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On Interval Estimation for Exponential Power Distribution Parameters

A. A. Olosunde[1] and A. T. Soyinka[2]

[1]Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria, akinolosunde@gmail.com

[2]Department of Research and Training, Federal Neuropsychiatric Hospital, Aro Abeokuta, Nigeria, soyinkataiwo@yahoo.ie

Abstract

Abstract: The probability that the estimator is equal to the value of the estimated parameter is zero. Hence in practical applications we provide together with the point estimates their estimated standard errors. Given a distribution of random variable which has heavier tails or thinner tails than a normal distribution, then the confidence interval common in the literature will not be applicable. In this study, we obtained some results on the confidence procedure for the parameters of generalized normal distribution which is robust in any case of heavier or thinner than the normal distribution using pivotal quantities approach, and on the basis of a random sample of fixed size n. Some simulation studies and applications are also examined.

Key words: Shape parameter, Short tails, exponential power distribution, confidence interval.

1 Introduction

The higher the degree of confidence, the larger the percentage of population values that the interval is to contain. Thus, if we want a high degree of confidence and a high confidence level, we are going far out into the tail of some distribution. For example, if the distribution has heavier tails than a normal distribution and we believe we are constructing a 99 percent confidence interval to contain at least 99 percent of the population, the percentage of the time we do that, say, 1000 randomly constructed intervals actually contain at least 99 percent of the population, the result obtained often times could be far less than 99 percent that we really required. Conversely, if the population distribution has much lighter tails than a normal distribution, the interval could be much wider than necessary. It is important to remember that if the population X is not normal, then sample mean $\overline{x}$ is not normal as well, and so the random variable $(\overline{x}-\mu)/(s/\sqrt{n})$ does not have the common $t-$distribution, implying that the commonly used t table values are not necessarily correct. In this paper we developed a robust confidence interval for the univariate family of elliptical density called exponential power distribution (EPD) which has a normal and laplace as its sub-family and it is well known for the flexibility of its tails. Using the widely known pivotal quantities method, the resulting robust confidence interval we anticipated will replace the common one in literature about the normal distribution especially when the observed sample data in an experiment has tail heavier or thinner than the usual normal distribution.

The uni-dimensional exponential power distribution is defined as

$$f(x;\mu,\sigma,\beta)=\frac{1}{\sigma\Gamma\left(1+\frac{1}{2\beta}\right)2^{1+\frac{1}{2\beta}}}\exp\left\{-\frac{1}{2}\left|\frac{x-\mu}{\sigma}\right|^{2\beta}\right\} \tag{1}$$

where the parameters $\mu\in\Re$ and $\sigma\in(0,\infty)$ are respectively scale and location parameters and $\beta\in(0,\beta)$ is the shape parameter which regulates the tails of the distribution such that when $\beta=1$ the density (1) is normal, but for $\beta=1/2$ we have double exponential distribution. The distribution (1) was first introduced by Subbotin (1923), it has been used in robust inference (see Box(1953)) where the parameters of the distribution were estimated via moments.

If a random variable X has the pdf (1) then its mth moments can be obtained from the relation

$$E(X^m)=\int_0^\infty\left(\left([-1^m(\sigma(2z)^{\frac{1}{2\beta}}-\mu)^m]+(\sigma(2z)^{\frac{1}{2\beta}}+\mu)^m\right)\left(\frac{z^{\frac{1}{2\beta}-1}\exp^{-z}}{2\Gamma(\frac{1}{2\beta})}\right)\right)dz \tag{2}$$

In addition, its central moment estimates Agro (1992, 1995) are:
$E(X)=\mu$; $E\left|X-E(X)\right|=\frac{\sigma2^{\frac{1}{2\beta}}\Gamma(\frac{1}{\beta})}{\Gamma(\frac{1}{2\beta})}$; $Var(X)=\frac{\sigma^2 2^{\frac{2}{2\beta}}\Gamma(\frac{3}{2\beta})}{\Gamma(\frac{1}{2\beta})}$;
$E(X-E(X))^3=0$; $E(X-E(X))^4=\frac{\sigma^4 2^{\frac{4}{2\beta}}\Gamma(\frac{5}{2\beta})}{\Gamma(\frac{1}{2\beta})}$; and Kurtosis$=\frac{\Gamma(\frac{5}{2\beta})\Gamma(\frac{1}{2\beta})}{\Gamma^2(\frac{3}{2\beta})}$.

