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Predicting Future CD_4 Cell Counts of HIV/AIDS Patients by Non Stationary Markov Chain: A Case Study of Anambra State

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Abstract: A total of 1094 HIV patients were involved in a cohort study (from January-December 2010) with follow-up in their CD_4 cell transition counts and grouped according to their immunological states into five(5) states developed by Guiseppe Di Biase *et al* (2007). The five states (5) considered were: State one (CD_4 > 500 cells/mm^3), State two (350 < CD_4 500 cells /mm^3) State three(200 < CD_4 350 cells/mm^3), State four(CD_4 200 cells/mm^3), State five(Death). These states de ne the seriousness of the sickness based on the epidemiological states of the patients CD_4 cell counts. We use the non-stationary Markov chain model for the prediction. The estimation of the non-stationary probabilities were done using the exponential smoothing technique. The result of the prediction showed a gradual decrease of the CD_4 cells as we move from Jan-Dec. Furthermore, the result showed that the patients in the study cannot survive death from the month Dec. 2011, if they are not subjected to therapy, using highly active antiretrovirals (HAART). The results also showed that the model can be used for the testing of the drug e efficacy administered to patients within a given period.

Key words: Non Stationary Markov chain model, Transition probability, Transition probability matrix, CD_4 cell counts, Exponential smoothing techniques.

1. Introduction

Since the outbreak of HIV/AIDS epidemic in Nigeria in 1981 and 1983 respectively, cases of the disease have been reported in all the thirty-six (36) states of Nigeria, including Abuja, the Federal capital territory. Many studies have been carried out on HIV/AIDS, e.g life expectancy of patients [See Osisiogu U. A. and Nwosu (2013)] Zero-prevalence rate of the disease [See. Bowler, Sheon, and D'Angelo(1992), Young 1992]. The infection has become the number one cause of death for persons between 25-44 years of age [Centre for disease control and prevention (CDC)(1995)]. The study provides an example of the use of epidemiological data such as the CD_4 cell counts for estimating and projecting the impact of HIV/AIDS epidemics using the non-stationary Markov chain models.

'The HIV's fatal effect arises from its attack of a person's CD_4 cell counts. The hallmark of the infection is the progressive depletion of the CD_4 cell counts which play a pivotal regulatory role in the immune response to infections and tumours (Anderson R. Met. al 1986). Infection by

the human immunodeficiency syndrome (AIDS) and which finally ends to death. A number of approaches have been used to quantify the magnitude of HIV/AIDS dynamics (See Osisiogu U. A., and Nwosu C. A (2013), Guiseppe Di Biase *et al*, (2007)) and the future predictive trend. We shall use the non-stationary Markov chain model in this paper for the analysis in predicting future *CD₄* cell counts of HIV/AIDS patients.

2. Notations

The notations used are defined as follows.

1. K = the number of states in the system.
2. $T =$ Calender times in months, $T = 0,1,2, \ldots$
3. $n_{ij}(T) =$ the number of patients in state i in month T who transited into state j in month T+1.
4. $n_{ij}(T) =$ number of patients in state i at the beginning of the month T.
5. $n_{i,k}(T) =$ number of patients who died from state i at month T.
6. $P_{ij} = \frac{n_{ij}(T)}{n_i(T)}$ is the probability that a patient in state i transits to state j at the end of the month T.
7. $N(T) = \sum_{i=1}^{K} n_i(T) =$ Total number of patients at the beginning of theperiod.
8. $W_i(T) = \frac{n_{i,k}(T)}{n_i(T)} =$ death rate of patients in state i at the month T.
9. $n_{0\cdot j}(T+1) =$ new entrants into states j at the beginning of the month T.
10. $\alpha =$ Smoothing constants

