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Comparison of estimation methods for unit-Gamma distribution

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Abstract

In this study we have considered different methods of estimation of the unknown parameters of a two-parameter unit-Gamma (UG) distribution from the frequentists point of view. First, we briefly describe different frequentists approaches: maximum likelihood estimators, moments estimators, least squares estimators, maximum product of spacings estimators, method of Cramer-von-Mises, methods of Anderson-Darling and four variants of Anderson-Darling test and compare them using extensive numerical simulations. Monte Carlo simulations are performed to compare the performances of the proposed methods of estimation for both small and large samples. The performances of the estimators have been compared in terms of their bias and root mean squared error using simulated samples. Also, for each method of estimation, we consider the interval estimation using the bootstrap method and calculate the coverage probability and the average width of the bootstrap confidence intervals. The study reveals that the maximum product of spacing estimators and Anderson-Darling 2 (AD2) estimators are highly competitive with the maximum likelihood estimators in small and large samples. Finally, two real data sets have been analyzed for illustrative purposes.

Keywords: Unit-Gamma distribution, Monte Carlo simulations, Estimation methods, Parametric bootstrap methods.

1 Introduction

Grassia (1977) introduced a new probability distribution which was later called by Ratnaparkhl and Mosimann (1990) as unit-Gamma (UG) distribution, since its support is on the unit interval (0, 1). A random variable X follows unit-Gamma distribution if its probability density function is given by:

$$f(x \mid \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\beta-1}(-\log x)^{\alpha-1} \qquad (1)$$

where $\Gamma(u) = \int_0^\infty u^{\alpha-1} e^{-u} du$ is the complete gamma function, $\alpha > 0$ and $\beta > 0$ are the shape parameters. Its corresponding cumulative distribution function (c.d.f.) is written as:

$$F(x|\alpha,\beta) = F_y(-\log(x)|\alpha,\beta) = \frac{\gamma(\alpha,\beta(-\log x))}{\Gamma(\alpha)} \qquad (2)$$

where $F_y(\cdot)$ denotes the c.d.f. of Gamma distribution with shape $(\alpha > 0)$ and scale $(\beta > 0)$ parameters and $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function, define as $\gamma(\mathrm{a},\mathrm{x}) = \int_0^{\mathrm{x}} \mathrm{t}^{\mathrm{a}-1}\, \mathrm{e}^{-\mathrm{t}} \mathrm{dt}$.

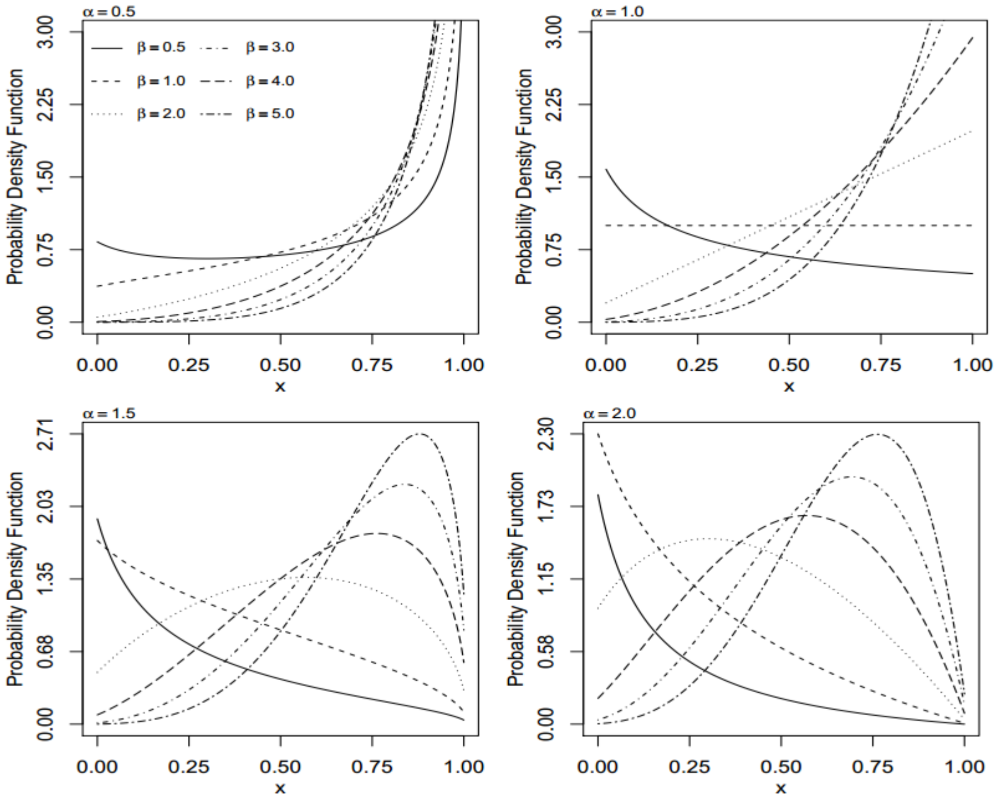

Figure 1: The unit-Gamma probability density function with different values of α and β.

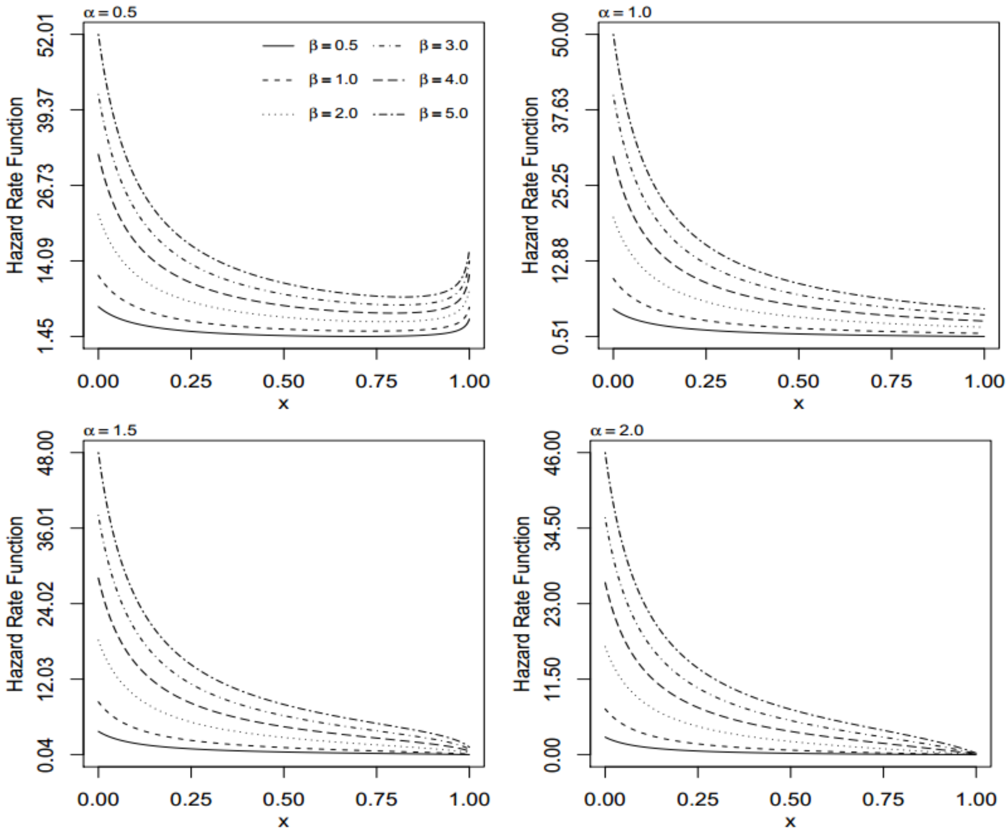

Figure 2: The unit-Gamma hazard rate function with di erent values of and .

The p.d.f (1) can have increasing, decreasing, constant and unimodal shapes, and the hazard rate function exhibits decreasing and bathtub shapes. Grassia (1977) gave a detailed account of UG distribution and its variants. Ratnaparkhl and Mosimann (1990) used this distribution for deriving some new distributions taking UG as a conditional distribution. Although, UG distribution has not been studied widely, but possesses some properties similar to that of the beta distribution. The applicability of the UG distribution has been found in areas like estimation of bacteria or virus density in dilution assays with host variability to infection using inoculation approach and for deriving other statistical distributions (see Grassia, 1977; Ratnaparkhl and Mosimann, 1990). Tadikamalla (1981) in his discussion paper pointed out that this distribution can be used as an alternative for Beta and Johnson SB distributions. He also investigated some of its properties. Ratnaparkhl and Mosimann (1990) studied the logarithmic and Tukey's lambda-type transformation on the unit-Gamma distribution. More recently, Mousa et al. (2016) formulated the UG regression model while Mazucheli et al. (2018) derived second order bias corrections for the parameters of UG distribution. Ho et al. (2019) considered the UG distribution to construct control charts to monitor rates and proportions. It is worth mentioning here that in studying real life situations we may come across distributions with bounded support such as percentages, proportions or fractions (see, Marshall and Olkin (2007)). In this respect, Papke and Wooldridge (1996) observed that variables bounded between zero and one arise naturally in many economic setting such as the fraction of total weekly hours spent on working, the proportion of income spent on non-durable consumption, pension plan participation rates, industry market shares, television rating, fraction of land area allocate to agriculture, etc. Various examples of proportions in the unit interval used in empirical finance are also discussed in Cook et al. (2008). Furthermore, when the reliability is measured as percentage or ratio, it is important to have models de ned on the unit interval (see, Genc (2013)) in order to have plausible results.

Parameter estimation is vital in the study of any probability distribution. Maximum likeli-hood estimation (MLE) is generally a starting point when it comes to estimating the parameters of any distribution due to its attractive properties. For example, they are asymptotically unbiased, consistent, and asymptotically normally distributed (Lehmann, 1999). However, there are other estimation methods developed over time for other distributions (see Gupta and Kundu (2001) for generalized Exponential distribution, Kundu and Raqab (2005) for generalized Rayleigh distributions, Teimouri et al. (2013) for Weibull distribution, Mazucheli et al. (2013) for weighted Lindley distribution, do Espirito Santo and Mazucheli (2015) for Marshall-Olkin extended Lindley distribution, Dey et al. (2015) for weighted Exponential distribution, Mazucheli et al. (2016) for Marshall-Olkin extended Exponential distribution and Dey et al. (2018) for Kumaraswamy distribution) which are based on different methodologies, such as method of moments estimation (MOM), method of L-moments estimation (LM), method of probability weighted moment estimation (PWM), method of least-squares estimation (LSE), method of weighted least-square estimation (WLSE), method of maximum product spacing estimation (MPS) and method of minimum distance estimation. Mazucheli and Menezes (2019) investigated the parameter estimation for the complementary Beta distribution considering the L-moments and maximum likelihood methods. Almetwally and Almongy (2019) used the maximum likelihood and maximum product spacing methods for estimating the parameters of generalized power Weibull distribution.

In this paper, we provide a comprehensive comparison of different methods of estimation for the unknown parameters for unit-Gamma distribution and to study the behaviour of these estimators for different sample sizes and for di erent parameter values. We mainly compare: the maximum likelihood estimators, maximum product of spacings estimators, moments estimators, least-squares estimators, weighted least-squares estimators, Cramer-von-Mises estimators and Anderson-Darling estimators and four of its variants. Since, it is difficult to compare theoretically the performances of the different methods of estimation, we perform extensive simulations to compare the performances of the different estimators based on bias and root mean squared error. Also, for each method of estimation, we consider the interval estimation using the bootstrap con dence interval (Efron, 1982a) and calculate the coverage probability and the average width of the con dence interval. The originality of this study comes from the fact that there has been no previous work comparing all of these estimation methods for the unit-Gamma distribution.

The final motivation of the paper is to show how different aforementioned frequentist estimators of this distribution perform for different sample sizes and different parameter values and to develop a guideline for choosing the best estimation method for the unit-Gamma distribution, which we think would be of interest to applied statisticians.

The remaining part of the paper is organized as follows: In Section 2 we discuss the eleven estimation methods considered in this paper. The comparison of these methods in terms of bias, root mean-squared error, coverage probability and average width is presented in Section 3. The eleven estimation methods are used for fitting two real data sets in Section 4. Some concluding remarks are presented in Section 5.

2 Estimation Methods

In this section, we describe seven estimation methods along with four variants of AD test for estimating the parameters, α and β, that index the unit-Gamma distribution. For all the methods of estimation, we assume that $\mathrm{x} = (\mathrm{x}_1, \dots, \mathrm{x}_\mathrm{n})^\mathrm{T}$ is a random sample of size n from unit-Gamma distribution, (1), with unknown parameters α and β. Besides, consider that $\mathrm{x}_{(1)} < \cdots < \mathrm{x}_{(\mathrm{n})}$ denote the corresponding order samples.

2.1 Method of Maximum Likelihood

The method of maximum likelihood (MLE) is the most popular estimation method in statistical inference, since its underlying motivation is simple and intuitive. Furthermore, the MLE enjoys several attractive properties (see, e.g, Lehmann and Casella, 1998; Pawitan, 2001; Rohde, 2014). For the unit-Gamma distribution, the log-likelihood function, apart from constant term, can be expressed as:

$$\mathrm{l}(\alpha, \beta | x) \propto n\alpha \log\beta - n\log\Gamma(\alpha) + \beta \sum_{i=1}^{n} x_i + \alpha \sum_{i=1}^{n} \log(-\log x_i) \quad (3)$$

The maximum likelihood estimators $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$, of the parameters α and β, respectively, can be obtained by maximizing (3), or equivalently solving the following nonlinear equations:

$$\frac{\partial}{\partial\alpha}\ell(\alpha,\beta\,|\,\mathbf{x}) = n\log\beta - n\psi(\alpha) + \sum_{i=1}^{n}\log\left(-\log x_i\right)$$
$$\frac{\partial}{\partial\beta}\ell(\alpha,\beta\,|\,\mathbf{x}) = \frac{n\beta}{\alpha} + \sum_{i=1}^{n}\log x_i$$

where $\psi(\cdot)$ denotes the digamma function, define as $\psi(x) = \frac{d}{dx}\log\Gamma(x)$

2.2 Method of Maximum Product of Spacings

The maximum product of spacing (MPS) method was introduced by Cheng and Amin (1979, 1983) as an alternative to MLE for estimating parameters of continuous univariate distributions. Ranneby (1984) independently derived the same method as an approximation to the Kullback-Leibler measure of information.

