 y.$$

In general, suppose

$$y = \theta_0 + \theta_1 x + \cdots + \theta_{p_n} x^{p_n} - \gamma_1 xy - \gamma_2 x^2 y - \cdots - \gamma_{p_d} x^{p_d} y,$$

where p_n and p_d are the degrees of the numerator and denominator, respectively, and the x and y contain the subset of points, not the full data set. Polynomials of substantial degrees can lead to models with a large number of parameters. The models considered in this paper have three parameters. Considering polynomials of degrees 1 and 2, we will have models with 4 parameters. The estimated coefficients from this fit obtained using the linear least squares algorithm are used as the starting values in the iterative procedure needed to fit the nonlinear model to a data set.

2 Tables with fitted model parameters

Table 1: Posterior summaries for the parameters of the functions $\eta(t)$, $Z(t)$ and $\sigma^2(t)$ of the Alpha AR(3) model fitted for daily number of new COVID-19 deaths in China from December 31, 2019 to July 31, 2020.

$\boldsymbol{\theta}$	Mean	Median	Sd $\times 10^{-2}$	CI 95%
θ_0	17685.95	17685.95	0.9858589	(17685.93, 17685.97)
θ_1	198.3128	198.3127	0.9989273	(198.2934, 198.3325)
θ_2	3.971313	3.971203	0.9936589	(3.951915, 3.990880)
ϕ_1	0.130898	0.130826	0.9943703	(0.111708, 0.150798)
ϕ_2	0.164828	0.164735	0.9840409	(0.145346, 0.184497)
ϕ_3	0.219145	0.218952	0.9849724	(0.199971, 0.238828)
α_0	0.476856	0.476864	0.4692601	(0.467627, 0.486045)
α_1	0.279076	0.279078	0.1979081	(0.275207, 0.283040)

Table 2: Posterior summaries for the parameters of the functions $\eta(t)$, $Z(t)$ and $\sigma^2(t)$ of the Rational function with AR(1) model fitted for the daily number of new COVID-19 deaths in USA from January 21, 2020 to July 31, 2020.

$\boldsymbol{\theta}$	Mean	Median	Sd $\times 10^{-6}$	CI 95%
θ_0	-0.6323471611	-0.6323471680	1.019774	(-0.632349162, -0.632345172)
θ_1	0.0127604043	0.0127603936	0.999001	(0.012758381, 0.012762348)
θ_2	-0.0214015735	-0.0214015663	1.001587	(-0.021403533, -0.021399624)
θ_3	0.0001245586	0.0001245444	0.461951	(0.000123672, 0.000125509)
ϕ_1	0.3438323947	0.3438324014	1.001761	(0.343830396, 0.343834336)
α_0	0.9645924112	0.9645924101	0.958305	(0.964590516, 0.964594313)
α_1	0.2440881724	0.2440881704	0.189783	(0.244087807, 0.244088545)

Table 3: Posterior summaries for the parameters of the functions $\eta(t)$, $Z(t)$ and $\sigma^2(t)$ of the Alpha with AR(1) model fitted for the daily number of new COVID-19 deaths in Spain from February 1, 2020 to July 31, 2020.

$\boldsymbol{\theta}$	Mean	Median	Sd	CI 95%
θ_0	79206.37	79206.37	0.019420608	(79206.33, 7920.641)
θ_1	312.2948	312.2948	0.019367352	(312.2573, 312.3323)
θ_2	4.505206	4.505060	0.015504662	(4.474906, 4.535462)
ϕ_1	0.319367	0.319170	0.019215324	(0.281787, 0.356541)
α_0	1.063502	1.062954	0.020785866	(1.022988, 1.104785)
α_1	0.194889	0.194911	0.003152706	(0.188690, 0.201105)

Table 4: Posterior summaries for the parameters of the functions $\eta(t)$, $Z(t)$ and $\sigma^2(t)$ of the Alpha with AR(1) model fitted for the daily number of new COVID-19 deaths in Italy from January 31, 2020 to July 31, 2020.

