Supplementary Material for "Q-Learning with Compound Outcome and Mixed Misclassification and Measurement Error in Covariates"

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In Section S1, we provide an example of constructing $S^*_{Kj}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^*_{Ki}, \overline{C}^*_{Ki}, \overline{Z}_{Ki})$ to demonstrate its explicit dependence on the degrees of misclassification and measurement error in covariates, reflected by π_{01}, π_{10}, and σ^2_K. In Section S2, we report the proportion of optimally treated future patients under the original simulation study of the main text. In Section S3, we present additional simulation results to evaluate the performance of correction strategies under a sample size different from that considered in Section 3.2 of the main text. To be specific, Tables S2–S5 summarize results for a reduced sample size ($n = 1000$), highlighting trends and occasional counterintuitive behavior of the EE-estimated method relative to the EE-known method. In Section S4, we explore the impact of a smaller validation subsample size on the performance of correction strategies. The results, presented in Tables S6–S8, show that reducing the validation subsample size significantly deteriorates the performance of both the RC and EE-estimated methods compared to cases with a 30% validation subsample size. Finally, Section S5 provides the full details of the data analysis introduced in Section 7 of the main text. This includes a description of the dataset, the data pre-processing procedure, and the implementation of the proposed methods. We also present the complete analysis results along with an in-depth interpretation.

S1 An Example of Constructing $S^{*}_{Kj}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^{*}_{Ki}, \overline{C}^{*}_{Ki}, \overline{Z}_{Ki})$

For illustration, we present explicitely how to construct $S^{*}_{Kj}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^{*}_{Ki}, \overline{C}^{*}_{Ki}, \overline{Z}_{Ki})$ based on given forms of H_{K0} and H_{K1}. Suppose H_{K0} and H_{K1} are specified as $H_{K0} = H_{K1} \triangleq \{A_{K-1}, X_K, C_K, Z_K\}$. Then, applying (13) gives

$$
\begin{aligned}
&S_{K\beta_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}_{Ki}, \overline{C}_{Ki}, \overline{Z}_{Ki}) \\
&= \big[Y_{Kji} - \big\{\beta_{Kj1}A_{K-1} + (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_K + \beta_{Kj3}C_K + \beta_{Kj4}Z_K \\
&\quad +(\psi_{Kj0} + \psi_{Kj1}A_{K-1} + \psi_{Kj3}C_K + \psi_{Kj4}Z_K)A_K\big\}\big]\begin{bmatrix} A_{K-1} \\ X_K \\ C_K \\ Z_K \end{bmatrix} \\
&= \begin{bmatrix}
Y_{Kji}A_{K-1} - \beta_{Kj1}A^2_{K-1} - (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_K A_{K-1} - \beta_{Kj3}C_K A_{K-1} - \beta_{Kj4}Z_K A_{K-1} \\
-\psi_{Kj0}A_K A_{K-1} - \psi_{Kj1}A^2_{K-1}A_K - \psi_{Kj3}C_K A_K A_{K-1} - \psi_{Kj4}Z_K A_K A_{K-1} \\
Y_{Kji}X_K - \beta_{Kj1}A_{K-1}X_K - \{(\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_K X_K^{\mathrm{T}}\}^{\mathrm{T}} - \beta_{Kj3}C_K X_K - \beta_{Kj4}Z_K X_K \\
-\psi_{Kj0}A_K X_K - \psi_{Kj1}A_{K-1}A_K X_K - \psi_{Kj3}C_K A_K X_K - \psi_{Kj4}Z_K A_K X_K \\
Y_{Kji}C_K - \beta_{Kj1}A_{K-1}C_K - (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_K C_K - \beta_{Kj3}C^2_K - \beta_{Kj4}Z_K C_K \\
-\psi_{Kj0}A_K C_K - \psi_{Kj1}A_{K-1}A_K C_K - \psi_{Kj3}C^2_K A_K - \psi_{Kj4}Z_K A_K C_K \\
Y_{Kji}Z_K - \beta_{Kj1}A_{K-1}Z_K - (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_K Z_K - \beta_{Kj3}C_K Z_K - \beta_{Kj4}Z^2_K \\
-\psi_{Kj0}A_K Z_K - \psi_{Kj1}A_{K-1}A_K Z_K - \psi_{Kj3}C_K A_K Z_K - \psi_{Kj4}Z^2_K A_K
\end{bmatrix}. \qquad \text{(S1)}
\end{aligned}
$$