The uniform spacing of a random sample from unit-Gamma distribution is defined as:

$D_i(\alpha,\beta) = F\left(x_{i:n}\,|\,\alpha,\beta\right) - F\left(x_{i-1:n}\,|\,\alpha,\beta\right)$ for $i = 1,\ldots,n, F\left(x_{0:n}\,|\,\alpha,\beta\right) = 0$ and $F\left(x_{n+1:n}\,|\,\alpha,\beta\right) = 1$. Clearly $\sum_{i=1}^{n+1} D_i(\alpha,\beta) = 1$.

From Cheng and Amin (1979, 1983), the MPSEs, $\hat{\alpha}_{MPS}$ and $\hat{\beta}_{MPS}$, are the values of α and β, which maximize the geometric mean of the spacing:

$$G(\alpha,\beta\,|\,\mathbf{x}) = \left[\prod_{i=1}^{n+1} D_i(\alpha,\beta)\right]^{\frac{1}{n+1}} \tag{6}$$

$$H(\alpha,\beta\,|\,\mathbf{x}) = \frac{1}{n+1}\sum_{i=1}^{n+1}\log D_i(\alpha,\beta) \tag{7}$$

The estimators $\hat{\alpha}_{MPS}$ and $\hat{\beta}_{MPS}$ of the parameters α and β can also be obtained by solving the nonlinear equations:

$$\frac{\partial}{\partial\alpha}H(\alpha,\beta)=\frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\left[\Delta_1\left(x_{i:n}\mid\alpha,\beta\right)-\Delta_1\left(x_{i-1:n}\mid\alpha,\beta\right)\right]=0$$

$$\frac{\partial}{\partial\beta}H(\alpha,\beta)=\frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\left[\Delta_2\left(x_{i:n}\mid\alpha,\beta\right)-\Delta_2\left(x_{i-1:n}\mid\alpha,\beta\right)\right]=0$$

where

$$\Delta_1\left(x_{i:n}\mid\alpha,\beta\right)=\frac{\partial}{\partial\alpha}F\left(x_{i:n}\mid\alpha,\beta\right) \qquad (8)$$

And

$$\Delta_2\left(x_{i:n}\mid\alpha,\beta\right)=\frac{\partial}{\partial\beta}F\left(x_{i:n}\mid\alpha,\beta\right) \qquad (9)$$

which must be obtained numerically, $F(\cdot)$ is defined in Equation (2).

It is noteworthy that the MPSE is as efficient as ML estimation and consistent under more general conditions than the ML estimators (Cheng and Amin, 1983)

2.3Method of Moments

Another technique fairly simple and commonly used in the parametric estimation is the method of moments (MOM). Grassia (1977) showed that the moment of order r about the origin of (1)is given by:

$$\mu_r=\mathbb{E}\left(X^r\right)=\left(\frac{\beta}{\beta+r}\right)^{\alpha} \qquad (10)$$

The moment estimators can be obtained by equating the first two moments (10) of unit-Gamma distribution to their counterparts sample moments, that is,

$$\mu_1=\left(\frac{\beta}{\beta+1}\right)^{\alpha}=m_1$$

$$\mu_2=\left(\frac{\beta}{\beta+2}\right)^{\alpha}=m_2$$

where $m_1=n^{-1}\sum_{i=1}^{n}x_i$ and $m_2=n^{-1}\sum_{i=1}^{n}x_i^2$

2.4 Methods of Least Squares

The least square methods were originally proposed by Swain et al. (1988) to estimate the parameters of the Beta distributions. Suppose that $F(X_{(i)})$ denotes the distribution function of the order statistics from the random sample $x=(x_1,x_2,...x_n)$. An important result from probability shows that $F(X_{(i)}) \sim Beta(i,n-i+1)$. Therefore, we have

$$\mathbb{E}\left[F\left(X_{(i)}\right)\right]=\frac{i}{n+1} \quad \text{and} \quad \mathrm{Var}\left[F\left(X_{(i)}\right)\right]=\frac{i(n-i+1)}{(n+1)^2(n+2)} \tag{11}$$

for further details see Johnson et al. (1995). Using the expectations and variances, we obtain two variants of the least squares methods.

2.4.1 Ordinary Least Squares

In case of unit-Gamma distribution, the ordinary least square estimators $\hat{\alpha}_{OLS}$ and $\hat{\beta}_{OLS}$ of the parameters α and β can be obtained by minimizing the function:

$$S(\alpha,\beta\,|\,\mathbf{x})=\sum_{i=1}^{n}\left[F\left(x_{i:n}\,|\,\alpha,\beta\right)-\frac{i}{n+1}\right]^2 \tag{12}$$

with respect to α and β. Alternatively, these estimates can also be obtained by solving the following nonlinear equations:

$$\sum_{i=1}^{n}\left[F\left(x_{i:n}\,|\,\alpha,\beta\right)-\frac{i}{n+1}\right]\Delta_1\left(x_{i:n}\,|\,\alpha,\beta\right)=0$$
$$\sum_{i=1}^{n}\left[F\left(x_{i:n}\,|\,\alpha,\beta\right)-\frac{i}{n+1}\right]\Delta_2\left(x_{i:n}\,|\,\alpha,\beta\right)=0$$

2.4.2 Weighted Least Squares

For the unit-Gamma distribution, the weighted least square estimators of α and β. say $\hat{\alpha}_{WLS}$ and $\hat{\beta}_{WLS}$, respectively are obtained by minimizing the function:

$$W(\alpha,\beta \mid \mathbf{x}) = \sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F\left(x_{i:n} \mid \alpha,\beta\right) - \frac{i}{n+1} \right]^2 \tag{13}$$

with respect to α and β. Equivalently, these estimators are the solution of the following nonlinear equations:

$$\sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F\left(x_{i:n} \mid \alpha,\beta\right) - \frac{i}{n+1} \right] \Delta_1\left(x_{i:n} \mid \alpha,\beta\right) = 0$$

$$\sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F\left(x_{i:n} \mid \alpha,\beta\right) - \frac{i}{n+1} \right] \Delta_2\left(x_{i:n} \mid \alpha,\beta\right) = 0$$

where $\Delta_1(\cdot \mid \alpha,\beta)$ and $\Delta_2(\cdot \mid \alpha,\beta)$ are defined in Equations (8) and (9), respectively.

2.5 Methods of Minimum Distances

Here, we will discuss some methods based on the test statistics of Cramer-von Mises, Anderson-distance between the theoretical and empirical cumulative distribution functions (see for further details e.g., D'Agostino and Stephens, 1986; Luce~no, 2006). The expressions for each method are presented in Table 1.

Table 1: Expression for the methods based on the minimum distances

Acronyms	Expressions
CvM	$W_n^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left(x_{i:n} - \frac{2i-1}{2n}\right)^2$
AD	$A_n^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\log x_{i:n} + \log\left(1 - x_{(n+1-i)}\right)\right]$
ADR	$R_n^2 = \frac{n}{2} - 2\sum_{i=1}^{m} x_{i:n} - \frac{1}{n}\sum_{i=1}^{N}(2i-1)\log\left(1 - x_{(n+1-i)}\right)$
ADR2	$r_n^2 = 2\sum_{i=1}^{n}\log\left(1 - x_{i:n}\right) + \frac{1}{n}\sum_{i=1}^{n}\frac{2i-1}{1 - x_{(n+1-i)}}$
AD2L	$l_n^2 = 2\sum_{i=1}^{n}\log x_{i:n} + \frac{1}{n}\sum_{i=1}^{n}\frac{2i-1}{x_{i:n}}$

AD2	$a_n^2 = 2\sum_{i=1}^{n}\left[\log x_{i:n} + \log\left(1 - x_{i:n}\right)\right] + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{2i-1}{x_{i:n}} + \frac{2i-1}{1-x_{(n+1-i)}}\right)$

For illustrative purposes, we have presented only the expressions used for the estimation of the parameters for the Cramer-von-Mises and Anderson-Darling methods.

2.5.1 Method of Cramer-von-Mises

In regard to unit-Gamma distribution, the Cramer-von- Mises estimates α_{CvM} and β_{CvM} are obtained by minimizing with respect to α and β the function:

$$C(\alpha, \beta \mid \mathbf{x}) = \frac{1}{12n} + \sum_{i=1}^{n}\left(F\left(x_{i:n} \mid \alpha, \beta\right) - \frac{2i-1}{2n}\right)^2 \qquad (14)$$

The estimators can also be obtained by solving the following nonlinear equations:

$$\sum_{i=1}^{n}\left(F\left(x_{i:n} \mid \alpha, \beta\right) - \frac{2i-1}{2n}\right)\Delta_1\left(x_{i:n} \mid \alpha, \beta\right) = 0$$

$$\sum_{i=1}^{n}\left(F\left(x_{i:n} \mid \alpha, \beta\right) - \frac{2i-1}{2n}\right)\Delta_2\left(x_{i:n} \mid \alpha, \beta\right) = 0$$

where $\Delta_1(\cdot \mid \alpha, \beta)$ and $\Delta_2(\cdot \mid \alpha, \beta)$ are speci ed in Equations (8) and (9), respectively.

2.5.2 Method of Anderson-Darling

Anderson and Darling (1952) developed a test, as an alternative to statistical tests for detecting sample distributions departure from normality. Using these test statistics, we can obtain the Anderson-Darling estimates, α_{ADE} and β_{ADE}, by minimizing the function

$$A(\alpha, \beta \mid \mathbf{x}) = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left\{\log F\left(x_{i:n} \mid \alpha, \beta\right) + \log \bar{F}\left(x_{(n+1-i)} \mid \alpha, \beta\right)\right\} \qquad (15)$$

with respect to α and β. Equivalently, these estimators are the solution of the following nonlinear equations:

$$\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_1\left(x_{i:n}\mid\alpha,\beta\right)}{F\left(x_{i:n}\mid\alpha,\beta\right)}-\frac{\Delta_1\left(x_{(n+1-i)}\mid\alpha,\beta\right)}{\bar{F}\left(x_{(n+1-i)}\mid\alpha,\beta\right)}\right]=0$$

$$\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_2\left(x_{i:n}\mid\alpha,\beta\right)}{F\left(x_{i:n}\mid\alpha,\beta\right)}-\frac{\Delta_2\left(x_{(n+1-i)}\mid\alpha,\beta\right)}{\bar{F}\left(x_{(n+1-i)}\mid\alpha,\beta\right)}\right]=0$$

where $\Delta_1(\cdot\mid\alpha,\beta)$ and $\Delta_2(\cdot\mid\alpha,\beta)$ are specified in Equations (8) and (9), respectively.

3 Monte Carlo Simulations

In this section, we conduct Monte Carlo simulation studies to compare the performance of the estimators discussed in the previous sections. We evaluate the performance of the estimators based on bias and root mean squared errors (RMSE), for different sample sizes and parameter values. Moreover, we also calculate the parametric bootstrap confidence intervals for each method and evaluate the coverage probability (CP) and the average length (AW) of the simulated confidence intervals. We have taken sample sizes of n = 20; 50; 100 and 200 and the following parameter values: α = 0:5; 1:0 and 2:0 and β = 0:5; 1:0; 2:0 and 3:0. For each scenario, the number of Monte Carlo simulations is set at 10,000 and the parametric bootstrap replications is fixed at 1000. To generates random samples from the UG distribution, we consider the transformation $X = e^{-Y}$, where $Y \sim \text{Gamma}(\alpha,\beta)$. Simulated bias, RMSE, CP and AW for the estimates are presented in Tables 2{13. Asuperscript indicate the rank of each of the estimators among all the estimators for that metric. For example, Table 2 shows the bias of MLE($\hat{\alpha}$) as 0:1259 for n = 20. This indicates, bias of $\hat{\alpha}$ obtained using the method of maximum likelihood ranks 9th among all other estimators. Table14 shows the partial and overall rank of the estimators. The Table 14 is used to find the over all performance of estimation techniques.

The following observations can be drawn from the Tables 2-13.

1. All the estimators show the property of consistency i.e., the RMSE decreases as sample size increases.

2. The bias of $\hat{\alpha}$ decreases with increasing n for all the methods of estimation.

3. The bias of $\hat{\beta}$ decreases with increasing n for all the methods of estimation.

4. The bias of $\hat{\alpha}$ generally increases with increasing α for any given α and n and for all methods of estimation $\hat{\beta}$.

5. In terms of RMSE, all the methods of estimation produces smaller RMSE or $\hat{\alpha}$ compared to that of β.

6. In terms of performance of the methods of estimation, we found that maximum product spacing (MPS) estimators is the best as it produces the least biases of the estimates with least RMSE for most of the configurations considered in our studies. The next best method is the AD2, followed by MLE. AD method ranked 4th while WLSE ranked 5th.AD2L ranked 11th among the eleven methods of estimation. The overall positions of the estimators are presented in Table 14, from which we confirm the superiority of MPS and AD2.