$\boldsymbol{\theta}$	Mean	Median	Sd	CI 95%
θ_0	53884.65	53884.65	0.028780468	(53884.59, 53884.70)
θ_1	224.1183	224.1181	0.028784640	(224.0632, 224.1758)
θ_2	3.115807	3.115816	0.015189382	(3.086481, 3.145358)
ϕ_1	0.153856	0.153972	0.027476467	(0.100922, 0.208041)
α_0	0.492669	0.492519	0.013709794	(0.465832, 0.520259)
α_1	0.122320	0.122293	0.003046826	(0.116449, 0.128249)

Table 5: Posterior summaries for the parameters of the functions $\eta(t)$, $Z(t)$ and $\sigma^2(t)$ of the Alpha with AR(6) model fitted for the daily number of new COVID-19 deaths in Brazil from February 26, 2020 to July 31, 2020.

$\boldsymbol{\theta}$	Mean	Median	Sd	CI 95%
θ_0	771748.0	771748.0	0.009922239	(771747.9, 771748.1)
θ_1	126.6897	126.6897	0.009940962	(126.6704, 126.7091)
θ_2	-1.139329	-1.139333	0.004523984	(-1.148266, -1.130345)
ϕ_1	0.252178	0.252154	0.009761873	(0.233521, 0.271272)
ϕ_2	-0.297425	-0.297338	0.009763612	(-0.316855, -0.278698)
ϕ_3	-0.245038	-0.244990	0.009643053	(-0.263573, -0.225994)
ϕ_4	-0.103228	-0.103125	0.009945473	(-0.123127, -0.083481)
ϕ_5	-0.356278	-0.356347	0.009731603	(-0.375237, -0.336665)
ϕ_6	0.267348	0.267078	0.010286647	(0.246730, 0.287712)
α_0	0.433427	0.433329	0.004230041	(0.425276, 0.441828)
α_1	0.152729	0.152720	0.001295245	(0.150224, 0.155263)

Table 6: Posterior summaries for the parameters of the functions $\eta(t)$ and $\sigma^2(t)$ of the Log-Normal model for the daily number of new COVID-19 deaths in India from January 31, 2020 to May 20, 2020.

$\boldsymbol{\theta}$	Mean	Median	Sd	CI 95%
θ_0	1126943.0	1126943.0	0.0030243474	(1126943.0, 1126949.0)
θ_1	7.658443	7.658432	0.0024011881	(7.653747, 7.663175)
θ_2	1.269896	1.269876	0.0023249796	(1.265246, 1.274575)
α_0	5.555355	5.555390	0.0166690133	(5.521975, 5.588194)
α_1	0.286328	0.286328	0.0006134334	(0.285109, 0.287533)

3 Residual analysis

In this section, we present the autocorrelation function (ACF), partial autocorrelation function (PACF) for the residuals $Z(t)$ and the squared residue $Z^2(t)$ of each fitted model, and the $lag = 1$

ARCH Lagrange Multiplier test.

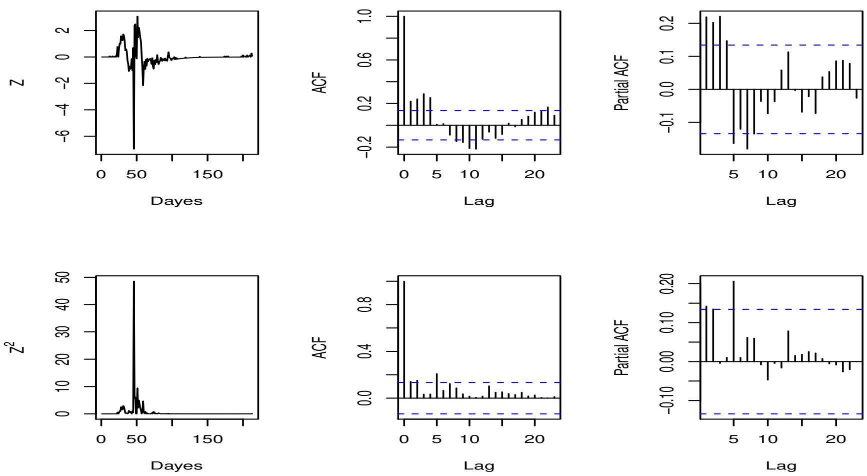

Figure 1: Analysis of residue $Z(t)$ from fitted Alpha with AR(3) model for China data set.