Let

$$
\begin{aligned}
U_{K\beta_{11}} &= Y_{Kji}A_{K-1} - \beta_{Kj1}A^2_{K-1} - \beta_{Kj4}Z_K A_{K-1} - \psi_{Kj0}A_K A_{K-1} - \psi_{Kj1}A^2_{K-1}A_K - \psi_{Kj4}Z_K A_K A_{K-1}, \\
U_{K\beta_{12}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)A_{K-1}, \\
U_{K\beta_{13}} &= \beta_{Kj3}A_{K-1} + \psi_{Kj3}A_K A_{K-1}, \\
U_{K\beta_{21}} &= Y_{Kji} - \beta_{Kj1}A_{K-1} - \beta_{Kj4}Z_K - \psi_{Kj0}A_K - \psi_{Kj1}A_{K-1}A_K - \psi_{Kj4}Z_K A_K, \\
U_{K\beta_{22}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K), \\
U_{K\beta_{23}} &= \beta_{Kj3} + \psi_{Kj3}A_K, \\
U_{K\beta_{31}} &= Y_{Kji} - \beta_{Kj1}A_{K-1} - \beta_{Kj4}Z_K - \psi_{Kj0}A_K - \psi_{Kj1}A_{K-1}A_K - \psi_{Kj4}Z_K A_K, \\
U_{K\beta_{32}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K), \\
U_{K\beta_{33}} &= \beta_{Kj3} + \psi_{Kj3}A_K, \\
U_{K\beta_{41}} &= Y_{Kji}Z_K - \beta_{Kj1}A_{K-1}Z_K - \beta_{Kj4}Z^2_K - \psi_{Kj0}A_K Z_K - \psi_{Kj1}A_{K-1}A_K Z_K - \psi_{Kj4}Z^2_K A_K, \\
U_{K\beta_{42}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)Z_K, \\
U_{K\beta_{43}} &= \beta_{Kj3}Z_K + \psi_{Kj3}A_K Z_K.
\end{aligned}
$$

Then (S1) becomes

$$
\Big[S_{K\beta_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}_{Ki}, \overline{C}_{Ki}, \overline{Z}_{Ki})\Big] = \begin{bmatrix}
U_{K\beta_{11}} - U_{K\beta_{12}}X_K - U_{K\beta_{13}}C_K \\
U_{K\beta_{21}}X_K - \{U_{K\beta_{22}}X_K X_K^{\mathrm{T}}\}^{\mathrm{T}} - U_{K\beta_{23}}C_K X_K \\
U_{K\beta_{31}}C_K - U_{K\beta_{32}}X_K C_K - U_{K\beta_{33}}C^2_K \\
U_{K\beta_{41}} - U_{K\beta_{42}}X_K - U_{K\beta_{43}}C_K
\end{bmatrix}. \qquad \text{(S2)}
$$

As defined in Section 5.1.1,

$$
X^{**}_k = \Pi^{-1}X^{*}_k,\ X^{***}_k = \sum_{t=1}^{2}\left\{X^{**\mathrm{T}}_k e_t\right\} e_t e_t^{\mathrm{T}},\ C^{**}_k = C^{*}_k, \text{ and } C^{***}_k = C^{*2}_k - \sigma^2_k \qquad \text{(S3)}
$$

represent the unbiased surrogates of X_k, $X_k X_k^{\mathrm{T}}$, C_k, and C^2_k, respectively. Then, replacing X_K, $X_K X_K^{\mathrm{T}}$, C_K, and C^2_K in (S2) with