Table 2: Simulation results for = 0.5 and = 0.5.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.058^{4}	-0.008^{1}	0.246^{11}	0.111^{8}	0.097^{6}	0.158^{10}	0.125^{9}	0.107^{7}	-0.070^{5}	0.036^{2}	0.046^{3}
	RMSE(α)	0.128^{5}	0.012^{1}	0.298^{9}	0.519^{11}	0.288^{8}	0.332^{10}	0.249^{7}	0.216^{6}	-0.103^{3}	0.095^{2}	0.109^{4}
	Bias(β)	0.336^{3}	0.302^{2}	0.818^{11}	0.498^{10}	0.394^{6}	0.491^{9}	0.360^{4}	0.416^{8}	0.277^{1}	0.394^{7}	0.377^{5}
	RMSE(β)	0.591^{3}	0.506^{2}	0.953^{10}	1.665^{11}	0.944^{9}	0.941^{8}	0.644^{4}	0.723^{7}	0.453^{1}	0.723^{6}	0.684^{5}
	CP(α)	0.937^{8}	0.942^{9}	0.891^{3}	0.933^{7}	0.921^{5}	0.890^{2}	0.884^{1}	0.921^{6}	0.902^{4}	0.945^{11}	0.944^{10}
	CP(β)	0.940^{8}	0.955^{11}	0.877^{2}	0.927^{7}	0.913^{5}	0.884^{3}	0.875^{1}	0.915^{6}	0.904^{4}	0.953^{10}	0.947^{9}
	AW(α)	0.670^{3}	0.577^{2}	1.605^{11}	0.969^{9}	0.796^{7}	0.992^{10}	0.740^{5}	0.850^{8}	0.474^{1}	0.745^{6}	0.713^{4}
	AW (β)	1.186^{3}	0.974^{2}	1.902^{8}	3.169^{11}	1.914^{9}	1.920^{10}	1.339^{5}	1.495^{7}	0.761^{1}	1.362^{6}	1.294^{4}
	Total	37^{4}	30^{2}	65^{10}	74^{11}	55^{7}	62^{9}	36^{3}	55^{7}	20^{1}	50^{6}	44^{5}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.020^{4}	-0.032^{5}	0.056^{11}	0.014^{2}	0.033^{6}	0.050^{10}	0.045^{8}	0.035^{7}	-0.050^{9}	0.008^{1}	0.017^{3}
	RMSE(α)	0.048^{4}	-0.041^{2}	0.061^{5}	0.129^{11}	0.100^{9}	0.111^{10}	0.091^{8}	0.076^{6}	-0.077^{7}	0.030^{1}	0.044^{3}
	Bias(β)	0.186^{4}	0.181^{2}	0.326^{11}	0.254^{10}	0.204^{6}	0.224^{9}	0.186^{3}	0.223^{8}	0.168^{1}	0.206^{7}	0.192^{5}
	RMSE(β)	0.313^{4}	0.282^{2}	0.386^{8}	0.715^{11}	0.433^{10}	0.410^{9}	0.308^{3}	0.369^{7}	0.265^{1}	0.366^{6}	0.334^{5}
	CP(α)	0.948^{8}	0.918^{2}	0.949^{10}	0.939^{5}	0.941^{6}	0.934^{4}	0.928^{3}	0.944^{7}	0.890^{1}	0.949^{9}	0.950^{11}
	CP(β)	0.948^{8}	0.932^{4}	0.950^{9}	0.954^{11}	0.935^{5}	0.919^{3}	0.917^{2}	0.936^{6}	0.895^{1}	0.950^{10}	0.947^{7}
	AW(α)	0.371^{4}	0.338^{2}	0.663^{11}	0.498^{10}	0.407^{6}	0.454^{9}	0.371^{3}	0.447^{8}	0.297^{1}	0.408^{7}	0.383^{5}
	AW (β)	0.620^{4}	0.532^{2}	0.794^{8}	1.381^{11}	0.848^{10}	0.806^{9}	0.619^{3}	0.730^{7}	0.464^{1}	0.702^{6}	0.652^{5}
	Total	40^{4}	21^{1}	73^{11}	71^{10}	58^{8}	63^{9}	33^{3}	56^{7}	22^{2}	47^{6}	44^{5}

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.009^{4}	-0.032^{10}	0.008^{3}	-0.008^{2}	0.015^{6}	0.023^{9}	0.021^{8}	0.016^{7}	-0.034^{11}	0.002^{1}	0.009^{5}
	RMSE(α)	0.020^{3}	-0.047^{9}	0.004^{1}	0.029^{5}	0.045^{8}	0.049^{10}	0.042^{7}	0.033^{6}	-0.056^{11}	0.011^{2}	0.020^{4}
	Bias(β)	0.128^{3}	0.131^{5}	0.212^{11}	0.174^{10}	0.137^{6}	0.149^{8}	0.124^{2}	0.153^{9}	0.119^{1}	0.143^{7}	0.131^{4}
	RMSE(β)	0.211^{4}	0.206^{3}	0.258^{9}	0.441^{11}	0.270^{10}	0.257^{8}	0.201^{2}	0.246^{7}	0.189^{1}	0.243^{6}	0.218^{5}
	CP(α)	0.947^{8}	0.900^{1}	0.952^{11}	0.932^{3}	0.944^{7}	0.939^{4}	0.942^{5}	0.944^{6}	0.902^{2}	0.948^{10}	0.948^{9}
	CP(β)	0.944^{8}	0.896^{1}	0.953^{11}	0.939^{5}	0.939^{5}	0.934^{4}	0.925^{3}	0.941^{7}	0.897^{2}	0.947^{10}	0.944^{9}
	AW(α)	0.251^{4}	0.243^{2}	0.427^{11}	0.339^{10}	0.270^{6}	0.292^{8}	0.244^{3}	0.299^{9}	0.215^{1}	0.277^{7}	0.257^{5}
	AW (β)	0.411^{4}	0.374^{2}	0.514^{9}	0.853^{11}	0.531^{10}	0.499^{8}	0.395^{3}	0.477^{7}	0.333^{1}	0.466^{6}	0.424^{5}
	Total	38^{4}	33^{2}	66^{11}	57^{7}	58^{8}	59^{10}	33^{2}	58^{8}	30^{1}	49^{6}	46^{5}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.004^{2}	-0.027^{11}	-0.011^{8}	-0.014^{9}	0.007^{4}	0.011^{7}	0.010^{6}	0.008^{5}	-0.022^{10}	0.001^{1}	0.004^{3}
	RMSE(α)	0.009^{2}	-0.042^{11}	-0.017^{6}	-0.012^{4}	0.021^{8}	0.023^{9}	0.020^{7}	0.016^{5}	-0.036^{10}	0.004^{1}	0.009^{3}
	Bias(β)	0.089^{3}	0.097^{6}	0.153^{11}	0.125^{10}	0.094^{5}	0.100^{8}	0.085^{2}	0.105^{9}	0.084^{1}	0.098^{7}	0.089^{4}
	RMSE(β)	0.142^{3}	0.149^{5}	0.186^{10}	0.304^{11}	0.178^{9}	0.166^{8}	0.134^{2}	0.163^{7}	0.131^{1}	0.161^{6}	0.144^{4}
	CP(α)	0.950^{11}	0.889^{1}	0.941^{4}	0.926^{3}	0.945^{6}	0.942^{5}	0.946^{7}	0.946^{7}	0.913^{2}	0.947^{9}	0.949^{10}
	CP(β)	0.954^{10}	0.890^{1}	0.938^{4}	0.932^{3}	0.951^{8}	0.946^{6}	0.945^{5}	0.949^{7}	0.906^{2}	0.955^{11}	0.953^{9}
	AW(α)	0.174^{3}	0.178^{5}	0.302^{11}	0.243^{10}	0.185^{6}	0.198^{8}	0.166^{2}	0.207^{9}	0.155^{1}	0.193^{7}	0.176^{4}
	AW (β)	0.282^{4}	0.273^{3}	0.363^{10}	0.588^{11}	0.355^{9}	0.331^{8}	0.265^{2}	0.326^{7}	0.240^{1}	0.320^{6}	0.287^{5}
	Total	38^{3}	43^{5}	64^{11}	61^{10}	55^{7}	59^{9}	33^{2}	56^{8}	28^{1}	48^{6}	42^{4}
	Overall Total	15^{4}	10^{2}	43^{11}	38^{10}	30^{7}	37^{9}	10^{2}	30^{7}	5^{1}	24^{6}	19^{5}

Table 4: Simulation results for α = 2.0 and β = 0.5.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.070^{4}	-0.011^{1}	0.209^{10}	0.140^{7}	0.116^{6}	0.195^{9}	0.166^{8}	0.251^{11}	-0.087^{5}	0.024^{2}	0.036^{3}
	RMSE(α)	0.085^{4}	-0.014^{1}	0.206^{8}	0.229^{9}	0.165^{6}	0.233^{10}	0.191^{7}	0.376^{11}	-0.103^{5}	0.030^{2}	0.046^{3}
	Bias(β)	0.388^{3}	0.344^{2}	0.649^{10}	0.557^{9}	0.461^{7}	0.545^{8}	0.429^{6}	0.673^{11}	0.320^{1}	0.424^{5}	0.402^{4}
	RMSE(β)	0.450^{3}	0.395^{2}	0.650^{9}	0.799^{10}	0.594^{7}	0.642^{8}	0.492^{6}	0.967^{11}	0.364^{1}	0.490^{5}	0.469^{4}
	CP(α)	0.939^{7}	0.941^{8}	0.913^{5}	0.944^{9}	0.930^{6}	0.893^{4}	0.877^{2}	0.867^{1}	0.881^{3}	0.956^{11}	0.951^{10}
	CP(β)	0.935^{7}	0.940^{8}	0.896^{5}	0.940^{9}	0.921^{6}	0.889^{4}	0.876^{2}	0.854^{1}	0.883^{3}	0.958^{11}	0.951^{10}
	AW(α)	2.998^{3}	2.563^{2}	4.732^{11}	4.232^{10}	3.531^{7}	4.042^{9}	3.323^{6}	4.001^{8}	2.078^{1}	3.217^{5}	3.105^{4}
	AW(β)	0.866^{3}	0.732^{2}	1.167^{8}	1.567^{11}	1.140^{7}	1.198^{9}	0.961^{6}	1.518^{10}	0.590^{1}	0.937^{5}	0.902^{4}
	Total	34^{3}	26^{2}	66^{10}	74^{11}	52^{7}	61^{8}	43^{5}	64^{9}	20^{1}	46^{6}	42^{4}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.025^{3}	-0.044^{6}	0.069^{8}	0.035^{4}	0.044^{5}	0.071^{10}	0.060^{7}	0.079^{11}	-0.069^{9}	0.003^{1}	0.022^{2}
	RMSE(α)	0.030^{3}	-0.053^{4}	0.059^{5}	0.064^{7}	0.063^{6}	0.088^{10}	0.069^{8}	0.114^{11}	-0.079^{9}	0.005^{1}	0.026^{2}
	Bias(β)	0.223^{4}	0.203^{2}	0.386^{11}	0.313^{9}	0.244^{7}	0.277^{8}	0.219^{3}	0.349^{10}	0.187^{1}	0.235^{6}	0.227^{5}
	RMSE(β)	0.254^{4}	0.233^{2}	0.372^{9}	0.438^{10}	0.307^{7}	0.321^{8}	0.248^{3}	0.506^{11}	0.213^{1}	0.271^{6}	0.260^{5}

CP(α)	0.941^{7}	0.905^{2}	0.944^{8}	0.950^{10}	0.939^{6}	0.926^{5}	0.916^{4}	0.915^{3}	0.871^{1}	0.956^{11}	0.948^{9}