3. The Markovian Model for the *CD₄* Counts

The difference equation

$$\bar{n}_j(T+1) = \sum_{i=1}^{N} \bar{n}_{ij}(T) + n_{0\cdot j}(T+1) \quad (i = 1,2, \ldots, N) \quad (T = 0,1,2, \ldots) \tag{1}$$

denotes the expected number of *CD₄* counts of patients, where bars denote the expected value($j = 1,2, \ldots, \mathrm{k}$); that is to say, the above equation is saying that patients in state j are made up of patients who transited to state j. Some of these variables may assume zero value. Take for example, the study of a cohort where no new entrants are allowed, in this case,

$$n_{0.c}(T+1) = 0 \text{ where } c = \text{cohort study}$$

Therefore, we can redefine the above model, by expressing the transition ow of the new entrants in terms of wastage i.e., death rate of patients within the month i.e.,

$$n_{0.j}(T+1) = W_j(T+1),$$

Then equation (1) becomes

$$\bar{n}_j(T+1) = \sum_{i=1}^{N} \bar{n}_{ij}(T) + W_j(T+1), \quad (j = 1,2,..,N) \ \ (T = 0,1,2,\dots) \tag{2}$$

If the transition probabilities are non-stationary, then we have that the transition probability given as:

$$P_{ij} = \frac{n_{ij}(T)}{n_i(T)} \quad (i,j = 1,2,\dots,k) \tag{3}$$

Substituting equation (3) in equation (2), we have

$$\bar{n}_j(T+1) = \sum_{i=1}^{N} P_{ij}\bar{n}_i(T) + W_j(T+1), \quad (T = 0,1,2,\dots) \ \ (j = 1,2,..,K) \tag{4}$$

In vector form equation (4) becomes

$$n(T+1) = n(T)P(T) + W(T+1) \tag{5}$$

The non-stationary Markov chain model show themselves in the structure of the T-step transition probability matrix, unlike the stationary Markov chain model that is associated with the power of the one step transition probability matrix P, which is a common estimate of the transition probability matrices over the past months on the assumption that they are stationary over time (Osisiogu 2004). We note that these estimate are weighted sum of the estimates for each month, where the weights are proportional to n(T). If however we vary these weights and put more weights on the current transition probability matrices depending on the stochastic variation prevalent at the time of study, then equation (3) becomes

$$\hat{P}_{ij}(T) = \sum_{T=0}^{T^*} \beta_i(\tau) P_{ij}(T) \tag{6}$$

where

$$\beta_i(\tau) = \alpha_i(1-\alpha_i)^{\tau} \text{ and } \tau = \mathrm{T}^* - T, \ \ (T = 0,1,2,\dots,T^*)$$

We can rewrite equation (6) in the form

$$\hat{P}_{ij}(T) = \alpha_i P_{ij}(T) + (1-\alpha_i)P_{ij}(T-1) \ (i,j = 1,2,\dots,k) \ \ (T = 0,1,2,\dots,T^*) \tag{7}$$

and $\hat{P}_{ij}(T^*)$ is as defined in equation (6). Thus the transition probability matrix P used in this model for future prediction is the one whose elements are derived from equation (7) given that

$$\hat{P} = \left[\hat{P}_{ij}\right] \ (i,j = 1,2,\dots,k) \tag{8}$$

[See Osisiogu U. A. and Nwosu (2013)] for more detail.

In determining the values of the exponential smoothing constants (α) the mean standard error is used (mean absolute error(MAE) is a measure of the forecast accuracy). The optimal values of the exponential smoothing constant to be used are determined through the method of trial and error. Using the different values of smoothing constants (α) within 0 to 1, we calculate their mean absolute error(MAE). The optimal value of the exponential smoothing constant is the value that generates the minimum mean absolute error. Another way which we may use is to run a retrospective simulation of the transition probabilities of the system using equation (5) for different values of $\alpha_i(i = 1,2,\dots,N)$, compare the forecast values with the actual values and

choose those values of $\alpha_i (i = 1,2,\dots,N)$, which are in some sense optimal. The following smoothing constants are tested in this paper.

Moodel I: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.25$
Moodel II: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.35$
Moodel III: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.40$
Moodel IV: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.45$
Moodel V: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.50$
Moodel VI: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.55$
Moodel VII: $\alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0.60$

4. Application

We apply these models to the cohort study of 1094 HIV-positive patients with follow-up in their transition counts from January-December 2010 to carry analysis of the model using seven(7) different smoothing constants (See Table I). The optimal smoothing constant is used to predict future *CD₄* cell counts for January-December 2011 and 2012.

Method

The *CD₄* cell counts of these patients were classified into five (5) states based on the classification developed by Guiseppe Di Biase *et al* (2007).