ARCH test from fitted Alpha AR(3) model for China: statistic = 4.1922,, lag = 1, p-value = 0.04061 alternative hypothesis: ARCH effects of order 1 are present.

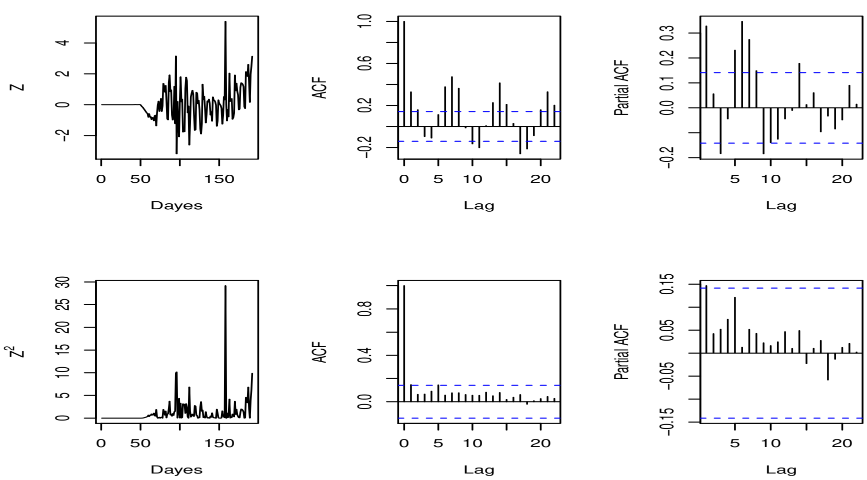

Figure 2: Analysis of residue $Z(t)$ from fitted Rational function with AR(1) model for the USA data set.

ARCH test from fitted Rational function with AR(1) model for USA: statistic = 4.2199, lag = 1, p-value = 0.03995 alternative hypothesis: ARCH effects of order 1 are present.

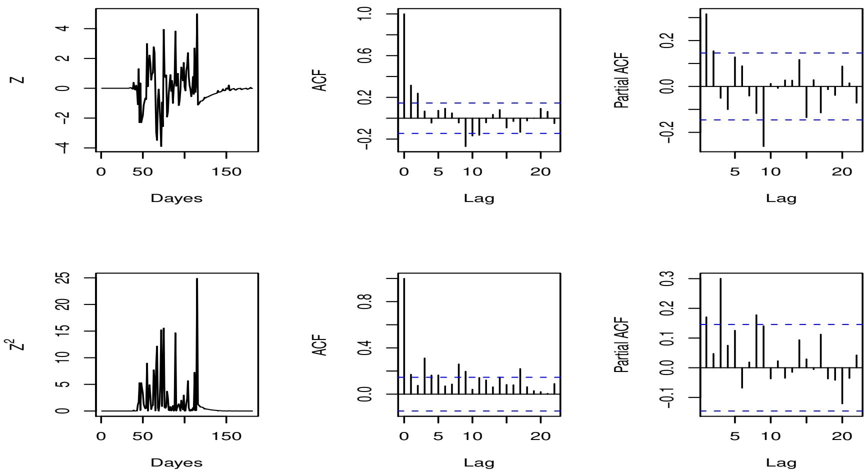

Figure 3: Analysis of residue $Z(t)$ from fitted Alpha with AR(1) model for Spain data set.

ARCH test from fitted Alpha with AR(1) model for Spain: statistic = 4.871, lag = 1, p-value = 0.02731, alternative hypothesis: ARCH effects of order 1 are present.