Table 3: Simulation results for α = 1.0 and β = 0.5.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.069^{4}	-0.000^{1}	0.252^{11}	0.158^{9}	0.120^{6}	0.187^{10}	0.156^{8}	0.141^{7}	-0.079^{5}	0.033^{2}	0.040^{3}
	RMSE(α)	0.098^{4}	0.007^{1}	0.261^{10}	0.357^{11}	0.206^{6}	0.260^{9}	0.219^{8}	0.207^{7}	-0.108^{5}	0.051^{2}	0.063^{3}
	Bias(β)	0.366^{3}	0.330^{2}	0.796^{11}	0.632^{10}	0.455^{7}	0.557^{9}	0.408^{5}	0.469^{8}	0.293^{1}	0.419^{6}	0.407^{4}
	RMSE(β)	0.497^{3}	0.435^{2}	0.809^{10}	1.241^{11}	0.704^{8}	0.757^{9}	0.551^{5}	0.652^{7}	0.381^{1}	0.564^{6}	0.545^{4}
20	CP(α)	0.939^{9}	0.939^{8}	0.891^{3}	0.925^{7}	0.924^{6}	0.891^{2}	0.875^{1}	0.908^{5}	0.893^{4}	0.948^{11}	0.946^{10}
	CP(β)	0.938^{8}	0.944^{9}	0.871^{2}	0.928^{7}	0.920^{6}	0.891^{4}	0.870^{1}	0.913^{5}	0.884^{3}	0.954^{11}	0.953^{10}
	AW(α)	1.468^{3}	1.247^{2}	3.142^{11}	2.351^{10}	1.807^{7}	2.227^{9}	1.656^{6}	1.977^{8}	1.000^{1}	1.632^{5}	1.549^{4}
	AW(β)	0.974^{3}	0.818^{2}	1.607^{10}	2.319^{11}	1.424^{8}	1.528^{9}	1.110^{6}	1.379^{7}	0.639^{1}	1.099^{5}	1.044^{4}
	Total	37^{3}	27^{2}	68^{10}	76^{11}	54^{7}	61^{9}	40^{4}	54^{7}	21^{1}	48^{6}	42^{5}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.025^{3}	-0.035^{5}	0.064^{10}	0.027^{4}	0.039^{6}	0.069^{11}	0.055^{8}	0.055^{7}	-0.058^{9}	0.009^{1}	0.021^{2}
	RMSE(α)	0.032^{3}	-0.045^{4}	0.059^{5}	0.077^{7}	0.069^{6}	0.094^{11}	0.077^{8}	0.078^{9}	-0.079^{10}	0.015^{1}	0.029^{2}
	Bias(β)	0.205^{3}	0.196^{2}	0.371^{11}	0.286^{10}	0.224^{6}	0.261^{9}	0.209^{4}	0.242^{8}	0.182^{1}	0.225^{7}	0.215^{5}
	RMSE(β)	0.266^{3}	0.250^{2}	0.386^{10}	0.514^{11}	0.341^{7}	0.344^{8}	0.272^{4}	0.345^{9}	0.234^{1}	0.303^{6}	0.278^{5}
50	CP(α)	0.942^{8}	0.914^{3}	0.945^{9}	0.946^{10}	0.938^{6}	0.920^{4}	0.913^{2}	0.930^{5}	0.878^{1}	0.954^{11}	0.942^{7}
	CP(β)	0.943^{7}	0.916^{4}	0.945^{8}	0.951^{10}	0.934^{6}	0.916^{3}	0.910^{2}	0.931^{5}	0.871^{1}	0.954^{11}	0.950^{9}
	AW(α)	0.805^{3}	0.718^{2}	1.466^{11}	1.131^{10}	0.899^{7}	1.019^{9}	0.806^{4}	0.962^{8}	0.627^{1}	0.894^{6}	0.836^{5}
	AW(β)	0.524^{3}	0.460^{2}	0.752^{10}	1.015^{11}	0.668^{7}	0.679^{8}	0.527^{4}	0.689^{9}	0.398^{1}	0.592^{6}	0.549^{5}
	Total	33^{3}	24^{1}	74^{11}	73^{10}	51^{7}	63^{9}	36^{4}	60^{8}	25^{2}	49^{6}	40^{5}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.012^{4}	-0.040^{11}	0.013^{5}	-0.006^{2}	0.021^{6}	0.034^{9}	0.028^{8}	0.024^{7}	-0.037^{10}	0.004^{1}	0.009^{3}
	RMSE(α)	0.014^{5}	-0.051^{10}	0.009^{3}	0.004^{1}	0.035^{6}	0.044^{9}	0.041^{8}	0.038^{7}	-0.052^{11}	0.006^{2}	0.014^{4}
	Bias(β)	0.138^{3}	0.141^{5}	0.242^{11}	0.195^{10}	0.152^{6}	0.166^{9}	0.135^{2}	0.159^{8}	0.127^{1}	0.153^{7}	0.139^{4}
	RMSE(β)	0.180^{3}	0.181^{4}	0.255^{10}	0.340^{11}	0.220^{8}	0.219^{7}	0.178^{2}	0.232^{9}	0.163^{1}	0.205^{6}	0.182^{5}
100	CP(α)	0.947^{8}	0.888^{1}	0.949^{9}	0.935^{5}	0.944^{7}	0.929^{3}	0.934^{4}	0.942^{6}	0.896^{2}	0.949^{10}	0.951^{11}
	CP(β)	0.946^{8}	0.886^{1}	0.948^{10}	0.939^{6}	0.944^{7}	0.933^{4}	0.928^{3}	0.938^{5}	0.886^{2}	0.946^{8}	0.951^{11}
	AW(α)	0.544^{4}	0.513^{2}	0.945^{11}	0.748^{10}	0.593^{6}	0.648^{9}	0.526^{3}	0.624^{8}	0.454^{1}	0.606^{7}	0.555^{5}
	AW(β)	0.352^{4}	0.328^{2}	0.491^{10}	0.651^{11}	0.432^{8}	0.427^{7}	0.341^{3}	0.452^{9}	0.289^{1}	0.399^{6}	0.363^{5}
	Total	39^{4}	36^{3}	69^{11}	56^{8}	54^{7}	57^{9}	33^{2}	59^{10}	29^{1}	47^{5}	48^{6}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.008^{3}	-0.034^{11}	-0.009^{4}	-0.017^{9}	0.010^{5}	0.015^{8}	0.014^{7}	0.011^{6}	-0.024^{10}	0.002^{1}	0.005^{2}
	RMSE(α)	0.009^{2}	-0.044^{11}	-0.013^{4}	-0.024^{9}	0.015^{5}	0.020^{8}	0.020^{7}	0.017^{6}	-0.034^{10}	0.002^{1}	0.009^{3}
	Bias(β)	0.097^{3}	0.105^{6}	0.173^{11}	0.139^{10}	0.104^{5}	0.110^{9}	0.092^{2}	0.110^{8}	0.090^{1}	0.108^{7}	0.099^{4}
200	RMSE(β)	0.125^{3}	0.134^{5}	0.181^{10}	0.240^{11}	0.149^{8}	0.145^{7}	0.120^{2}	0.160^{9}	0.115^{1}	0.142^{6}	0.128^{4}

	CP(α)	0.947^{9}	0.874^{1}	0.933^{4}	0.923^{3}	0.947^{10}	0.940^{7}	0.937^{5}	0.939^{6}	0.905^{2}	0.949^{11}	0.941^{8}
	CP(β)	0.946^{9}	0.875^{1}	0.933^{4}	0.923^{3}	0.947^{10}	0.944^{7}	0.935^{5}	0.941^{6}	0.898^{2}	0.944^{8}	0.947^{11}
	AW(α)	0.377^{3}	0.377^{4}	0.668^{11}	0.531^{10}	0.405^{6}	0.434^{9}	0.358^{2}	0.425^{8}	0.328^{1}	0.421^{7}	0.381^{5}
	AW(β)	0.244^{4}	0.241^{3}	0.348^{10}	0.460^{11}	0.291^{8}	0.285^{7}	0.230^{2}	0.309^{9}	0.210^{1}	0.276^{6}	0.248^{5}
	Total	36^{3}	42^{4}	58^{8}	66^{11}	57^{7}	62^{10}	32^{2}	58^{8}	28^{1}	47^{6}	42^{4}
	Overall Total	13^{4}	10^{2}	40^{10}	40^{10}	28^{7}	37^{9}	12^{3}	33^{8}	5^{1}	23^{6}	20^{5}
	CP(β)	0.941^{7}	0.905^{2}	0.947^{8}	0.951^{10}	0.941^{6}	0.919^{5}	0.914^{4}	0.907^{3}	0.873^{1}	0.961^{11}	0.951^{9}
	AW(α)	1.707^{3}	1.501^{2}	2.982^{11}	2.481^{10}	1.949^{7}	2.185^{8}	1.712^{4}	2.393^{9}	1.305^{1}	1.904^{6}	1.783^{5}
	AW(β)	0.488^{4}	0.426^{2}	0.719^{9}	0.877^{10}	0.607^{7}	0.633^{8}	0.488^{3}	0.882^{11}	0.370^{1}	0.548^{6}	0.511^{5}
	Total	35^{3}	22^{1}	69^{9}	70^{11}	51^{7}	62^{8}	36^{4}	69^{9}	24^{2}	48^{6}	42^{5}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.012^{3}	-0.044^{10}	0.013^{5}	0.002^{2}	0.024^{7}	0.036^{9}	0.031^{8}	0.022^{6}	-0.045^{11}	0.002^{1}	0.012^{4}
	RMSE(α)	0.012^{4}	-0.052^{11}	0.009^{2}	0.010^{3}	0.033^{7}	0.045^{9}	0.036^{8}	0.029^{6}	-0.051^{10}	0.002^{1}	0.014^{5}
	Bias(β)	0.148^{3}	0.150^{4}	0.258^{11}	0.215^{10}	0.165^{7}	0.177^{8}	0.143^{2}	0.205^{9}	0.134^{1}	0.162^{6}	0.151^{5}
	RMSE(β)	0.169^{3}	0.172^{4}	0.252^{9}	0.297^{10}	0.202^{7}	0.206^{8}	0.164^{2}	0.305^{11}	0.153^{1}	0.187^{6}	0.173^{5}
	CP(α)	0.941^{8}	0.881^{1}	0.945^{9}	0.934^{6}	0.941^{7}	0.933^{5}	0.932^{4}	0.928^{3}	0.884^{2}	0.951^{11}	0.946^{10}
	CP(β)	0.946^{10}	0.870^{1}	0.943^{8}	0.938^{6}	0.940^{7}	0.931^{5}	0.930^{4}	0.927^{3}	0.883^{2}	0.952^{11}	0.946^{9}
	AW(α)	1.153^{4}	1.081^{2}	1.993^{11}	1.656^{10}	1.283^{6}	1.392^{8}	1.114^{3}	1.555^{9}	0.950^{1}	1.297^{7}	1.185^{5}
	AW(β)	0.329^{4}	0.307^{2}	0.485^{9}	0.579^{11}	0.395^{7}	0.402^{8}	0.318^{3}	0.572^{10}	0.270^{1}	0.373^{6}	0.339^{5}
	Total	39^{4}	35^{3}	64^{11}	58^{9}	55^{7}	60^{10}	34^{2}	57^{8}	29^{1}	49^{6}	48^{5}
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.007^{4}	-0.035^{11}	-0.011^{5}	-0.013^{7}	0.012^{6}	0.018^{9}	0.015^{8}	-0.001^{1}	-0.028^{10}	0.002^{2}	0.006^{3}
	RMSE(α)	0.007^{4}	-0.042^{11}	-0.012^{5}	-0.015^{6}	0.016^{7}	0.022^{9}	0.018^{8}	-0.006^{2}	-0.032^{10}	0.002^{1}	0.007^{3}
	Bias(β)	0.103^{4}	0.111^{6}	0.180^{11}	0.155^{10}	0.109^{5}	0.120^{9}	0.097^{2}	0.118^{8}	0.094^{1}	0.116^{7}	0.103^{3}
	RMSE(β)	0.118^{4}	0.127^{5}	0.175^{9}	0.216^{11}	0.134^{7}	0.138^{8}	0.110^{2}	0.181^{10}	0.107^{1}	0.133^{6}	0.117^{3}
	CP(α)	0.944^{7}	0.871^{1}	0.936^{4}	0.923^{3}	0.948^{10}	0.941^{6}	0.939^{5}	0.948^{9}	0.905^{2}	0.947^{8}	0.948^{11}
	CP(β)	0.947^{8}	0.863^{1}	0.935^{5}	0.922^{3}	0.948^{11}	0.940^{6}	0.940^{6}	0.931^{4}	0.905^{2}	0.947^{9}	0.948^{10}
	AW(α)	0.799^{3}	0.800^{4}	1.414^{11}	1.169^{10}	0.873^{6}	0.932^{8}	0.755^{2}	1.004^{9}	0.690^{1}	0.901^{7}	0.810^{5}
	AW(β)	0.228^{4}	0.227^{3}	0.344^{9}	0.409^{11}	0.267^{7}	0.268^{8}	0.215^{2}	0.378^{10}	0.196^{1}	0.259^{6}	0.231^{5}
	Total	38^{3}	42^{4}	59^{8}	61^{10}	59^{8}	63^{11}	35^{2}	53^{7}	28^{1}	46^{6}	43^{5}
	Overall Total	13^{3}	10^{2}	38^{10}	41^{11}	29^{7}	37^{9}	13^{3}	33^{8}	5^{1}	24^{6}	19^{5}

Table 5: Simulation results for α = 0.5 and β = 1.0.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.0574	-0.0011	0.25511	0.0937	0.0906	0.15010	0.1288	0.1339	-0.0715	0.0262	0.0443
	RMSE(α)	0.1245	0.0241	0.30810	0.39811	0.2648	0.3079	0.2477	0.2346	-0.1053	0.0842	0.1114
	Bias(β)	0.3263	0.2992	0.77211	0.4479	0.3877	0.46510	0.3565	0.4328	0.2821	0.3766	0.3564
20	RMSE(β)	0.5843	0.5132	0.8678	1.25011	0.8889	0.90210	0.6344	0.7027	0.4411	0.6776	0.6595