X_K^{**}, X_K^{***}, C_K^{**}, and C_K^{***}, respectively, yields

$$\left[S^*_{K\beta_{K_j}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^*_{Ki}, \overline{C}^*_{Ki}, \overline{Z}_{Ki})\right] = \begin{bmatrix} U_{K\beta_{11}} - U_{K\beta_{12}}X_K^{**} - U_{K\beta_{13}}C_K^{**} \\ U_{K\beta_{21}}X_K^{**} - \{U_{K\beta_{22}}X_K^{***}\}^{\mathrm{T}} - U_{K\beta_{23}}C_K^{**}X_K^{**} \\ U_{K\beta_{31}}C_K^{**} - U_{K\beta_{32}}X_K^{**}C_K^{**} - U_{K\beta_{33}}C_K^{***} \\ U_{K\beta_{41}} - U_{K\beta_{42}}X_K^{**} - U_{K\beta_{43}}C_K^{**} \end{bmatrix}.$$

Similarly, applying (14) gives

$$\begin{aligned}
& S_{K\psi_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}_{Ki}, \overline{C}_{Ki}, \overline{Z}_{Ki}) \\
=& \big[Y_{Kji} - \big\{\beta_{Kj1}A_{K-1} + (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_K + \beta_{Kj3}C_K + \beta_{Kj4}Z_K \\
& +(\psi_{Kj0} + \psi_{Kj1}A_{K-1} + \psi_{Kj3}C_K + \psi_{Kj4}Z_K)A_K\big\}\big]\, A_K \begin{bmatrix} A_{K-1} \\ X_K \\ C_K \\ Z_K \end{bmatrix} \\
=& \begin{bmatrix} Y_{Kji}A_KA_{K-1} - \beta_{Kj1}A_{K-1}A_KA_{K-1} - (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_KA_KA_{K-1} - \beta_{Kj3}C_KA_KA_{K-1} - \beta_{Kj4}Z_KA_KA_{K-1} \\ -\psi_{Kj0}A_K^2A_{K-1} - \psi_{Kj1}A_{K-1}^2A_K^2 - \psi_{Kj3}C_KA_K^2A_{K-1} - \psi_{Kj4}Z_KA_K^2A_{K-1} \\ Y_{Kji}A_KX_K - \beta_{Kj1}A_{K-1}A_KX_K - \{(\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_KX_K^{\mathrm{T}}A_K\}^{\mathrm{T}} - \beta_{Kj3}C_KA_KX_K - \beta_{Kj4}Z_KA_KX_K \\ -\psi_{Kj0}A_K^2X_K - \psi_{Kj1}A_{K-1}A_K^2X_K - \psi_{Kj3}C_KA_K^2X_K - \psi_{Kj4}Z_KA_K^2X_K \\ Y_{Kji}A_KC_K - \beta_{Kj1}A_{K-1}A_KC_K - (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_KA_KC_K - \beta_{Kj3}C_K^2A_K - \beta_{Kj4}Z_KA_KC_K \\ -\psi_{Kj0}A_K^2C_K - \psi_{Kj1}A_{K-1}A_K^2C_K - \psi_{Kj3}C_K^2A_K^2 - \psi_{Kj4}Z_KA_K^2C_K \\ Y_{Kji}A_KZ_K - \beta_{Kj1}A_{K-1}A_KZ_K - (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)X_KA_KZ_K - \beta_{Kj3}C_KA_KZ_K - \beta_{Kj4}Z_K^2A_K \\ -\psi_{Kj0}A_K^2Z_K - \psi_{Kj1}A_{K-1}A_K^2Z_K - \psi_{Kj3}C_KA_K^2Z_K - \psi_{Kj4}Z_K^2A_K^2 \end{bmatrix}.
\end{aligned} \tag{S4}$$