The results indicate that the sample mean $\overline{X}$ is the estimate of the true mean μ while the shape parameter can be numerically obtained from the estimate of the kurtosis. Substituting shape parameter estimate into Var(X) we estimate the scale parameter σ. Also the log-likelihood function for random samples $x_1,x_2,..,x_n$ from (1) is:

$$LogL(\mu,\sigma,\beta)=n\ln\left(\frac{1}{\sigma\Gamma(1+\frac{1}{2\beta})2^{1+\frac{1}{2\beta}}}\right)-\sum_{i=1}^{i=n}\frac{1}{2}\left|\frac{x_i-\mu}{\sigma}\right|^{2\beta} \tag{3}$$

The derivatives of (8) with respect to μ, σ, and β are
$\frac{\partial LogL}{\partial\mu}=\frac{\beta}{\sigma^{2\beta}}\left(\sum_{x_i\geq\mu}(x_i-\mu)-\sum_{x_i<\mu}(x_i-\mu)\right)$; $\frac{\partial LogL}{\partial\sigma}=-\frac{n}{\sigma}+\frac{\beta}{\sigma}\left|\frac{x-\mu}{\sigma}\right|^{2\beta}$; and
$\frac{\partial LogL}{\partial\beta}=\frac{n}{2\beta^2}[\Psi(1+\frac{1}{2\beta})+1]-\sum_{i=1}^{i=n}\left|\frac{x_i-\mu}{\sigma}\right|^{2\beta}\ln\left|\frac{x_i-\mu}{\sigma}\right|$

Finally the expected fisher information matrix of EPD are
$E(-\frac{\partial^2 LogL}{\partial\mu^2}) = \frac{n\beta(2\beta-1)2^{1-\frac{1}{\beta}}\Gamma(1-\frac{1}{2\beta})}{\sigma^2\Gamma\frac{1}{2\beta}}$; $E(-\frac{\partial^2 LogL}{\partial\sigma^2}) = \frac{2\beta n}{\sigma^2}$;
$E(-\frac{\partial^2 LogL}{\partial\sigma\partial\beta}) = -\frac{1}{\sigma\beta}(1+\Psi(1+\frac{1}{2\beta})\ln 2)$; and
$E(-\frac{\partial^2 LogL}{\partial\beta^2}) = \frac{n}{\beta^3}\left(1+\Psi(1+\frac{1}{2\beta})+\frac{\Psi'(1+\frac{1}{2\beta})}{2\beta}\right)+n\frac{(\ln 2)^2}{4\beta^3}\left(\Psi^2(1+\frac{1}{2\beta})+\Psi'(1+\frac{1}{2\beta})\right)$

Mineo and Ruggieri (2003a,b) developed codes in R programming environment to estimate these parameters from any given sample from (1), this also includes the parameter β which has no explicit solution.

2 Confidence Interval

Definition 2.0.(Confidence Interval). Let $X_1, X_2, ..X_n$ be a random sample from the density $f(X|\theta)$. Let $a = t_1(X_1, X_2, ..X_n)$ and $b = t_2(X_1, X_2, ..X_n)$ be two statistics satisfying the relation $a \le b$ for which $Pr\{a < Z(X|\theta) < b\} \equiv \gamma$, where γ does not depend on θ; the random interval (a,b) is called 100γ percent confidence interval for $Z(X|\theta)$; γ is called the confidence coefficient; a and b are called the lower and upper confidence limits, respectively for $Z(X|\theta)$. A value (t_1, t_2) of the random interval (a, b) is also called a 100γ percent confidence interval for $Z(X|\theta)$. Ram et.al. (2010) in his paper provided new confidence intervals (CIs) based on F-approximations as well as normal approximations. Riggs (2015), showed from the simulation results which indicated that the score and likelihood ratio intervals are generally preferable over the Wald interval.