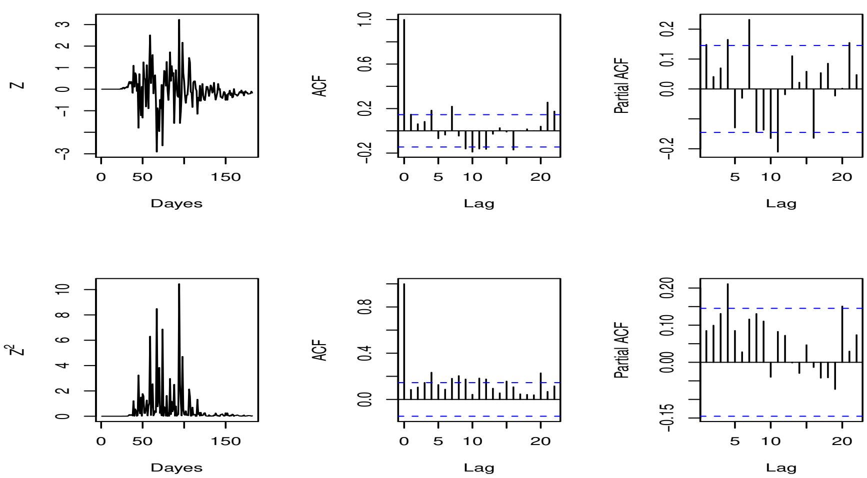

Figure 4: Analysis of residue $Z(t)$ from fitted Alpha with AR(1) model for Italy data set.

ARCH test from fitted Alpha with AR(1) model for Italy: statistic = 4.6593, lag = 1, p-value = 0.03089, alternative hypothesis: ARCH effects of order 1 are present.

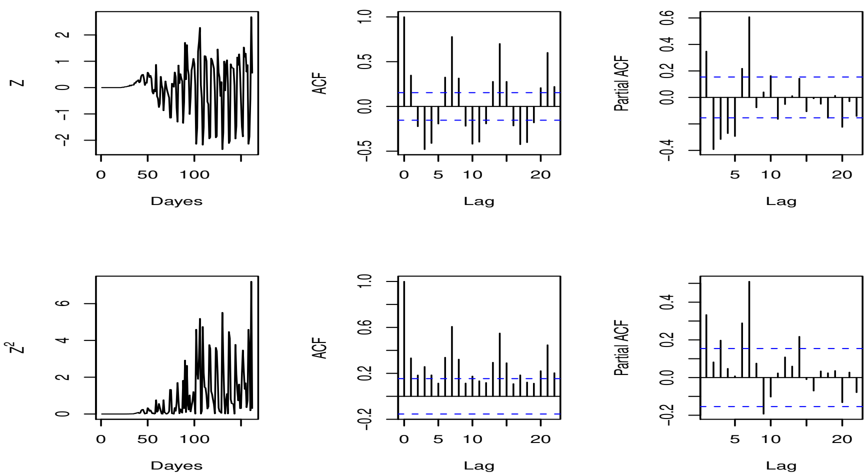

Figure 5: Analysis of residue $Z(t)$ from fitted Alpha with AR(6) model for Brazil data set.

ARCH test from fitted Alpha with AR(6) model for Brazil: statistic = 18.44, lag = 1, p-value = 1.75e-05, alternative hypothesis: ARCH effects of order 1 are present.

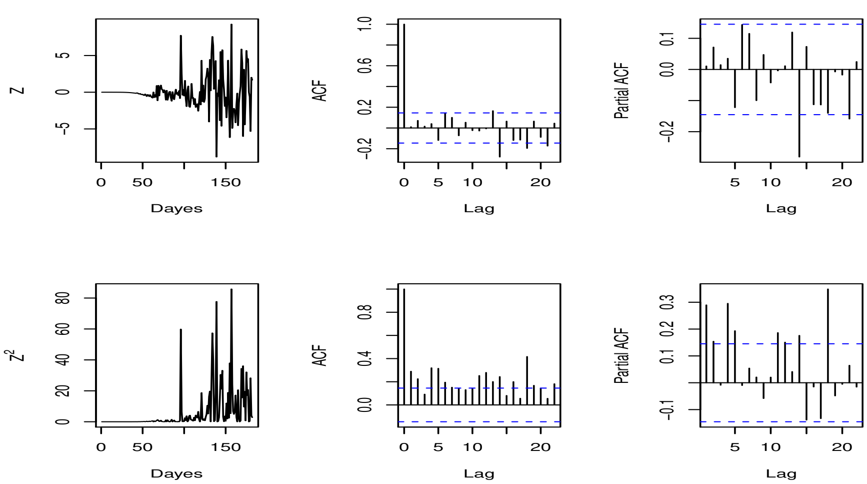

Figure 6: Analysis of residue $Z(t)$ from fitted Log-Normal model for India data set.