The classifications are stated below:

State I: $\mathrm{CD}_4 > 500\mathrm{cells/mm}^3$
State II: $350 < \mathrm{CD}_4 > 500\mathrm{cells/mm}^3$
State III: $200 < \mathrm{CD}_4 < 350\mathrm{cells/mm}^3$
State IV: $\mathrm{CD}_4 \leq 200\ \mathrm{cells/mm}^3$
State V: Death(Absorbing State)

The transition matrices and the transition probabilities of the *CD₄* cell counts of the 1094 HIV/AIDS patients of the cohort study for the twelve (12) observable months, January-December 2010 were computed as follows:

Table I: Transition counts of *CD₄* cells in January 2010

JANUARY		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	95	99	48	18	12	272
$350 < CD_4 \leq 500$ cells$/mm^3$	II	100	96	50	20	4	270
$200 < CD_4 \leq 350$ cells$/mm^3$	III	70	50	95	55	7	277
$CD_4 \leq 200$ cells $/mm^3$	IV	76	95	30	64	10	275
DEATH	V	0	0	0	0	0	0
TOTAL							1094

Table I: Represents the transition counts of the *CD₄* cells of the 1094 patients of the cohort study in the month of January 2010. Similar classifications were done for the month of February-December 2010 (See Appendix A). Model I

Table II: Transition probability matrix for the month of Jan 2010:

$$P = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0.349 & 0.364 & 0.176 & 0.066 & 0.044 \\ 0.370 & 0.356 & 0.185 & 0.074 & 0.015 \\ 0.252 & 0.181 & 0.343 & 0.199 & 0.025 \\ 0.276 & 0.345 & 0.109 & 0.233 & 0.036 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The transition probability matrix for the month of January 2010 were computed using equation (3) and similar computations were done for the months February-June 2010, using the smoothing constant, $\alpha = 0.25$

Table III: Transition probability matrix, $\hat{P}$ for the months Jan-June 2010.

$$P = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0.322 & 0.344 & 0.200 & 0.110 & 0.023 \\ 0.321 & 0.348 & 0.198 & 0.114 & 0.018 \\ 0.239 & 0.287 & 0.326 & 0.128 & 0.018 \\ 0.237 & 0.327 & 0.284 & 0.131 & 0.021 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

By equation (5), we predict the *CD₄* cell counts for the months of July-December 2010, using the matrix $\hat{P}$ (in table III). The mean absolute errors are computed from the difference between predicted *CD₄* cell counts and the actual.

Table IV: Predicted *CD₄* cell counts for the months of July-December 2010 with the mean absolute error (MAE)

MODEL I	STATES	PREDICTED CD_4 CELL COUNTS FOR THE MONTHS OF JULY-DEC. 2010						MEAN ABS ERROR(MAE)
		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.25$	I	239	234	229	224	220	215	12.20
	II	241	235	230	225	221	216	
	III	239	235	230	225	221	216	
	IV	240	235	230	225	220	216	
	V	0	0	0	0	0	0	

5. Model II

Using equation (8), we compute the transition probability matrix $\hat{P}$ for the months of January-June 2010, with the smoothing constant $(\alpha) = 0.35$

Table V: Transition probability matrix $\hat{P}$ for the months of Jan-June 2010

$$P = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0.314 & 0.342 & 0.203 & 0.121 & 0.019 \\ 0.317 & 0.345 & 0.199 & 0.121 & 0.017 \\ 0.240 & 0.302 & 0.324 & 0.118 & 0.016 \\ 0.231 & 0.323 & 0.312 & 0.117 & 0.018 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Using equation (5), we predict the *CD₄* cell counts for the month of July-December 2010 for model II. The mean absolute error are computed from the difference between the predicted *CD₄* cell counts and the actual.