2.1 Pivotal Quantity

In this present study we used the pivotal quantity approach to derive the confidence interval for the exponential power distribution.
Definition 2.1**.(Pivotal Quantity)**. A pivotal quantity (Z) for a parameter θ is a random variable $Z(X|\theta)$ whose value depends both on (the data) X and on the value of the unknown parameter θ but whose distribution is known to be independent of θ. For the case of the normal distribution $N(\mu, \sigma^2)$, the pivotal quantity $Z = \frac{X-\mu}{\sigma}$ and $Z^2 = \frac{(X-\mu)^2}{\sigma^2}$ has distribution $N(0, 1)$ and χ^2_1 that are independent of μ and σ and as such both are pivotal quantity for μ and σ respectively. We establish some important theorem in this section that served as a foundation to the results obtained
Proposition 2.1: Let X has a pdf (1), then $\left|\frac{X-\mu}{\sigma}\right|^{\beta} \sim \Gamma(\frac{1}{2\beta}, 2)$. Where μ,σ,and β, and δ are location, scale and shape parameters respectively. Values for $Gamma(.)$ for various β can be obtained from Abramowitz and Stegun (1963), Paris (2010) and Winitzki (2003).

Proof: By transformation techniques, we have that
$f_Y(y) = |\frac{d}{dy}g^{-1}(y)|f_X(g^{-1}(y)) = \Gamma(\frac{1}{2\beta}, 2), y > 0$; the pdf (1) is a three parameter family, $\underline{\theta} = (\mu, \sigma, \beta)$. We can deduce from above proposition 2.1 that

Corollary 2.1: Let X has a pdf (1) then,

$$Z = \sqrt{\frac{\beta}{n}}\left|\frac{x-\mu}{\sigma}\right|^{\beta} \sim EPD(0, \frac{1}{n}, \beta) \tag{4}$$

and $\left|\frac{x-\mu}{\sigma}\right|^{\beta} \sim \Gamma(\frac{1}{2\beta}, 2)$ are pivotal quantities and independent.

Proposition 2.2: If the random variable X has the pdf (1). Then the proposed pivotal quantity (4) has the pdf

$$g(Z) = \frac{(\frac{n}{\beta})^{\frac{1}{2\beta}}}{2^{\frac{1}{2\beta}}\Gamma(\frac{1}{2\beta})} Z^{\frac{1}{\beta}-1} \exp(-\frac{n}{2\beta}Z^2); \quad -\infty < Z < \infty, \ \beta > 0. \tag{5}$$

Proof. Substituting (4) into (1) and using change of variable techniques; the pdf (5) obtained is independent of μ and σ and thus (4) is a pivotal quantity for μ and σ.

Remark 2.1: pdf (5) generalizes Normal, Laplace and Weibull distributions. By simply substituting for β in (5) and simplify then the results obtained reflect the targeted distribution.

2.1.1 Generalized $\mathbf{t}_1 - distribution$

Proposition 2.3: Let $Z \sim EPD(0, \frac{1}{n}, \beta)$ from (5) and let V denotes a random variable which is $\Gamma(\frac{r}{2}, 2)$; a new random variable t_1 define as

$$T_1 = \frac{Z}{\sqrt{\frac{V}{r}}} \tag{6}$$

has the pdf

$$f_r(t_1) = \sqrt{\frac{n}{\beta r}} \left\{\frac{\Gamma(\frac{r}{2}+\frac{1}{2\beta})}{\Gamma(\frac{r}{2})\Gamma(\frac{1}{2\beta})}\right\} \left\{\sqrt{\frac{n}{\beta r}}t_1\right\}^{\frac{1}{\beta}-1} \left\{(1 + \left(\sqrt{\frac{n}{\beta r}}t_1\right)^2\right\}^{-(\frac{r}{2}+\frac{1}{2\beta})} \tag{7}$$

and is a pivotal quantity.