ARCH test from fitted Log-Normal model for India: statistic = 15.283, lag = 1, p-value = 0.00009256, alternative hypothesis: ARCH effects of order 1 are present.

4 Report for each of the countries we consider in the paper

The COVID-19 pandemic is a huge public health problem. Three elements have been shown to be fundamental to differentiate the response to the crisis caused by the coronavirus:

1. public attitudes of the authorities to the uncertainties brought about by the new virus;

2. the ability of the government to implement preventive measures;

3. the structure and competence of public health systems to serve patients.

We briefly report on these attitudes for each of the countries we consider in the paper.

- China: It is evident that by February 8, more than 90% of the Chinese population had taken in-home isolation following a rigorous quarantine protocol enforced in the entire country. In response, there was a pandemic cycle that started in January 1 and practically ended at the end of March 30, with a total number of deaths much lower than the other countries considered in this paper.

- Italy: Political conflicts in Italy caused 15 days of conflicting instructions given by the central and state governments, before Italy went into complete lockdown on March 8. It is from that date that Italy takes the lead in cases in Europe, creating a peculiar and dramatic trajectory in public health. The failures listed ranged from errors in the hospital protocol for patient care to the government's lethargy in initiating measures of social isolation and mass testing of the population.

- Spain: In the first days of March, the Spanish government declared a state of alert throughout the country, taking the measures, the most drastic measures approved in the country in recent decades, to try to control the propagation of the COVID-19. In these measures, the movement of people across the country became restricted, except for exceptional reasons, determined the closure of schools, as well as non-essential companies, suspended sporting and cultural events, and leisure facilities. However, the measures were adopted late, when Spain already had more than 5,700 confirmed cases of COVID-19 and registered 136 deaths until the date of the decree.

- Brazil: In Brazil, the clash between the President of the Republic and the 27 state governors over the best form of isolation generated conflict of information for the population. The authorities of each unit of the Federation act in different ways to ensure the dissemination of COVID-19. These actions resulted in conflicts of opinion, few restrictive measures, low social isolation. Consequently, in Brazil, the population's little knowledge about the seriousness of the situation caused a death rate that arrived on June 10, with an average rate of more than 1000 deaths per day, and that continues to the present day (August 2020).

- USA: USA is now the country with the largest number of confirmed cases and the highest number of accumulated deaths in the world. Several factors explain the current situation in the US, such as the size and heterogeneity of the country, and a delay in adopting restrictive rules. The government announced rules of social distancing in the country on March 16. Medical officials say the United States could have had fewer deaths if it had adopted restrictive measures earlier to contain the spread of the new coronavirus. Commercial interests have caused precipitous relaxation of restrictive rules, and as a consequence, the US has seen an increase in the number of cases in some states.

- India: Learning from the experience of other countries, on March 24, the Indian government initiated strict isolation of the population and ordered the closure of trade and non-essential services, the number of positive cases of coronavirus added up to around 500. With a population of 1.3 billion and densely populated cities, the authorities knew that the spread of the disease could take on a catastrophic dimension. Two months later, 280,000 cases and 8,000 deaths had been reported. India has begun to ease the confinement, in response to pressure from the economic impact of the isolation measures and a humanitarian crisis, with millions of people without an income and job. Daily records of positive diagnoses and scarcity of beds started to be registered in the country. But there are several warnings that the numbers of cases and deaths are underreported. India now ranks alongside the countries most plagued by the virus, such as the United States and Brazil, and displaced Italy in the number of cases. On the other hand, with the circulation of people tightly controlled, and the dissemination of information by the government, the Indians managed to keep the number of deaths low. The government of India maintains a website to inform the population saying:

"Government of India is taking all necessary steps to ensure that we are prepared well to face the challenge and threat posed by the growing pandemic of COVID-19 the Corona Virus. With active support of the people of India, we have been able to contain the spread of the Virus in our country. The most important factor in preventing the spread of the Virus locally is to empower the citizens with the right information and taking precautions as per the advisories being issued by Ministry of Health & Family Welfare." (`https://www.mygov.in/covid-19`)