	CP(α)	0.9387	0.9469	0.8912	0.94610	0.9286	0.9004	0.8901	0.9185	0.8963	0.94911	0.9468
	CP(β)	0.9447	0.95410	0.8701	0.9458	0.9146	0.8873	0.8752	0.9045	0.9024	0.95611	0.9509
	aw(α)	0.6643	0.5802	1.47811	0.8538	0.7587	0.93210	0.7366	0.8949	0.4731	0.7215	0.6994
	AW(β)	2.3273	1.9542	3.43910	4.69811	3.4209	3.4168	2.6256	2.9497	1.5201	2.5995	2.5104
	Total	35 3	292	649	7511	588	649	394	567	191	486	415
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0183	-0.0245	0.06511	0.0142	0.0326	0.05310	0.0478	0.0489	-0.0467	0.0101	0.0234
	RMSE(α)	0.0413	-0.0342	0.0685	0.12811	0.0999	0.10210	0.0928	0.0767	-0.0766	0.0311	0.0474
	Bias(β)	0.1853	0.1812	0.33811	0.25110	0.2036	0.2248	0.1914	0.2369	0.1691	0.2047	0.1935
	RMSE(β)	0.3083	0.2852	0.4079	0.71811	0.43810	0.3918	0.3174	0.3375	0.2701	0.3567	0.3386
	CP(α)	0.9468	0.9263	0.9436	0.9479	0.9447	0.9324	0.9232	0.9375	0.8971	0.94910	0.95211
	CP(β)	0.9458	0.9334	0.9509	0.95110	0.9366	0.9223	0.9122	0.9355	0.8831	0.95311	0.9457
	aw(α)	0.3703	0.3412	0.67011	0.49310	0.4076	0.4558	0.3714	0.4709	0.2991	0.4087	0.3865
	AW(β)	1.2343	1.0702	1.5948	2.67811	1.69310	1.5979	1.2394	1.4097	0.9271	1.4056	1.3035
	Total	343	222	7010	7411	608	608	364	567	191	506	475
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0093	-0.02911	0.0114	-0.0072	0.0176	0.0249	0.0207	0.0228	-0.02910	0.0011	0.0125
	RMSE(α)	0.0235	-0.0428	0.0041	0.0203	0.04910	0.0439	0.0427	0.0356	-0.05111	0.0132	0.0214
	Bias(β)	0.1293	0.1325	0.21811	0.17110	0.1356	0.1458	0.1252	0.1559	0.1201	0.1387	0.1294
	RMSE(β)	0.2114	0.2063	0.2629	0.42011	0.27410	0.2468	0.2032	0.2186	0.1891	0.2377	0.2155
	CP(α)	0.9457	0.9052	0.9499	0.9414	0.9445	0.9446	0.9313	0.9498	0.9051	0.95411	0.95210
	CP(β)	0.9446	0.9022	0.95210	0.9509	0.9405	0.9384	0.9313	0.9468	0.8931	0.95311	0.9447
	aw(α)	0.2524	0.2442	0.42811	0.33910	0.2706	0.2938	0.2443	0.3119	0.2161	0.2777	0.2585
	AW(β)	0.8254	0.7522	1.0279	1.68911	1.06610	0.9928	0.7893	0.9056	0.6681	0.9347	0.8475
	Total	364	353	6411	608	587	608	302	608	271	536	455
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0052	-0.02911	-0.0074	-0.0159	0.0105	0.0138	0.0106	0.0107	-0.02110	0.0001	0.0073
	RMSE(α)	0.0113	-0.04211	-0.0144	-0.0155	0.0249	0.0207	0.0218	0.0176	-0.03510	0.0081	0.0092
	Bias(β)	0.0893	0.0987	0.15311	0.12610	0.0945	0.1018	0.0862	0.1069	0.0851	0.0986	0.0894
	RMSE(β)	0.1443	0.1526	0.18510	0.30211	0.1819	0.1658	0.1392	0.1505	0.1331	0.1657	0.1464
	CP(α)	0.9456	0.8841	0.9455	0.9253	0.95010	0.9456	0.9394	0.9468	0.9142	0.95010	0.9509
	CP(β)	0.95111	0.8801	0.9448	0.9293	0.9479	0.9447	0.9344	0.9425	0.9052	0.94810	0.9446
	aw(α)	0.1753	0.1775	0.30311	0.24210	0.1866	0.1988	0.1662	0.2139	0.1551	0.1937	0.1774
	AW(β)	0.5664	0.5463	0.72810	1.17111	0.7119	0.6608	0.5312	0.6126	0.4821	0.6437	0.5735
	Total	35 3	455	6311	629	629	608	302	557	281	496	374
	Overall Total	134	122	4111	3910	328	339	122	297	41	246	195

Table 6: Simulation results for α = 1.0 and β = 1.0.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS

20	Bias(α)	0.0684	-0.0031	0.26711	0.1358	0.1176	0.17810	0.1469	0.1197	-0.0905	0.0292	0.0513
	RMSE(α)	0.0974	-0.0031	0.26710	0.31211	0.2218	0.2579	0.2087	0.1756	-0.1195	0.0442	0.0743
	Bias(β)	0.3663	0.3272	0.82611	0.55410	0.4418	0.5399	0.4014	0.4035	0.2931	0.4217	0.4136
	RMSE(β)	0.4793	0.4242	0.81710	1.04311	0.7118	0.7379	0.5344	0.5525	0.3781	0.5587	0.5546
	CP(α)	0.9449	0.9428	0.8903	0.9387	0.9246	0.8934	0.8841	0.9145	0.8872	0.95111	0.94410
	CP(β)	0.9428	0.9459	0.8772	0.9307	0.9156	0.8894	0.8681	0.9105	0.8833	0.95611	0.94810
	aw(α)	1.4593	1.2432	3.08411	2.0909	1.7588	2.14910	1.6346	1.7037	0.9861	1.6005	1.5524
	AW(β)	1.9403	1.6182	3.11210	3.91911	2.7738	2.9309	2.1886	2.2847	1.2641	2.1475	2.0764
	Total	373	272	6810	7411	588	649	384	476	191	507	465
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0192	-0.0335	0.06911	0.0223	0.0396	0.0609	0.0538	0.0507	-0.06210	0.0101	0.0234
	RMSE(α)	0.0282	-0.0444	0.0625	0.0737	0.0738	0.08511	0.0769	0.0696	-0.08310	0.0151	0.0333
	Bias(β)	0.2003	0.1942	0.37411	0.28810	0.2248	0.2509	0.2054	0.2237	0.1781	0.2216	0.2125
	RMSE(β)	0.2613	0.2492	0.38210	0.51111	0.3378	0.3379	0.2694	0.2906	0.2321	0.2987	0.2805
	CP(α)	0.94910	0.9172	0.9406	0.9437	0.9448	0.9274	0.9203	0.9365	0.8841	0.95211	0.9448
	CP(β)	0.9478	0.9183	0.9467	0.95311	0.9406	0.9224	0.9122	0.9385	0.8771	0.95010	0.9489
	aw(α)	0.8003	0.7192	1.47311	1.12110	0.9008	1.0109	0.8044	0.8806	0.6241	0.8967	0.8385
	AW(β)	1.0453	0.9202	1.50810	2.01711	1.3438	1.3509	1.0524	1.1596	0.7941	1.1847	1.1025
	Total	343	221	7111	7010	608	649	384	486	262	507	445
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0062	-0.03610	0.0114	-0.0073	0.0216	0.0289	0.0268	0.0227	-0.03911	0.0061	0.0145
	RMSE(α)	0.0124	-0.04810	0.0051	0.0072	0.0387	0.0399	0.0388	0.0326	-0.05411	0.0073	0.0215
	Bias(β)	0.1373	0.1404	0.24011	0.19010	0.1517	0.1619	0.1342	0.1476	0.1281	0.1548	0.1445
	RMSE(β)	0.1804	0.1803	0.24810	0.33311	0.2209	0.2128	0.1762	0.1916	0.1651	0.2017	0.1895
	CP(α)	0.94810	0.8972	0.9469	0.9394	0.9427	0.9395	0.9353	0.9438	0.8931	0.95111	0.9426
	CP(β)	0.95011	0.8992	0.9499	0.9447	0.9426	0.9394	0.9293	0.9425	0.8821	0.95010	0.9448
	aw(α)	0.5404	0.5152	0.94311	0.74710	0.5937	0.6449	0.5253	0.5806	0.4531	0.6078	0.5595
	AW(β)	0.7044	0.6562	0.98010	1.30711	0.8679	0.8508	0.6813	0.7616	0.5781	0.7997	0.7305
	Total	424	353	6511	588	588	6110	322	506	281	557	445
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0011	-0.03111	-0.0116	-0.0159	0.0083	0.0148	0.0137	0.0115	-0.02710	0.0022	0.0084
	RMSE(α)	0.0032	-0.04211	-0.0165	-0.0177	0.0166	0.0188	0.0199	0.0164	-0.03610	0.0021	0.0123
	Bias(β)	0.0953	0.1026	0.17411	0.13910	0.1037	0.1109	0.0932	0.0995	0.0901	0.1098	0.0994
	RMSE(β)	0.1243	0.1326	0.17910	0.24311	0.1479	0.1448	0.1202	0.1285	0.1161	0.1427	0.1284
	CP(α)	0.94911	0.8901	0.9304	0.9273	0.9479	0.9447	0.9305	0.94810	0.9012	0.9468	0.9446
	CP(β)	0.95011	0.8841	0.9335	0.9243	0.9477	0.9446	0.9324	0.9479	0.8972	0.94810	0.9478
	aw(α)	0.3743	0.3794	0.66711	0.53210	0.4047	0.4339	0.3572	0.3986	0.3271	0.4218	0.3835
	AW(β)	0.4854	0.4833	0.69410	0.92411	0.5849	0.5708	0.4602	0.5216	0.4181	0.5527	0.4985
	Total	383	435	629	6411	578	6310	332	506	281	517	394

Overall Total	134	112	4111	4010	328	389	123	246	51	287	195

Table 7: Simulation results for α = 2.0 and β = 1.0.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.0704	-0.0071	0.22011	0.1357	0.1216	0.1819	0.1538	0.18910	-0.0865	0.0262	0.0503
	RMSE(α)	0.0804	-0.0031	0.2108	0.22110	0.1686	0.2209	0.1827	0.23411	-0.1005	0.0332	0.0603
	Bias(β)	0.3913	0.3592	0.66611	0.56210	0.4627	0.5399	0.4074	0.5188	0.3271	0.4336	0.4235
	RMSE(β)	0.4433	0.4182	0.65910	0.81911	0.5937	0.6419	0.4764	0.6218	0.3701	0.5106	0.4905
	CP(α)	0.9408	0.9367	0.9095	0.9439	0.9276	0.9004	0.8903	0.8892	0.8731	0.95511	0.94410
	CP(β)	0.9438	0.9417	0.8965	0.9449	0.9246	0.8944	0.8842	0.8933	0.8721	0.95311	0.94910
	aw(α)	2.9923	2.5672	4.74011	4.20110	3.5447	4.0149	3.3056	3.8388	2.0791	3.2095	3.1274
	AW(β)	1.7223	1.4772	2.3248	3.08911	2.2787	2.38110	1.9186	2.3299	1.1811	1.8715	1.8174
	Total	363	242	6910	7711	527	639	404	598	161	486	445
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0324	-0.0416	0.06510	0.0252	0.0385	0.06811	0.0608	0.0659	-0.0597	0.0121	0.0263
	RMSE(α)	0.0363	-0.0464	0.0557	0.0515	0.0536	0.08111	0.0698	0.08110	-0.0719	0.0131	0.0302
	Bias(β)	0.2224	0.2072	0.38411	0.31010	0.2406	0.2759	0.2143	0.2578	0.1961	0.2437	0.2325
	RMSE(β)	0.2524	0.2362	0.37210	0.44111	0.2997	0.3189	0.2433	0.3148	0.2241	0.2816	0.2625
	CP(α)	0.9406	0.9152	0.9459	0.95111	0.9427	0.9285	0.9193	0.9244	0.8731	0.95110	0.9438
	CP(β)	0.9417	0.9072	0.9489	0.95611	0.9416	0.9234	0.9173	0.9234	0.8611	0.95110	0.9478
	aw(α)	1.7194	1.5052	2.96711	2.45610	1.9397	2.1789	1.7133	2.0548	1.3191	1.9216	1.7895
	AW(β)	0.9824	0.8572	1.42910	1.73411	1.2037	1.2609	0.9773	1.2528	0.7461	1.1056	1.0275
	Total	364	221	7711	7110	517	679	343	598	221	476	415
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0144	-0.04111	0.0123	-0.0061	0.0206	0.0308	0.0307	0.0329	-0.04110	0.0062	0.0155
	RMSE(α)	0.0164	-0.04710	0.0073	0.0021	0.0286	0.0368	0.0357	0.0409	-0.04811	0.0052	0.0175
	Bias(β)	0.1483	0.1504	0.25811	0.20910	0.1596	0.1749	0.1442	0.1667	0.1351	0.1688	0.1535
	RMSE(β)	0.1683	0.1734	0.25110	0.29411	0.1977	0.2008	0.1622	0.2069	0.1571	0.1936	0.1745
	CP(α)	0.94911	0.8892	0.9489	0.9386	0.94810	0.9335	0.9313	0.9324	0.8841	0.9478	0.9457
	CP(β)	0.94811	0.8852	0.94710	0.9416	0.9448	0.9375	0.9283	0.9304	0.8751	0.9459	0.9417
	aw(α)	1.1564	1.0832	1.99011	1.64210	1.2776	1.3819	1.1133	1.3228	0.9541	1.3037	1.1895
	AW(β)	0.6604	0.6162	0.96610	1.14911	0.7857	0.7968	0.6343	0.8139	0.5411	0.7486	0.6805
	Total	444	373	6711	567	567	6010	302	599	271	486	444
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.0062	-0.03711	-0.0115	-0.0169	0.0114	0.0147	0.0148	0.0126	-0.02710	0.0011	0.0093
	RMSE(α)	0.0072	-0.04111	-0.0134	-0.0189	0.0145	0.0178	0.0167	0.0146	-0.03010	-0.0001	0.0103
	Bias(β)	0.1013	0.1116	0.18211	0.15310	0.1127	0.1179	0.0962	0.1105	0.0951	0.1168	0.1074
200	RMSE(β)	0.1153	0.1275	0.17910	0.21411	0.1379	0.1347	0.1091	0.1368	0.1092	0.1316	0.1214

	CP(α)	0.95111	0.8711	0.9354	0.9273	0.9458	0.94710	0.9446	0.9447	0.9002	0.9469	0.9385
	CP(β)	0.94810	0.8681	0.9314	0.9273	0.9438	0.9459	0.9385	0.9437	0.9002	0.94911	0.9426
	aw(α)	0.7993	0.7994	1.41411	1.16510	0.8726	0.9289	0.7552	0.8907	0.6911	0.9008	0.8135
	AW(β)	0.4564	0.4553	0.68810	0.81611	0.5337	0.5348	0.4292	0.5499	0.3921	0.5166	0.4645
	Total	384	425	599	6610	547	6711	332	558	291	506	353
	Overall Total	154	112	4111	389	287	3910	112	338	41	246	175