Proof. Since Z has a symmetric distribution about zero, so does the t_1 and its pdf will satisfy $f_r(t_1) = f_r(-t_1)$. Assuming both Z and V are stochastically independent then for $t_1 > 0$ we have,

$$P\left[\frac{Z}{\sqrt{V/r}} > t_1\right] = \frac{1}{2}P\left[\frac{Z^2}{V/r} > t_1^2\right] = \frac{1}{2}P\left[\frac{Z^2}{2} > \frac{V}{2}\frac{t_1^2}{r}\right]$$

taking the negative derivative wrt t_1. The marginal distribution $f(t_1)$ is the pdf in 7 which we called a generalized t_1 distribution.

Remarks 2.2:

1. But (7) is a generalized version of the usual student $t-$distribution because $f(t_1)$ becomes usual students t-distribution when $\beta = 1$ and $n = 1$ thus reducing (4) to $Z_1 = \left|\frac{x-\mu}{\sigma}\right|$.
$$\left[\frac{\Gamma(\frac{r+1}{2})}{\sqrt{\pi r}\Gamma(r/2)}\right]\frac{1}{(1+t^2/r)^{(r+1)/2}} \tag{8}$$
because of the present parameter β which further regulate its tails.

2. Since $f(t_1)$ is independent of μ and σ then it is a pivotal quantity.

3. $f_r(t_1) \propto \left\{\sqrt{\frac{n}{\beta r}}t_1\right\}^{\frac{1}{\beta}-1}\left\{(1+\left(\sqrt{\frac{n}{\beta r}}t_1\right)^2\right\}^{-(\frac{r}{2}+\frac{1}{2\beta})}$ is symmetric and bell shaped, but falls off to zero as $t \to \pm\infty$ more slowly than the normal and exponential power densities. They have $f(.) \approx e^{-z^2/2}$ and $g(.) \approx e^{-|z|^\beta/2}$ respectively.

4. To get the cdf of (7), the transformation $\tan\theta = \sqrt{\frac{n}{\beta r}}T_1$ shows that the cdf of (7) is an incomplete beta $F_r(t_1) = Beta\left(\sqrt{\frac{n}{\beta r}}T_1; \frac{1}{2\beta}, \frac{r}{2}\right)$.

Corollary 2.2 (Generalized t_2): Let $Z \sim EPD(0, \frac{1}{n}, \beta)$ from (4) and let V denotes a random variable which is $\Gamma(\frac{1}{2\beta}, 2)$ see proposition 2.1; a new random variable T_2 given as
$$T_2 = \frac{Z}{V} \tag{9}$$
has the pdf
$$f(t_2) = n^{\frac{1}{2\beta}}\frac{\Gamma(\frac{1}{\beta})}{(\Gamma(\frac{1}{2\beta})^2)}\frac{t_2^{\frac{1}{\beta}-1}}{(1+nt_2^2)^{\frac{1}{\beta}}} \tag{10}$$
Using the transformation $\tan\theta = \sqrt{n}T_2$, we obtain the CDF has $F_{\frac{1}{\beta}}(t_2) = Beta\left(\sqrt{n}T_2; \frac{1}{2\beta}, \frac{1}{2\beta}\right)$
Proof. Substitute $r = \frac{1}{\beta}$ into (7) and (10) follows.

Having established that (6) and (9) are pivotal quantities for μ and σ, with their confidence limits distributed as $F_r(t_1){=}Beta\left(\sqrt{\frac{n}{\beta r}}T_1; \frac{1}{2\beta}, \frac{r}{2}\right)$ and $F_{\frac{1}{\beta}}(t_2){=}Beta\left(\sqrt{n}T_2; \frac{1}{2\beta}, \frac{1}{2\beta}\right)$ respectively then we proceed to obtain its confidence intervals while restricting our application to confidence interval via t_2.