Table VI: Predicted *CD₄* cell counts. For the months of July-Dec 2010 with the mean absolute error (MAE)

MODEL II	STATES	PREDICTED CD_4 CELL COUNTS FOR THE MONTHS OF JULY-DEC 2010						MEAN ABS ERROR(MAE)
		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.35$	I	240	236	231	227	223	219	10.70
	II	241	236	232	228	224	220	
	III	240	237	232	228	224	220	
	IV	240	236	232	228	224	220	
	V	0	0	0	0	0	0	

Model III

We compute the transition probability matrix $\hat{P}$ for the months of Jan-June 2010, using equation (8) and a smoothing constant $(\alpha) = 0.40$

Table VII: Transition probability matrix, $\hat{P}$ for the months of Jan-June 2010 for Model III

$$P = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0.310 & 0.342 & 0.203 & 0.126 & 0.017 \\ 0.317 & 0.344 & 0.198 & 0.124 & 0.017 \\ 0.239 & 0.306 & 0.324 & 0.118 & 0.015 \\ 0.228 & 0.321 & 0.321 & 0.113 & 0.017 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Using (5), we make predictions of the *CD₄* cell counts for the months of July-December 2010 for model III. The mean absolute errors are computed from the difference between the predicted *CD₄* cell counts and actual

Table VIII: Predicted *CD₄* cell counts for the months of July-Dec 2010 with the absolute error (MAE)

MODEL III	STATES	PREDICTED CD_4 CELL COUNTS FOR THE MONTHS OF JULY-DEC 2010						MEAN ABS ERROR(MAE)
		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.40$	I	241	236	232	228	225	221	9.97
	II	241	237	233	229	225	221	
	III	241	238	234	230	226	222	
	IV	240	237	233	229	225	222	
	V	0	0	0	0	0	0	

Model IV

We compute the transition probability matrix $\hat{P}$ for model IV is obtained using equation (8) and a smoothing constant $(\alpha) = 0.45$ for the months of January - June 2010:

Table IX: Transition probability matrix, $\hat{P}$ for the months of Jan-June 2010 for model IV.

$$P = \begin{array}{c} \\ I \\ II \\ III \\ IV \\ V \end{array} \begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \begin{bmatrix} 0.308 & 0.342 & 0.203 & 0.130 & 0.016 \\ 0.318 & 0.343 & 0.197 & 0.126 & 0.016 \\ 0.238 & 0.309 & 0.323 & 0.115 & 0.015 \\ 0.227 & 0.320 & 0.327 & 0.111 & 0.016 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Using equation (5), we make predictions of the CD_4 cell counts for the months of July-December 2010 for model IV. The mean absolute errors are computed from the difference between the predicted values of the CD_4 cell counts and actual.

Table X: Predicted CD_4 cell counts for the months of the July-Dec 2010 with the mean absolute error (MAE)

MODEL IV	STATES	PREDICTED CD_4 CELL COUNTS						MEAN ABS ERROR(MAE)
		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.45$	I	241	237	233	229	226	222	9.70
	II	241	237	233	230	226	222	
	III	240	237	234	230	226	223	
	IV	241	237	234	230	226	223	
	V	0	0	0	0	0	0	

MODEL V

We compute the transition probability matrix $\hat{P}$ for the months of January - June 2010 using equation (8) and smoothing constant $(\alpha) = 0.50$ for model V.

Table XI: Transition probability matrix $\hat{P}$ for the months of Jan-June 2010 for model V.

$$P = \begin{array}{c} \\ I \\ II \\ III \\ IV \\ V \end{array} \begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \begin{bmatrix} 0.306 & 0.342 & 0.203 & 0.134 & 0.014 \\ 0.319 & 0.342 & 0.196 & 0.128 & 0.015 \\ 0.238 & 0.311 & 0.322 & 0.114 & 0.015 \\ 0.226 & 0.318 & 0.332 & 0.109 & 0.015 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

We use equation (5) to make predictions of the CD_4 cell counts for the months of July-Dec. 2010 for model V. The mean absolute errors are computed from the difference between the predicted CD_4 cell counts and the actual.