Table 8: Simulation results for $\alpha = 0.5$ and $\beta = 2.0$.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.0595	-0.0031	0.19911	0.0384	0.0757	0.1309	0.1198	0.15210	-0.0736	0.0212	0.0353
	RMSE(α)	0.1365	0.0231	0.2438	0.1986	0.2087	0.25811	0.2499	0.25110	-0.0954	0.0502	0.0793
	Bias(β)	0.3313	0.2962	0.61911	0.3708	0.3546	0.4139	0.3515	0.49610	0.2731	0.3557	0.3494
	RMSE(β)	0.5643	0.5062	0.7057	0.84211	0.7169	0.71710	0.6166	0.7088	0.4501	0.5834	0.5855
20	CP(α)	0.9447	0.9468	0.9034	0.96511	0.9396	0.9225	0.8973	0.8871	0.8942	0.95210	0.9489
	CP(β)	0.9417	0.9549	0.8863	0.99611	0.9396	0.9095	0.8712	0.8651	0.9064	0.96710	0.9538
	aw(α)	0.6333	0.5632	1.13211	0.6948	0.6716	0.7979	0.6807	0.85210	0.4661	0.6655	0.6464
	aw (β)	4.1464	3.6222	4.9238	5.86711	5.05810	5.0419	4.4187	4.1053	2.9571	4.3206	4.2025
	Total	373	272	639	7011	578	6710	476	537	201	465	414
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.0204	-0.0315	0.05711	0.0021	0.0336	0.0509	0.0468	0.0437	-0.05110	0.0112	0.0143
	RMSE(α)	0.0454	-0.0383	0.0655	0.0888	0.10010	0.10511	0.0939	0.0746	-0.0747	0.0291	0.0322
	Bias(β)	0.1884	0.1802	0.32511	0.2399	0.2056	0.2278	0.1853	0.26010	0.1661	0.2127	0.1895
	RMSE(β)	0.3093	0.2862	0.3928	0.60811	0.42710	0.4019	0.3124	0.3426	0.2661	0.3527	0.3245
50	CP(α)	0.9488	0.9193	0.9447	0.94910	0.9416	0.9305	0.9284	0.9192	0.8961	0.9489	0.95411
	CP(β)	0.9487	0.9295	0.9508	0.96311	0.9356	0.9224	0.9152	0.9183	0.8911	0.95410	0.9509
	aw(α)	0.3703	0.3392	0.64811	0.4589	0.4026	0.4488	0.3704	0.49310	0.2971	0.4077	0.3815
	aw (β)	2.4683	2.1342	3.0908	4.51111	3.26910	3.1269	2.4754	2.5405	1.8591	2.7707	2.5586
	Total	363	242	6910	7011	608	639	384	496	231	507	465
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.0112	-0.03110	0.0136	-0.0124	0.0167	0.0249	0.0238	0.0123	-0.03311	0.0051	0.0135
	RMSE(α)	0.0204	-0.0467	0.0111	0.0183	0.0478	0.05311	0.0489	0.0235	-0.05310	0.0132	0.0246
	Bias(β)	0.1273	0.1314	0.22111	0.17410	0.1406	0.1508	0.1242	0.1619	0.1181	0.1457	0.1325
	RMSE(β)	0.2075	0.2033	0.2669	0.42911	0.27210	0.2568	0.2034	0.1912	0.1861	0.2387	0.2186
100	CP(α)	0.94911	0.8991	0.9448	0.9334	0.9397	0.9356	0.9355	0.9283	0.9052	0.9469	0.94710
	CP(β)	0.95311	0.9112	0.9499	0.9376	0.9407	0.9294	0.9263	0.9295	0.8951	0.94910	0.9458
	aw(α)	0.2524	0.2432	0.42911	0.33310	0.2706	0.2928	0.2443	0.3229	0.2151	0.2787	0.2585
	aw (β)	1.6444	1.4982	2.0689	3.29911	2.12610	2.0018	1.5873	1.6525	1.3351	1.8657	1.6986
	Total	445	312	6411	598	619	6210	373	414	281	506	517
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS

200	Bias(α)	0.0063	-0.02711	-0.0085	-0.0199	0.0096	0.0128	0.0117	0.0032	-0.02010	0.0031	0.0064
	RMSE(α)	0.0124	-0.04311	-0.0123	-0.0216	0.0258	0.0269	0.0237	0.0062	-0.03310	0.0061	0.0145
	Bias(β)	0.0893	0.0966	0.15611	0.12710	0.0965	0.1018	0.0842	0.1019	0.0821	0.0997	0.0914
	RMSE(β)	0.1444	0.1516	0.18810	0.30311	0.1829	0.1718	0.1373	0.1001	0.1322	0.1627	0.1485
	CP(α)	0.9479	0.8901	0.9364	0.9233	0.9479	0.9458	0.9457	0.9375	0.9192	0.95111	0.9436
	CP(β)	0.95110	0.8861	0.9334	0.9223	0.9468	0.9376	0.9365	0.9377	0.9082	0.95511	0.9489
	aw(α)	0.1753	0.1785	0.30311	0.24110	0.1856	0.1988	0.1672	0.2059	0.1551	0.1937	0.1774
	aw (β)	1.1325	1.0914	1.45910	2.33311	1.4249	1.3298	1.0653	1.0262	0.9651	1.2827	1.1516
	Total	414	456	588	6310	609	6310	362	373	291	527	435
	Overall Total	153	122	389	4011	348	3910	153	205	41	257	216

Table 9: Simulation results for α = 1.0 and β = 2.0.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.0684	-0.0111	0.23511	0.0836	0.0927	0.16510	0.1398	0.1409	-0.0835	0.0302	0.0333
	RMSE(α)	0.1004	-0.0071	0.23711	0.2009	0.1546	0.23310	0.1988	0.1937	-0.1085	0.0473	0.0452
	Bias(β)	0.3603	0.3252	0.68511	0.4439	0.4097	0.49510	0.3956	0.4278	0.2861	0.3925	0.3784
	RMSE(β)	0.4773	0.4262	0.68310	0.76311	0.5998	0.6499	0.5205	0.5497	0.3771	0.5206	0.4854
	CP(α)	0.9478	0.9427	0.9013	0.96611	0.9396	0.9064	0.8851	0.9085	0.8932	0.95710	0.9539
	CP(β)	0.9437	0.9478	0.8832	0.96811	0.9326	0.9025	0.8731	0.8964	0.8913	0.96010	0.9579
	aw(α)	1.3883	1.2012	2.44111	1.6269	1.5197	1.79710	1.5066	1.6148	0.9851	1.4755	1.4194
	aw (β)	3.6193	3.1102	4.75110	5.40411	4.4398	4.6299	3.9136	4.0797	2.5181	3.8545	3.7024
	Total	35 3	252	6910	7711	557	679	415	557	191	466	394
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0254	-0.0356	0.06611	0.0152	0.0355	0.0559	0.0457	0.0528	-0.05710	0.0111	0.0193
	RMSE(α)	0.0393	-0.0434	0.0616	0.0575	0.0627	0.08211	0.0698	0.0709	-0.07410	0.0161	0.0262
	Bias(β)	0.2034	0.1942	0.37011	0.27010	0.2226	0.2469	0.2013	0.2298	0.1801	0.2267	0.2105
	RMSE(β)	0.2694	0.2502	0.38010	0.47111	0.3238	0.3329	0.2643	0.2866	0.2361	0.3017	0.2745
	CP(α)	0.9458	0.9122	0.9406	0.95211	0.9427	0.9325	0.9253	0.9253	0.8851	0.9519	0.95210
	CP(β)	0.9468	0.9163	0.9446	0.95911	0.9447	0.9275	0.9122	0.9244	0.8801	0.95710	0.9509
	aw(α)	0.8054	0.7172	1.42811	1.06410	0.8916	0.9989	0.7963	0.8988	0.6271	0.8937	0.8335
	aw (β)	2.1094	1.8412	2.92610	3.74011	2.6428	2.6689	2.0923	2.2726	1.6011	2.3587	2.1845
	Total	394	231	7110	7110	548	669	323	527	262	496	445
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0125	-0.03610	0.0092	-0.0093	0.0176	0.0258	0.0227	0.0259	-0.03911	0.0061	0.0124
	RMSE(α)	0.0205	-0.04510	0.0042	0.0041	0.0336	0.0399	0.0358	0.0337	-0.05211	0.0103	0.0164
	Bias(β)	0.1373	0.1404	0.24611	0.18910	0.1456	0.1599	0.1332	0.1507	0.1281	0.1548	0.1415
	RMSE(β)	0.1825	0.1803	0.25710	0.32911	0.2159	0.2138	0.1752	0.1826	0.1651	0.2067	0.1814
	CP(α)	0.9489	0.8962	0.9416	0.9384	0.94810	0.9437	0.9385	0.9343	0.8921	0.9448	0.94911
	CP(β)	0.9468	0.8932	0.9416	0.9457	0.9479	0.9385	0.9313	0.9384	0.8831	0.94811	0.94710

	aw(α)	0.5444	0.5152	0.94011	0.74410	0.5916	0.6419	0.5233	0.5927	0.4541	0.6088	0.5575
	aw (β)	1.4174	1.3162	1.95810	2.59911	1.7289	1.7018	1.3583	1.4786	1.1581	1.6017	1.4515
	Total	434	353	589	578	6110	6311	332	496	281	537	485
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0062	-0.03411	-0.0105	-0.0169	0.0094	0.0106	0.0117	0.0138	-0.02410	0.0021	0.0073
	RMSE(α)	0.0093	-0.04511	-0.0154	-0.0199	0.0165	0.0178	0.0167	0.0166	-0.03310	0.0041	0.0092
	Bias(β)	0.0963	0.1057	0.17711	0.13710	0.1016	0.1099	0.0912	0.0995	0.0901	0.1088	0.0984
	RMSE(β)	0.1264	0.1346	0.18210	0.23911	0.1479	0.1458	0.1173	0.1161	0.1162	0.1427	0.1285
	CP(α)	0.9509	0.8701	0.9293	0.9314	0.95111	0.9468	0.9376	0.9365	0.9062	0.95110	0.9467
	CP(β)	0.94810	0.8761	0.9294	0.9263	0.9479	0.9435	0.9436	0.9447	0.9002	0.95211	0.9448
	aw(α)	0.3773	0.3774	0.66811	0.53210	0.4057	0.4329	0.3562	0.4036	0.3291	0.4218	0.3815
	aw (β)	0.9774	0.9633	1.39010	1.84611	1.1689	1.1388	0.9192	1.0036	0.8391	1.1067	0.9905
	Total	383	445	588	6711	609	6110	352	445	291	537	394
	Overall Total	144	112	379	4011	348	3910	123	256	51	267	185

Table 10: Simulation results for $\alpha = 2.0$ and $\beta = 2.0$.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.0704	-0.0101	0.21911	0.1176	0.1227	0.17810	0.1599	0.1478	-0.0875	0.0362	0.0463
	RMSE(α)	0.0844	-0.0071	0.21010	0.1898	0.1626	0.21211	0.1909	0.1757	-0.1005	0.0432	0.0583
	Bias(β)	0.3813	0.3402	0.64911	0.5109	0.4588	0.51710	0.4205	0.4336	0.3181	0.4417	0.4144
	RMSE(β)	0.4383	0.3972	0.63210	0.70511	0.5758	0.5949	0.4825	0.4996	0.3621	0.5087	0.4804
	CP(α)	0.9458	0.9457	0.9095	0.96011	0.9326	0.9044	0.8802	0.8933	0.8721	0.95310	0.9509
	CP(β)	0.9488	0.9457	0.8974	0.95811	0.9296	0.9005	0.8762	0.8913	0.8711	0.9489	0.95010
	aw(α)	2.9373	2.5372	4.59711	3.6649	3.3298	3.79410	3.2136	3.2647	2.0691	3.1205	3.0284
	aw (β)	3.3693	2.9052	4.42410	5.05511	4.1538	4.3689	3.6866	3.7487	2.3511	3.5905	3.4884
	Total	363	242	7210	7611	578	689	445	476	161	476	414
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0263	-0.0415	0.06811	0.0374	0.0466	0.06710	0.0588	0.0517	-0.0589	0.0131	0.0232
	RMSE(α)	0.0333	-0.0464	0.0606	0.0668	0.0605	0.08211	0.0679	0.0617	-0.07010	0.0161	0.0292
	Bias(β)	0.2164	0.2072	0.38511	0.31410	0.2437	0.2729	0.2153	0.2215	0.1961	0.2508	0.2266
	RMSE(β)	0.2514	0.2342	0.37510	0.43811	0.3018	0.3159	0.2433	0.2555	0.2231	0.2887	0.2606
	CP(α)	0.9447	0.9112	0.9479	0.94810	0.9386	0.9274	0.9263	0.9305	0.8731	0.95011	0.9458
	CP(β)	0.9457	0.9092	0.94810	0.95211	0.9356	0.9223	0.9254	0.9275	0.8631	0.9469	0.9458
	aw(α)	1.7084	1.5072	2.96911	2.41210	1.9468	2.1669	1.7073	1.7735	1.3201	1.9187	1.7826
	aw (β)	1.9584	1.7152	2.86210	3.38711	2.4118	2.5079	1.9493	2.0445	1.4921	2.2127	2.0496
	Total	363	211	7811	7510	548	649	363	445	252	517	445
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
	Bias(α)	0.0145	-0.04010	0.0093	-0.0011	0.0227	0.0349	0.0288	0.0216	-0.04011	0.0052	0.0134
	RMSE(α)	0.0164	-0.04410	0.0052	0.0041	0.0307	0.0419	0.0318	0.0266	-0.04811	0.0053	0.0175