2.1.2 Confidence Interval for μ for a known σ^2

Given the t_2 distribution, (and its cdf $F_{\frac{1}{\beta}}(t_2)$ which has no close form), for any independent and identically distributed random sample $(\mathbf{x})=\{x_1, ..., x_n\}\sim$EPD$(\mu, \sigma^2, \beta)$; we obtain the sufficient statistics $\overline{x}_n = \frac{\sum x_i}{n}$, $\hat{\sigma}_n^2 = \frac{s_n^2}{n} = \left(\frac{\beta}{n}\right)^{\frac{1}{\beta}} \sum_{i=1}^{n}(x-\mu)^2$ and for any $0 < \gamma < 1$, we compute t_2^* such that $F_{\frac{1}{\beta}}(t_2^*) = (1+\gamma)/2$ from

$\gamma = P\left[-t_2^* \leq \sqrt{\frac{\beta}{n}}\left|\frac{x-\mu}{\sigma}\right|^\beta \leq t_2^*\right] \sim EPD(0, \frac{1}{n}, \beta)$ owing that $F_{\frac{1}{\beta}}(t_2^*){=}Beta\left(t_2^*; \frac{1}{2\beta}, \frac{1}{2\beta}\right)$

Proposition 2.4: The confidence interval for the μ of the EPD when variance $E(\sigma^2) = s^2$ can be obtained as

$$\left\{\overline{x} - \frac{s}{n}\left(\frac{n}{\beta}\right)^{\frac{1}{2\beta}} (t_2^*)^{\frac{1}{\beta}} < \mu < \overline{x} + \frac{s}{n}\left(\frac{n}{\beta}\right)^{\frac{1}{2\beta}} (t_2^*)^{\frac{1}{\beta}}\right\} \tag{11}$$

Proof. Simplify

$$P\left\{-t_2^* < \sqrt{\frac{\beta}{n}}\left|\frac{x-\mu}{\sigma}\right|^\beta < t_2^*\right\} \equiv \gamma \tag{12}$$

for μ.

2.1.3 Confidence Interval for σ^2 with μ unknown

Proposition 2.5: Given the interval $(-t_2^*, t_2^*)$. Then the confidence interval for the σ^2 of the EPD is given as

$$\frac{s^2}{(2beta[(F_{\frac{1}{\beta}}(t_2^*)), \frac{1}{2\beta}, \frac{1}{2\beta}])^{\frac{1}{\beta}}}; \frac{s^2}{(2beta[1-(F_{\frac{1}{\beta}}(t_2^*)), \frac{1}{2\beta}, \frac{1}{2\beta}])^{\frac{1}{\beta}}} \tag{13}$$

where $s^2 = \frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}$

Proof. Simplify

$$Pr\left\{(-t_2^*)^2 < \frac{\beta}{n}\left\{\frac{x-\mu}{\sigma}\right\}^{2\beta} < (t_2^*)^2\right\} \equiv \gamma \tag{14}$$

for σ^2 and recall from proposition 2.1 that $\frac{1}{2}\left|\frac{X-\mu}{\sigma}\right|^\beta \sim \Gamma(\frac{1}{2\beta})$.

3 Numerical Illustration

3.1 Evaluating confidence limits

Given interval $(-t_2^*, t_2^*)$, we defined the 100γ percent confidence interval for pivot (9) as

$$Pr\{-t_2^* < T_2 < t_2^*\} \equiv \gamma \tag{15}$$

We then evaluate the different values of $-t_2^*$ and t_2^* at various values of γ from $F_{t_2^*}(t_2){=}Beta\left(\sqrt{n}T_2; \frac{1}{2\beta}, \frac{1}{2\beta}\right)$ for known values of β. Using the code $Rbeta.inv(\gamma, \frac{1}{2\beta}, \frac{1}{2\beta})$ written in R environment. We thus have