Table XII: Predicted CD_4 cell counts for the month of JULY-DEC 2010 WITH THE MEAN ABSOLUTE ERROR (MAE)

MODEL V	STATES	PREDICTED CD_4 CELL COUNTS						MEAN ABS ERROR(MAE)
SMOOTHING CONSTANT		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.50$	I	242	238	234	231	227	224	9.23
	II	242	238	234	231	227	224	
	III	240	238	234	231	227	224	
	IV	240	238	234	231	227	224	
	V	0	0	0	0	0	0	

MODEL VI

The transition probability matrix $\hat{P}$ is computed for the months of January-June 2010 using equation (8) and a smoothing constant $(\alpha) = 0.55$

Table XIII: Transition probability matrix, $\hat{P}$ for the months of Jan-June 2010

$$P = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0.304 & 0.343 & 0.202 & 0.137 & 0.013 \\ 0.321 & 0.341 & 0.194 & 0.129 & 0.014 \\ 0.237 & 0.312 & 0.321 & 0.115 & 0.015 \\ 0.225 & 0.317 & 0.335 & 0.109 & 0.014 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Using equation (5) we make predictions of the CD_4 cell counts for the months of July-Dec 2010. The mean absolute errors (MAE) are computed from the difference between the predicted CD_4 cell counts and the actual.

Table XIV: Predicted CD_4 cell counts for the months of July-Dec 2010 with the mean of absolute error (MAE)

MODEL VI	STATES	PREDICTED CD_4 CELL COUNTS						MEAN ABS ERROR(MAE)
SMOOTHING CONSTANT		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.50$	I	242	238	234	231	228	224	9.20
	II	242	238	234	231	227	224	
	III	240	238	234	231	227	224	
	IV	241	238	234	231	228	224	
	V	0	0	0	0	0	0	

MODEL VII

The transition probability matrix $\hat{P}$ is obtained for the months of January-June 2010 for model VII is computed by making use of equation (8) with a smoothing constant $(\alpha) = 0.60$

Table XV: TRANSITION PROBABILITY MATRIX $\hat{P}$ FOR THE MONTHS OF JAN-JUNE 2010

$$P = \begin{array}{c} \\ I \\ II \\ III \\ IV \\ V \end{array} \begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \begin{bmatrix} 0.303 & 0.343 & 0.201 & 0.126 & 0.012 \\ 0.323 & 0.341 & 0.192 & 0.130 & 0.014 \\ 0.237 & 0.313 & 0.320 & 0.115 & 0.015 \\ 0.225 & 0.316 & 0.337 & 0.109 & 0.013 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Using equation (5), we make predictions of the CD_4 cell counts for the months of July-Dec 2010. The mean absolute errors (MAE) are computed from the difference between the predicted CD_4 cell counts and the actual.

Table XVI: Predicted CD_4 cell counts for the month of July-Dec 2010 with the mean absolute error(MAE)

MODEL I	STATES	PREDICTED CD_4 CELL COUNTS FOR THE MONTHS OF JULY-DEC 2010.						MEAN ABS ERROR(MAE)
		JULY	AUG	SEPT	OCT	NOV	DEC	
$\alpha = 0.60$	I	239	234	230	226	222	218	10.43
	II	242	237	233	229	225	221	
	III	241	237	233	229	225	221	
	IV	241	237	233	229	225	221	
	V	0	0	0	0	0	0	

6. Prediction

We have seen the efficacy of the model in making predictions. The results of the predicted CD_4 cell counts and their mean absolute errors have shown that $\alpha = 0.55$ is the optimal smoothing constant with a mean absolute error (MAE) of 9.2. We shall use the optimal smoothing constant $\alpha = 0.55$ to make predictions of the future CD_4 cell counts for the months of Jan-Dec, 2011 and 2012 (see Table XVII). The predicted and observed CD_4 cell counts from January to December 2010 is shown in Table XVIII

Table XVII: Predicted CD_4 cell count in 2011 (Jan-Dec)

STATES	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
I	234	230	226	223	220	216	213	210	207	204	201	198
II	227	223	220	217	213	210	207	204	201	198	195	192
III	231	229	225	222	219	215	212	209	206	203	200	197
IV	231	228	225	222	218	215	212	209	206	203	200	197
V	0	0	0	0	0	0	0	0	0	0	0	0

Table XVII: Predicted CD_4 cell count in 2012 (Jan-Dec)

STATES	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
I	195	160	132	109	80	74	61	50	41	34	28	23
II	189	156	128	101	87	72	59	48	40	33	27	22
III	194	160	131	108	89	73	60	50	41	34	28	23
IV	194	159	131	108	89	73	60	50	41	34	28	23
V	0	0	0	0	0	0	0	0	0	0	0	0