100	Bias(β)	0.1474	0.1505	0.25511	0.20910	0.1607	0.1799	0.1392	0.1433	0.1351	0.1698	0.1526
	RMSE(β)	0.1694	0.1695	0.24810	0.28611	0.1998	0.2059	0.1582	0.1663	0.1541	0.1947	0.1746
	CP(α)	0.9489	0.8872	0.9436	0.9447	0.94810	0.9343	0.9344	0.9365	0.8821	0.94811	0.9458
	CP(β)	0.94610	0.8922	0.9457	0.9416	0.9459	0.9313	0.9345	0.9334	0.8781	0.94811	0.9458
	aw(α)	1.1564	1.0862	1.98311	1.64710	1.2797	1.3879	1.1123	1.1605	0.9551	1.3028	1.1876
	aw (β)	1.3214	1.2372	1.93110	2.29911	1.5748	1.5999	1.2643	1.3365	1.0821	1.4967	1.3596
	Total	445	384	609	577	6311	609	352	373	281	577	496
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0073	-0.03611	-0.0126	-0.0147	0.0115	0.0179	0.0158	0.0094	-0.02610	0.0001	0.0062
	RMSE(α)	0.0092	-0.03911	-0.0125	-0.0167	0.0156	0.0209	0.0168	0.0104	-0.03010	-0.0011	0.0093
	Bias(β)	0.1045	0.1117	0.18311	0.15110	0.1106	0.1219	0.0963	0.0911	0.0952	0.1158	0.1044
	RMSE(β)	0.1205	0.1266	0.17810	0.21111	0.1358	0.1379	0.1103	0.1051	0.1082	0.1327	0.1184
	CP(α)	0.9427	0.8691	0.9304	0.9263	0.95010	0.9335	0.9406	0.9448	0.9022	0.95411	0.9459
	CP(β)	0.9395	0.8711	0.9324	0.9283	0.94810	0.9416	0.9417	0.9438	0.8972	0.95511	0.9479
	aw(α)	0.7995	0.7994	1.41211	1.16710	0.8727	0.9329	0.7562	0.7913	0.6911	0.9008	0.8116
	aw (β)	0.9125	0.9124	1.37610	1.63411	1.0688	1.0719	0.8582	0.9113	0.7841	1.0327	0.9286
	Total	373	456	619	6210	608	6511	394	322	301	547	435
	Overall Total	143	132	3911	389	358	389	143	165	51	277	206

Table 11: Simulation results for $\alpha = 0.5$ and $\beta = 3.0$.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.0366	-0.0143	0.13911	0.0021	0.0457	0.10110	0.1019	0.0164	-0.0798	-0.0042	0.0215
	RMSE(α)	0.0796	-0.0081	0.1419	0.0475	0.0967	0.16310	0.17911	0.0193	-0.1198	-0.0092	0.0404
	Bias(β)	0.3003	0.2872	0.50011	0.3368	0.3215	0.3869	0.3257	0.39210	0.2691	0.3246	0.3184
	RMSE(β)	0.4754	0.4473	0.5369	0.61811	0.5258	0.55510	0.4887	0.3051	0.4182	0.4875	0.4876
	CP(α)	0.9577	0.9486	0.9405	0.96811	0.9588	0.9394	0.9163	0.9032	0.8891	0.9589	0.96110
	CP(β)	0.9746	0.9725	0.9404	0.99811	0.99610	0.9757	0.9242	0.9283	0.9091	0.9868	0.9869
	aw(α)	0.5763	0.5292	0.90911	0.6057	0.5965	0.6869	0.6118	0.73110	0.4511	0.5976	0.5854
	aw (β)	4.9854	4.6533	5.3468	6.28111	5.63810	5.5349	5.0625	3.5091	4.0052	5.1577	5.0666
	Total	394	252	6810	65 9	608	6810	527	343	241	455	486
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0194	-0.0337	0.04910	-0.0113	0.0266	0.0479	0.0408	-0.0041	-0.05211	0.0042	0.0205
	RMSE(α)	0.0466	-0.0455	0.0477	0.0343	0.0738	0.09211	0.0799	-0.0021	-0.08310	0.0112	0.0454
	Bias(β)	0.1854	0.1802	0.31311	0.23110	0.1957	0.2229	0.1833	0.1936	0.1671	0.2038	0.1935
	RMSE(β)	0.3135	0.2813	0.3738	0.52311	0.38010	0.3779	0.2984	0.1211	0.2672	0.3357	0.3206
	CP(α)	0.9487	0.9202	0.9519	0.9486	0.9498	0.9374	0.9363	0.98211	0.8901	0.95610	0.9475
	CP(β)	0.9436	0.9272	0.9579	0.97411	0.9558	0.9314	0.9273	0.9415	0.8891	0.95710	0.9497
	aw(α)	0.3634	0.3362	0.59411	0.4179	0.3826	0.42510	0.3623	0.3857	0.2961	0.3928	0.3745
	aw (β)	3.5375	3.1293	4.0988	5.37811	4.30310	4.1619	3.5094	1.0831	2.7432	3.8407	3.6636

	Total	41 5	261	7311	649	638	6510	374	333	292	547	436
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0104	-0.03510	0.0083	-0.0157	0.0136	0.0229	0.0198	0.0011	-0.03511	0.0032	0.0115
	RMSE(α)	0.0245	-0.05110	0.0011	0.0054	0.0377	0.0459	0.0388	-0.0022	-0.05811	0.0053	0.0276
	Bias(β)	0.1284	0.1336	0.21811	0.17310	0.1357	0.1489	0.1233	0.1222	0.1181	0.1428	0.1295
	RMSE(β)	0.2125	0.2044	0.2639	0.41511	0.26510	0.2528	0.1993	0.0341	0.1872	0.2317	0.2186
	CP(α)	0.9487	0.8992	0.9509	0.9363	0.9456	0.9384	0.9435	0.98211	0.8981	0.9498	0.95310
	CP(β)	0.9405	0.9022	0.9509	0.9467	0.9446	0.9333	0.9374	0.95211	0.8921	0.95010	0.9478
	aw(α)	0.2525	0.2423	0.42111	0.31910	0.2677	0.2909	0.2434	0.2322	0.2141	0.2778	0.2576
	aw (β)	2.4725	2.2383	3.0249	4.45611	3.10310	2.9408	2.3564	0.0431	1.9932	2.7597	2.5466
	Total	404	404	6210	6311	598	598	393	312	301	537	526
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0043	-0.02911	-0.0086	-0.0209	0.0064	0.0128	0.0107	0.0011	-0.02310	0.0022	0.0075
	RMSE(α)	0.0153	-0.04411	-0.0164	-0.0259	0.0196	0.0228	0.0217	-0.0001	-0.03810	0.0012	0.0165
	Bias(β)	0.0884	0.0987	0.15811	0.12610	0.0936	0.1019	0.0853	0.0842	0.0831	0.0998	0.0905
	RMSE(β)	0.1474	0.1496	0.19210	0.30311	0.1809	0.1708	0.1353	0.0021	0.1302	0.1607	0.1485
	CP(α)	0.9498	0.8861	0.9374	0.9213	0.9509	0.9435	0.9446	0.97411	0.9112	0.95210	0.9487
	CP(β)	0.9468	0.8871	0.9334	0.9243	0.9479	0.9375	0.9426	0.96411	0.9132	0.94910	0.9447
	aw(α)	0.1754	0.1776	0.30311	0.23910	0.1857	0.1989	0.1673	0.1632	0.1551	0.1938	0.1775
	aw (β)	1.7035	1.6364	2.18010	3.43011	2.1259	1.9858	1.5943	0.0221	1.4422	1.9137	1.7306
	Total	394	476	609	6611	598	609	383	301	301	547	455
	Overall Total	174	133	4010	4010	328	379	174	92	51	267	236

Table 12: Simulation results for α = 1.0 and β = 3.0.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.0445	-0.0233	0.13711	0.0344	0.0626	0.1249	0.13210	0.1118	-0.0817	-0.0001	0.0232
	RMSE(α)	0.0644	-0.0252	0.1308	0.0755	0.1036	0.16910	0.17111	0.1379	-0.1077	0.0051	0.0303
	Bias(β)	0.3273	0.3022	0.51211	0.3758	0.3465	0.4039	0.3657	0.40410	0.2971	0.3454	0.3516
	RMSE(β)	0.4193	0.3812	0.5059	0.56011	0.4848	0.51510	0.4546	0.4707	0.3761	0.4425	0.4354
	CP(α)	0.9567	0.9455	0.9526	0.97811	0.9598	0.9424	0.9033	0.8972	0.8811	0.96610	0.9599
	CP(β)	0.9657	0.9566	0.9384	0.99611	0.9719	0.9495	0.9042	0.9093	0.8871	0.98110	0.9708
	aw(α)	1.2353	1.1192	1.87211	1.3468	1.2946	1.47210	1.3197	1.3889	0.9531	1.2825	1.2574
	aw (β)	4.5704	4.2062	5.22510	5.90611	5.1908	5.2199	4.7106	4.5083	3.5861	4.7837	4.6665
	Total	363	242	7011	6910	568	669	527	516	201	435	414
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0224	-0.0406	0.0558	0.0132	0.0345	0.06211	0.05910	0.0417	-0.0569	0.0081	0.0223
	RMSE(α)	0.0343	-0.0485	0.0526	0.0454	0.0588	0.08711	0.07610	0.0557	-0.0719	0.0131	0.0282
	Bias(β)	0.2023	0.1972	0.34511	0.26410	0.2236	0.2469	0.2084	0.2278	0.1821	0.2237	0.2185
	RMSE(β)	0.2643	0.2512	0.35810	0.44711	0.3278	0.3319	0.2664	0.2685	0.2331	0.3007	0.2796

	CP(α)	0.9478	0.9073	0.9519	0.95611	0.9416	0.9345	0.9204	0.8992	0.8801	0.95510	0.9437
	CP(β)	0.9498	0.9112	0.95410	0.97011	0.9436	0.9275	0.9164	0.9143	0.8821	0.9529	0.9477
	aw(α)	0.7913	0.7102	1.29611	0.96210	0.8526	0.9539	0.7974	0.8668	0.6271	0.8667	0.8195
	aw (β)	3.0855	2.7342	3.95610	4.78211	3.7018	3.7359	3.0844	3.0313	2.4061	3.3947	3.1906
	Total	373	241	7511	7010	538	689	446	435	241	497	414
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0114	-0.04111	0.0063	-0.0041	0.0167	0.0329	0.0308	0.0135	-0.03710	0.0062	0.0136
	RMSE(α)	0.0175	-0.05211	0.0011	0.0072	0.0257	0.0449	0.0398	0.0176	-0.04710	0.0093	0.0154
	Bias(β)	0.1373	0.1415	0.23611	0.19410	0.1537	0.1639	0.1362	0.1374	0.1291	0.1548	0.1436
	RMSE(β)	0.1784	0.1795	0.24710	0.33311	0.2209	0.2148	0.1753	0.1481	0.1662	0.2057	0.1846
	CP(α)	0.95110	0.8932	0.95111	0.9396	0.9428	0.9395	0.9304	0.9053	0.8861	0.9499	0.9417
	CP(β)	0.95211	0.8872	0.9508	0.9447	0.9396	0.9355	0.9274	0.9193	0.8861	0.9509	0.95110
	aw(α)	0.5434	0.5122	0.92711	0.72710	0.5887	0.6449	0.5273	0.5736	0.4551	0.6078	0.5575
	aw (β)	2.1205	1.9642	2.89010	3.74211	2.5599	2.5528	2.0444	2.0093	1.7441	2.3947	2.1726
	Total	465	404	6511	588	609	6210	363	312	271	537	506
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0063	-0.03411	-0.0137	-0.0126	0.0064	0.0179	0.0158	0.0021	-0.02410	0.0022	0.0075
	RMSE(α)	0.0094	-0.04211	-0.0177	-0.0166	0.0125	0.0249	0.0198	0.0052	-0.03210	0.0041	0.0073
	Bias(β)	0.0954	0.1057	0.17411	0.14310	0.1056	0.1119	0.0933	0.0791	0.0912	0.1088	0.0965
	RMSE(β)	0.1234	0.1346	0.18010	0.24511	0.1509	0.1468	0.1183	0.0721	0.1182	0.1437	0.1245
	CP(α)	0.95310	0.8701	0.9324	0.9243	0.9478	0.9406	0.9335	0.96011	0.9032	0.9457	0.9539
	CP(β)	0.95210	0.8751	0.9304	0.9243	0.9427	0.9416	0.9428	0.9335	0.8952	0.9459	0.95211
	aw(α)	0.3764	0.3775	0.66511	0.53310	0.4037	0.4359	0.3583	0.3302	0.3281	0.4218	0.3816
	aw (β)	1.4665	1.4494	2.07810	2.76711	1.7459	1.7178	1.3813	0.8661	1.2592	1.6587	1.4846
	Total	444	465	6410	609	558	6410	413	241	312	496	507
	Overall Total	154	122	4311	379	338	3810	195	143	51	257	216

Table 13: Simulation results for α = 2.0 and β = 3.0.