Table 1: Limits $(-t_2^*, t_2^*)$ for different γ values from $F_{t_2^*}(t_2)$ at various β values

γ	limits	$F_{t_2^*}(t_2)$, $\beta = 0.1$	$F_{t_2^*}(t_2)$, $\beta = \frac{1}{2}$	$F_{t_2^*}(t_2)$, $\beta = 1$	$F_{t_2^*}(t_2)$, $\beta = 1.5$	$F_{t_2^*}(t_2)$, $\beta = 2$
0.1	-t_2^*	0.3009688	0.1	0.0245	0.0055	0.0012
0.9	t_2^*	0.6990312	0.9	0.9755	0.99945	0.9988
0.05	-t_2^*	0.2513676	0.05	0.0062	0.0007	7.38×10^{-5}
0.95	t_2^*	0.7486324	0.95	0.9938	0.9993	0.9999
0.025	-t_2^*	0.2120085	0.025	0.0015	0.00009	4.616×10^{-6}
0.975	t_2^*	0.7879915	0.975	0.9985	0.9999	0.99999
0.005	-t_2^*	0.1460562	0.005	6.17×10^{-5}	9×10^{-7}	7.36×10^{-9}
0.995	t_2^*	0.8539438	0.995	0.9999	1	1

3.2 Simulation Results

We simulated 1000,000 sample from population having an exponential power distribution with parameters location parameter $\mu = 4.000$, scale parameter $\sigma = 2.500$ and shape parameter = 3.546, the simulation is from R package called normalp see Mineo (2003a,b). With new code written in R environment, we have table 2, the estimated confidence interval for the mean. In the table 2 we also have confidence interval for the normal case, that is, suppose we assume the data might have come from normal population, then the estimated mean $\mu_{normal} = 4.000863$ and the variance $\sigma_{normal} = 2.112163$. We note from the table that the confidence length for the normal case is bigger compare with exponential power distribution at 90%, 95% and 99%. This shows a better result for the latter.

Table 2: Confidence Interval for the estimate of mean

Limits Distribution	Exponential Power Normal Distribution	
90%	1.238116e-05	2.7078e-03
95%	1.238119e-05	4.454e-03
99%	1.238119e-05	0.0104

In Table 3 we have confidence interval for the variance of exponential power compare with normal distribution. Just like the case of mean, the confidence length for the exponential power is closer compare with normal

Table 3: Confidence Interval for the estimate of variance

Limits Distribution	Exponential Power Normal Distribution	
90%	3.977735-5.741168	2.348020526-44.61239
95%	3.972031-6.717694	2.287814872-89.22478
99%	3.967626-10.208426	2.241828643-446.1239

4 Data sets

Example 1. Forty-eight pairs of poultry birds were fed with inorganic and organic poultry feed Olosunde (2013), The comprehensive analysis and summary of the estimated parameters have been carried out in, Olosunde (2013). The data set are the average weights (wi, wo) and average cholesterol (ci, co) contents of the eggs produced by the birds, the data are reproduce in the appendix.

Table 4: Confidence interval for the means of egg weights and its cholesterol contents assuming exponential power distribution when beta is estimated

γ	wi	Ci	wo	Co
90%	±0.0972	±0.0455	±0.53	±0.9621
95%	±0.0976	±0.0456	±0.5315	±0.9655
99%	±0.0977	±0.0457	±0.532	±0.9667

Table 3: Confidence i nterval f or t he m eans o f e gg w eights a nd i ts c holesterol contents assuming normal distribution

γ	wi	Ci	wo	Co
90%	±0.1233	±0.0631	±0.7586	±1.2847
95%	±0.1469	±0.0752	±0.9038	±1.5307
99%	±0.1930	±0.0988	±1.1879	±2.0117

Table 4: Confidence interval for the variance of egg weights and its cholesterol content assuming exponential power distribution when β is estimated