Table XVIII: Showing the predicted and obsoleted values for the month of Jan – Dec 2010

STATES		JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
I	PREDICTED	234	230	226	223	220	216	213	210	207	204	201	198
	OBSERVED	272	267	260	273	238	237	258	231	232	256	242	244
II	PREDICTED	227	223	220	217	213	210	207	204	201	198	195	192
	OBSERVED	270	275	260	262	259	257	259	256	251	252	223	223
III	PREDICTED	231	229	225	222	219	215	212	209	206	203	200	197
	OBSERVED	277	263	275	228	245	234	228	228	237	231	243	231
IV	PREDICTED	231	228	225	222	218	215	212	209	206	203	200	197
	OBSERVED	275	250	250	250	245	252	224	249	238	238	237	242
V	PREDICTED	0	0	0	0	0	0	0	0	0	0	0	0
	OBSERVED	0	0	0	0	0	0	0	0	0	0	0	0

7. Conclusion

We have used different values of exponential smoothing constants to estimate non-stationary transition probabilities in this paper. We observed that as the values of the exponential smoothing constant increases, the optimal smoothing constant tends towards having a minimal mean absolute error $(\text{MAE})[\alpha = 0.55,\ (\text{MAE}) = 9.2, \text{See table XIV}]$. The prediction of the CD_4 cell counts with the optimal smoothing constant for the months of Jan-Dec 2011, showed some interesting results. We observe a gradual decrease of the CD_4 cell counts from the months of Jan-Dec, 2011. The decrease is as a result of the effect of the attack

of the human immunodeficiency virus (HIV) on the CD_4 cells of patients. But with the administration of highly active antiretroviral therapy (HAART) the decrease of the CD_4 cells can be reduced. The results also suggest that patients cannot survive death from the month of Dec 2011 (See table XVII) if not placed on therapy (HAART). The use of highly active antiretroviral therapy (HAART) decreases the patients burden of HIV (Human immunodeficiency Virus), maintain function of the immune system, and prevents opportunistic infections that often lead to death. The beauty of this model is that it can be used to determine the effect of HAART on patients, using the predicted CD_4 cell counts as a base. Therefore we suggest that health management personnel should use the model for the prediction of CD_4 cell counts, so as to ascertain when to administer HAART, especially in the rural areas where quasi-health centres are the only health institutions available to HIV/AIDS patients.

Appendix

Appendix A

Year 2010

JANUARY		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	95	99	48	18	12	272
$350 < CD_4 \leq 500$ cells$/mm^3$	II	100	96	50	20	4	270
$200 < CD_4 \leq 350$ cells$/mm^3$	III	70	50	95	55	7	277
$CD_4 \leq 200$ cells $/mm^3$	IV	76	95	30	64	10	275
DEATH	V	0	0	0	0	0	0

FEBRUARY		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	110	90	50	15	2	267
$350 < CD_4 \leq 500$ cells$/mm^3$	II	80	120	55	15	5	275
$200 < CD_4 \leq 350$ cells$/mm^3$	III	65	90	70	30	8	263
$CD_4 \leq 200$ cells $/mm^3$	IV	60	84	85	20	1	250
DEATH	V	0	0	0	0	0	0

MARCH		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	95	86	50	20	9	260
$350 < CD_4 \leq 500$ cells$/mm^3$	II	75	90	50	37	8	260
$200 < CD_4 \leq 350$ cells$/mm^3$	III	70	95	80	24	6	275
$CD_4 \leq 200$ cells $/mm^3$	IV	55	85	80	24	6	250
DEATH	V	0	0	0	0	0	0

APRIL		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	60	80	70	50	13	273
$350 < CD_4 \leq 500$ cells$/mm^3$	II	60	70	65	57	10	262
$200 < CD_4 \leq 350$ cells$/mm^3$	III	50	75	80	21	2	228
$CD_4 \leq 200$ cells $/mm^3$	IV	60	70	85	25	10	250
DEATH	V	0	0	0	0	0	0