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
20	Bias(α)	0.0605	-0.0232	0.14410	0.0594	0.0896	0.1379	0.15411	0.1208	-0.0987	0.0111	0.0313
	RMSE(α)	0.0754	-0.0302	0.1298	0.1015	0.1187	0.15810	0.17111	0.1359	-0.1136	0.0111	0.0353
	Bias(β)	0.3563	0.3252	0.52511	0.4119	0.3958	0.44610	0.3947	0.3856	0.3091	0.3845	0.3754
	RMSE(β)	0.4063	0.3692	0.49810	0.54811	0.4788	0.4929	0.4357	0.4235	0.3531	0.4306	0.4214
	CP(α)	0.9557	0.9475	0.9526	0.98511	0.9578	0.9374	0.8972	0.9183	0.8731	0.96710	0.9639
	CP(β)	0.9557	0.9516	0.9404	0.99611	0.9568	0.9425	0.8942	0.9203	0.8701	0.97510	0.9639
	aw(α)	2.6143	2.3532	3.75711	2.9169	2.7737	3.08410	2.7818	2.7596	1.9761	2.7255	2.6634
	aw (β)	4.3853	3.9492	5.15410	5.63911	4.9468	5.0589	4.5436	4.5245	3.3471	4.5827	4.4734
	Total	35 3	232	7010	7111	608	669	547	45 5	191	455	404

n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
50	Bias(α)	0.0274	-0.0435	0.0548	0.0233	0.0436	0.06310	0.0619	0.0517	-0.06611	0.0101	0.0222
	RMSE(α)	0.0323	-0.0506	0.0475	0.0474	0.0598	0.07510	0.0699	0.0597	-0.08011	0.0141	0.0282
	Bias(β)	0.2123	0.2042	0.35911	0.29210	0.2417	0.2649	0.2164	0.2165	0.1891	0.2438	0.2286
	RMSE(β)	0.2445	0.2362	0.34710	0.40611	0.3028	0.3029	0.2434	0.2423	0.2171	0.2807	0.2636
	CP(α)	0.9477	0.9062	0.9519	0.96211	0.9396	0.9295	0.9163	0.9274	0.8681	0.95510	0.9488
	CP(β)	0.9478	0.9072	0.95610	0.96111	0.9356	0.9335	0.9213	0.9254	0.8621	0.9569	0.9447
	aw(α)	1.6893	1.4982	2.70611	2.15210	1.8648	2.0549	1.6924	1.7325	1.3071	1.8637	1.7466
	aw (β)	2.8984	2.5512	3.90310	4.43811	3.4458	3.5409	2.8883	2.9455	2.2161	3.2127	3.0036
	Total	373	231	7411	7110	578	669	394	405	282	507	436
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
100	Bias(α)	0.0145	-0.04210	0.0113	-0.0021	0.0226	0.0308	0.0319	0.0227	-0.04311	0.0072	0.0124
	RMSE(α)	0.0175	-0.04910	0.0072	0.0061	0.0307	0.0369	0.0358	0.0256	-0.05111	0.0093	0.0154
	Bias(β)	0.1464	0.1485	0.25611	0.20810	0.1617	0.1719	0.1423	0.1372	0.1341	0.1648	0.1546
	RMSE(β)	0.1684	0.1695	0.25110	0.29211	0.1989	0.1978	0.1613	0.1531	0.1542	0.1907	0.1746
	CP(α)	0.94810	0.8912	0.9459	0.9437	0.9438	0.9416	0.9293	0.9334	0.8851	0.95511	0.9395
	CP(β)	0.94710	0.8862	0.9405	0.9459	0.9436	0.9448	0.9303	0.9364	0.8771	0.95211	0.9437
	aw(α)	1.1575	1.0822	1.96411	1.61110	1.2777	1.3799	1.1143	1.1504	0.9521	1.3028	1.1866
	aw (β)	1.9835	1.8442	2.86410	3.37311	2.3558	2.3819	1.9023	1.9554	1.6171	2.2477	2.0346
	Total	486	384	6110	609	588	6611	353	322	291	577	445
n	Qtd	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
200	Bias(α)	0.0083	-0.03611	-0.0136	-0.0157	0.0105	0.0158	0.0169	0.0094	-0.02810	0.0021	0.0062
	RMSE(α)	0.0094	-0.04211	-0.0146	-0.0177	0.0135	0.0188	0.0199	0.0093	-0.03410	0.0031	0.0082
	Bias(β)	0.1014	0.1116	0.18211	0.15310	0.1127	0.1169	0.0983	0.0831	0.0942	0.1148	0.1045
	RMSE(β)	0.1164	0.1276	0.17810	0.21311	0.1389	0.1338	0.1103	0.0901	0.1082	0.1317	0.1185
	CP(α)	0.94810	0.8731	0.9334	0.9283	0.9468	0.9469	0.9376	0.9365	0.9032	0.95111	0.9447
	CP(β)	0.9458	0.8661	0.9294	0.9283	0.9427	0.9479	0.9385	0.9406	0.8982	0.95311	0.95010
	aw(α)	0.8005	0.7994	1.40911	1.16610	0.8717	0.9299	0.7562	0.7773	0.6891	0.9008	0.8106
	aw (β)	1.3705	1.3644	2.05910	2.44811	1.6008	1.6039	1.2902	1.3253	1.1711	1.5527	1.3886
	Total	434	446	629	629	568	6911	393	261	302	547	434
	Overall Total	164	132	4010	399	328	4010	175	132	61	267	196

Table 14: Overall performance of the estimation methods

Scenario	AD	AD2	AD2L	AD2R	ADR	CvM	MLE	MOM	MPS	OLS	WLS
(α=0.5,β=0.5)	154	102	4311	3810	307	379	102	307	51	246	195
(α=1.0, β=0.5)	134	102	4010	4010	287	379	123	338	51	236	205
(α=2.0, β=0.5)	133	102	3810	4111	297	379	133	338	51	246	195

(α=0.5, β=1.0)	134	122	4111	3910	328	339	122	297	41	246	195
(α=1.0, β=1.0)	134	112	4111	4010	328	389	123	246	51	287	195
(α=2.0,β=1.0)	154	112	4111	389	287	3910	112	338	41	246	175
(α=0.5, β=2.0)	153	122	389	4011	348	3910	153	205	41	257	216
(α=1.0, β=2.0)	144	112	379	4011	348	3910	123	256	51	267	185
(α=2.0, β=2.0)	143	132	3911	389	358	389	143	165	51	277	206
(α=0.5, β=3.0)	174	133	4010	4010	328	379	174	92	51	267	236
(α=1.0,β=3.0)	154	122	4311	379	338	3810	195	143	51	257	216
(α=2.0, β=3.0)	164	132	4010	399	328	4010	175	132	61	267	196
Total	1734	1382	48111	47010	3798	4529	1643	2796	581	3027	2355

4 Applications

In this section, we use two real data sets to demonstrate the performance of different methods of estimation considered in this paper. The first data set is from Dumonceaux and Antle (1973) and corresponds to 20 observations of the maximum flood level (in millions of cubic feet per second) for Susquehanna River at Harrisburg, Pennsylvania. The second data set corresponds to twelve core samples from petroleum reservoirs that were sampled by four cross-sections, and there are 48 observations. Each core sample was measured for permeability and each cross-section has the following variables: the total area of pores, the total perimeter of pores and shape. It should be noted that this data can be found in R Core Team (2017), especially on data. frame named rock. Both data sets are in the Table 15 and some descriptive measures are reported in Table 16. Further, we note that these data sets were studied by Mazucheli et al.(2018) to illustrate the applicability of the unit-Gamma distribution in order to compare the second-order bias corrections of the maximum likelihood estimators.

Table 15: Maximum flood level data and petroleum reservoirs data.

Data set I
0.265, 0.269, 0.297, 0.315, 0.3235, 0.338, 0.379, 0.379, 0.392, 0.402, 0.412, 0.416, 0.418,

0.423, 0.449, 0.484, 0.494, 0.613, 0.654, 0.740
Data set II
0.090, 0.149,0.183,0.117,0.122,0.167,0.190,0.164,0.204,0.162,0.151,0.148,0.229,
0.232, 0.173,0.153,0.204,0.263,0.200,0.145,0.114,0.291,0.240,0.162,0.281,0.179,
0.192, 0.133,0.225,0.341,0.312,0.276,0.198,0.327,0.154,0.276,0.177,0.439,0.164,
0.254, 0.329,0.230,0.464,0.420,0.201,0.263,0.182,0.200

Table 16: Descritive measures for data sets I and II.

	Data set I	Data set II
n	20	48
mean	0.4321	0.2181
std-dev	0.1253	0.0835
min	0.2650	0.0903
median	0.4070	0.1989
max	0.7400	0.4641

The parameter estimates and their corresponding bootstrap confidence intervals under various methods considered in this paper for the two data sets are summarized in Tables 17 and 18. We also present the results of formal goodness-of-fit tests, the Kolmogorov-Smirnov (KS) statistic along with p-value, in order to show that the unit-Gamma can be used to model these data sets.

From Table 17 we can see that all estimates provides a good fit to the data sets. We also observe that the MPS and AD2 estimators give the shortest confidence intervals for both the parameters α and β, respectively.

The results in Table 18 indicate that the CvM estimates do not provide a good fit to this data set as per K-S statistic is concerned. We also observe that OLS has the lowest value of K-S. It is also noteworthy, that AD2 and ADR have the shortest confidence intervals for both α and β.

The overall performance of the estimators of a and /3 with respect to width of the parametric bootstrap confidence intervals are presented in Table 17. We considered the p-bootstrap method based on B = 1000 resamples, Efron (1982b), to construct the confidence intervals for α and β .

Table 17: Parameter estimates, 95% confidence intervals based on parametric Bootstrap and K-S test: data set I

Method	α	LCL	UCL	β	LCL	UCL	KS (p-value)

MLE	8.7332	5.4251	18.8766	9.7275	5.9152	21.0504	0.1955	(0.4294)
MPS	6.2605	3.0916	10.3881	6.9502	3.2673	11.3440	0.2081	(0.3519)
MOM	8.3938	5.1258	17.1975	9.2678	5.3816	20.3412	0.1874	(0.4837)
OLS	12.8657	6.0086	30.0672	13.7425	6.2067	31.3893	0.1274	(0.9017)
WLS	12.6654	6.3662	26.9534	13.5638	6.4821	30.5281	0.1260	(0.9085)
CvM	15.2902	8.1805	40.6867	16.3930	8.6397	43.6603	0.1357	(0.8552)
AD	9.1007	4.9838	19.0116	9.8457	5.3477	20.6942	0.1592	(0.6909)
ADR	6.6690	3.7733	16.3230	7.0020	3.8323	17.4604	0.1504	(0.7559)
AD2R	4.7501	2.1651	15.3588	4.7887	1.9332	17.0276	0.2066	(0.3605)
AD2L	17.7944	7.7244	61.0730	18.9778	8.3134	62.2243	0.1416	(0.8172)
AD2	6.2011	3.0918	11.9963	7.0293	3.2679	13.8680	0.2289	(0.2454)

L(U) CL lower (upper) confidence limit.

Table 18: Parameter estimates, 95% confidence intervals based on parametric Bootstrap and K-S test: data set II

Method	α	LCL	UCL	β	LCL	UCL	KS (p-value)	
MLE	17.9541	12.8359	29.0063	11.3115	8.0495	18.2736	0.1365	(0.3331)
MPS	15.3776	7.1628	27.0135	9.6316	4.3030	16.9532	0.1298	(0.3937)
MOM	15.8115	9.3381	34.5760	9.8926	5.7186	21.5892	0.1275	(0.4162)
OLS	19.6011	8.8391	42.6476	12.0676	5.3705	26.4850	0.0947	(0.7829)
WLS	19.8300	9.9533	43.1044	12.3017	6.0218	26.9789	0.1077	(0.6341)
CvM	15.2857	8.2430	40.5391	16.3882	8.3749	43.2302	0.7456	(0.0000)
AD	18.4562	12.0864	29.3272	11.4635	7.4358	18.5932	0.1118	(0.5862)
ADR	15.0523	9.9705	25.1797	9.2342	5.9850	16.0918	0.1183	(0.5131)
AD2R	11.0919	5.9193	22.5897	6.6442	3.2657	14.4050	0.1706	(0.1221)
AD2L	30.3925	15.2310	57.8613	18.6428	9.3745	35.2371	0.1124	(0.5794)
AD2	14.5794	8.3875	21.3646	9.3427	5.4558	14.0784	0.1655	(0.1441)

L(U)CL lower (upper) confidence limit.

Table 19: Width of the parametric Bootstrap confidence intervals.

Method	Data set I		Data set II	
	Width of α	Width of β	Width of α	Width of β
MLE	13.4515	15.1353	16.1704	10.2241
MPS	7.2965	8.0768	19.8507	12.6501
MOM	12.0717	14.9595	25.2379	15.8707
OLS	24.0586	25.1825	33.8084	21.1146
WLS	20.5872	24.0460	33.1511	20.9571
CvM	32.5062	35.0207	32.2960	34.8553
AD	14.0277	15.3465	17.2408	11.1574
ADR	12.5497	13.6281	15.2093	10.1068
AD2R	13.1937	15.0944	16.6704	11.1393
AD2L	53.3486	53.9109	42.6302	25.8625
AD2	8.9046	10.6001	12.9771	8.6226

5 Concluding Remarks

In this paper, we have performed an extensive simulation study to compare eleven aforementioned methods of estimation. We have compared estimators with respect to bias and rootmean-squared error. Besides, we have obtained the coverage probability and the average widthof the Bootstrap confidence intervals. To illustrate the use of these methods of estimation,we provided two real data examples where the parameters of a two-parameter unit-Gammadistribution have been obtained. Furthermore, we have obtained estimates for the parameters α and β along with 95% confidence intervals and width of CIs based on parametric Bootstrap confidence intervals. The simulation results show that MPS estimators is the best performing estimator in terms of biases and RMSE. The next best performing estimators is the AD2, followed by MLE. The real data applications show that the MPS and AD2 estimators give the shortest confidence intervals for α and β, respectively for the data set-I and AD2 and ADR have the shortest confidence intervals for the data set-II. Hence, we can argue that the MPS estimators, AD2, ADR and ML estimators are among the best performing estimators for unit-Gamma distribution. From both simulation study and real data examples, we observe that performance of AD2L estimators is the worst among all methods of estimation.

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