γ	wi	Ci	wo	Co
0.05	11.726	455.612	3.128	1286.121
0.95	15.156	518.78	3.678	1573.268
0.025	11.715	455.419	3.115	1285.213
0.975	17.061	555.687	4.024	1734.75
0.005	11.707	455.27	3.113	1284.51
0.995	20.622	680.608	5.154	2283.813

Table 5: Confidence interval for the variance of egg weights and its cholesterol content assuming normal distribution

γ	wi	Ci	wo	Co
0.05	9.3313	353.3471	2.4449	1013.3887
0.95	18.3735	695.7474	4.814	1995.3822
0.025	8.8106	333.6285	2.3084	956.8363
0.975	19.7736	748.7651	5.1808	2147.4353
0.005	7.901	299.1850	2.0701	858.0535
0.995	22.9391	868.63	6.0102	2491.2052

Further examples attached.

5 Conclusion

From the application, obviously from practical experience it common to have data with density function that has heavier or thinner tail than the usual normal distribution. Estimating confidence interval with assumption of normal distribution will always give confidence l ength w ider t han n ecessary a nd o ften t he c onfidence be comes unreliable in hypothesis, as this is shown in both simulation and real life data from exponential power distribution. In this case assuming normal distribution in the estimation of confidence interval may lead to erroneous conclusion, especially for those who will be interest in going further to hypothesis testing. This article presented a robust parametric method of evaluating confidence interval for the mean and the variance with known shape parameters. The results further generalized what is obtainable in normal and the double-exponential case.

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APPENDIX

LIVESTOCK PRODUCTION, TEACHING AND RESEARCH FARM FEDERAL UNIVERSITY OF AGRICULTURE ABEOKUTA (Poultry Unit)

KEY

w – Egg weight
C – Cholesterol /egg (mg/egg)
i – Inorganic subscript
o – Organic subscript

S/NO	wi	Ci	wo	Co
1	52.67	164.23	56.08	60.73
2	53.17	167.42	56.34	66.03
3	53.67	170.60	56.61	71.33
4	54.17	173.78	56.87	76.63
5	54.67	176.96	57.13	81.93
6	55.17	180.14	57.39	87.22
7	55.67	183.32	57.65	92.52
8	56.17	186.51	57.92	97.82
9	56.67	189.69	58.18	103.11
10	57.17	192.87	58.44	108.41
11	57.67	196.05	58.70	113.70
12	58.17	199.24	58.96	119.0
13	58.67	202.42	59.23	124.30
14	59.17	205.60	59.45	129.60
15	59.67	208.78	59.75	134.89
16	60.17	211.96	60.01	140.19
17	60.67	215.14	60.27	145.48
18	61.17	218.33	60.54	150.78
19	61.67	221.52	60.80	156.08
20	62.17	224.69	61.06	161.37
21	62.67	224.85	61.32	166.67
22	63.17	227.88	61.58	171.97
23	63.43	228.03	61.85	177.26
24	65.67	231.06	62.34	182.56
25	65.15	228.01	62.11	182.56
26	63.43	224.83	61.85	187.86
27	62.93	221.65	61.58	193.16
28	62.43	218.46	61.32	187.86
29	61.93	215.28	61.06	182.56
30	61.43	212.10	60.80	177.26
31	60.93	208.92	60.54	171.96
32	60.43	205.74	60.27	166.66
33	59.93	202.56	60.01	161.36
34	59.43	199.37	59.75	156.06
35	58.93	196.19	59.49	150.76
36	58.43	193.01	59.23	145.46
37	57.93	189.83	59.00	140.16
38	57.43	186.65	58.70	134.86
39	56.93	183.46	58.44	129.56
40	56.43	180.28	58.18	124.26
41	55.93	177.10	57.92	118.96
42	55.43	173.92	57.65	113.66
43	54.93	170.74	57.39	108.36
44	54.43	167.55	57.13	103.06
45	53.93	164.37	56.87	97.76
46	53.43	161.19	56.61	92.46
47	52.93	158.01	56.34	87.16
48	52.43	154.83	56.08	81.86

Figure 1: data used