MAY		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	80	90	50	17	1	238
$350 < CD_4 \leq 500$ cells$/mm^3$	II	90	100	55	10	4	259
$200 < CD_4 \leq 350$ cells$/mm^3$	III	60	70	90	23	2	245
$CD_4 \leq 200$ cells $/mm^3$	IV	50	90	85	19	1	245
DEATH	V	0	0	0	0	0	0

JUNE		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	70	80	45	40	2	237
$350 < CD_4 \leq 500$ cells$/mm^3$	II	85	85	45	40	2	257
$200 < CD_4 \leq 350$ cells$/mm^3$	III	55	75	70	30	4	234
$CD_4 \leq 200$ cells $/mm^3$	IV	58	75	85	31	3	252
DEATH	V	0	0	0	0	0	0

NOVEMBER		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	80	65	55	40	2	242
$350 < CD_4 \leq 500$ cells$/mm^3$	II	72	65	44	40	2	223
$200 < CD_4 \leq 350$ cells$/mm^3$	III	50	80	70	42	1	243
$CD_4 \leq 200$ cells $/mm^3$	IV	46	80	70	40	1	237
DEATH	V	0	0	0	0	0	0

DECEMBER		I	II	III	IV	V	TOTAL
$CD_4 > 500$ cells $/mm^3$	I	83	75	50	35	1	244
$350 < CD_4 \leq 500$ cells$/mm^3$	II	75	60	50	35	3	223
$200 < CD_4 \leq 350$ cells$/mm^3$	III	56	65	78	30	2	231
$CD_4 \leq 200$ cells $/mm^3$	IV	58	86	60	35	3	242
DEATH	V	0	0	0	0	0	0

Appendix B: Transition Probability Matrix from Feb-Dec 2010

$P(T)$ For FEB.

	I	II	III	IV	V
I	0.412	0.337	0.187	0.056	0.007
II	0.290	0.436	0.200	0.058	0.007
III	0.247	0.342	0.266	0.114	0.030
IV	0.240	0.336	0.340	0.080	0.004
V	0	0	0	0	1

MARCH $P_{ij}(T)$

	I	II	III	IV	V
I	0.365	0.331	0.192	0.077	0.027
II	0.289	0.346	0.192	0.142	0.031
III	0.255	0.346	0.291	0.089	0.022
IV	0.220	0.340	0.320	0.096	0.024
V	0	0	0	0	1

APRIL $P_{ij}(T)$

	I	II	III	IV	V
I	0.220	0.293	0.256	0.183	0.048
II	0.229	0.267	0.248	0.218	0.038
III	0.219	0.329	0.351	0.092	0.009
IV	0.240	0.280	0.340	0.100	0.040
V	0	0	0	0	1

MAY $P_{ij}(T)$

	I	II	III	IV	V
I	0.336	0.378	0.210	0.071	0.004
II	0.345	0.386	0.212	0.039	0.015
III	0.245	0.286	0.367	0.094	0.008
IV	0.204	0.367	0.347	0.078	0.004
V	0	0	0	0	1

JUNE $P_{ij}(T)$

	I	II	III	IV	V
I	0.295	0.337	0.190	0.169	0.008
II	0.331	0.331	0.175	0.155	0.008
III	0.235	0.321	0.299	0.128	0.017
IV	0.230	0.298	0.337	0.123	0.012
V	0	0	0	0	1

JULY $P_{ij}(T)$

	I	II	III	IV	V
I	0.349	0.310	0.182	0.155	0.004
II	0.270	0.309	0.232	0.182	0.008
III	0.219	0.285	0.329	0.162	0.004
IV	0.223	0.313	0.268	0.192	0.005
V	0	0	0	0	1

NOV $P_{ij}(T)$

	I	II	III	IV	V
I	0.331	0.269	0.227	0.165	0.008
II	0.323	0.292	0.197	0.179	0.009
III	0.206	0.329	0.288	0.173	0.004
IV	0.194	0.338	0.295	0.169	0.004
V	0	0	0	0	1

DEC $P_{ij}(T)$

	I	II	III	IV	V
I	0.340	0.307	0.205	0.143	0.004
II	0.336	0.269	0.224	0.157	0.014
III	0.242	0.281	0.338	0.130	0.009
IV	0.240	0.355	0.248	0.145	0.012
V	0	0	0	0	1

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