Let

$$\begin{aligned}
U'_{K\beta_{11}} &= Y_{Kji}A_KA_{K-1} - \beta_{Kj1}A_{K-1}A_KA_{K-1} - \beta_{Kj4}Z_KA_KA_{K-1} - \psi_{Kj0}A_K^2A_{K-1} - \psi_{Kj1}A_{K-1}^2A_K^2 - \psi_{Kj4}Z_KA_K^2A_{K-1}, \\
U'_{K\beta_{12}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)A_KA_{K-1}, \\
U'_{K\beta_{13}} &= \beta_{Kj3}A_KA_{K-1} + \psi_{Kj3}A_K^2A_{K-1}, \\
U'_{K\beta_{21}} &= Y_{Kji}A_K - \beta_{Kj1}A_{K-1}A_K - \beta_{Kj4}Z_KA_K - \psi_{Kj0}A_K^2 - \psi_{Kj1}A_{K-1}A_K^2 - \psi_{Kj4}Z_KA_K^2, \\
U'_{K\beta_{22}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)A_K, \\
U'_{K\beta_{23}} &= \beta_{Kj3}A_K + \psi_{Kj3}A_K^2, \\
U'_{K\beta_{31}} &= Y_{Kji}A_K - \beta_{Kj1}A_{K-1}A_K - \beta_{Kj4}Z_KA_K - \psi_{Kj0}A_K^2 - \psi_{Kj1}A_{K-1}A_K^2 - \psi_{Kj4}Z_KA_K^2, \\
U'_{K\beta_{32}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)A_K, \\
U'_{K\beta_{33}} &= \beta_{Kj3}A_K + \psi_{Kj3}A_K^2, \\
U'_{K\beta_{41}} &= Y_{Kji}A_KZ_K - \beta_{Kj1}A_{K-1}A_KZ_K - \beta_{Kj4}Z_K^2A_K - \psi_{Kj0}A_K^2Z_K - \psi_{Kj1}A_{K-1}A_K^2Z_K - \psi_{Kj4}Z_K^2A_K^2, \\
U'_{K\beta_{42}} &= (\beta_{Kj0}, \beta_{Kj0} + \beta_{Kj2} + \psi_{Kj2}A_K)A_KZ_K, \\
U'_{K\beta_{43}} &= \beta_{Kj3}A_KZ_K + \psi_{Kj3}A_K^2Z_K.
\end{aligned}$$

Then (S4) becomes

$$\left[S_{K\psi_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}_{Ki}, \overline{C}_{Ki}, \overline{Z}_{Ki})\right] = \begin{bmatrix} U'_{K\beta_{11}} - U'_{K\beta_{12}}X_K - U'_{K\beta_{13}}C_K \\ U'_{K\beta_{21}}X_K - \{U'_{K\beta_{22}}X_KX_K^{\mathrm{T}}\}^{\mathrm{T}} - U'_{K\beta_{23}}C_KX_K \\ U'_{K\beta_{31}}C_K - U'_{K\beta_{32}}X_KC_K - U'_{K\beta_{33}}C_K^2 \\ U'_{K\beta_{41}} - U'_{K\beta_{42}}X_K - U'_{K\beta_{43}}C_K \end{bmatrix},$$

and therefore, we construct

$$\left[S^*_{K\psi_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^*_{Ki}, \overline{C}^*_{Ki}, \overline{Z}_{Ki})\right] = \begin{bmatrix} U'_{K\beta_{11}} - U'_{K\beta_{12}} X^{**}_K - U'_{K\beta_{13}} C^{**}_K \\ U'_{K\beta_{21}} X^{**}_K - \{U'_{K\beta_{22}} X^{***}_K\}^{\mathrm{T}} - U'_{K\beta_{23}} C^{**}_K X^{**}_K \\ U'_{K\beta_{31}} C^{**}_K - U'_{K\beta_{32}} X^{**}_K C^{**}_K - U'_{K\beta_{33}} C^{***}_K \\ U'_{K\beta_{41}} - U'_{K\beta_{42}} X^{**}_K - U'_{K\beta_{43}} C^{**}_K \end{bmatrix}.$$

Consequently, $S^*_{Kj}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^*_{Ki}, \overline{C}^*_{Ki}, \overline{Z}_{Ki})$ is defined to be

$$\left(S^{*\mathrm{T}}_{K\beta_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^*_{Ki}, \overline{C}^*_{Ki}, \overline{Z}_{Ki}), S^{*\mathrm{T}}_{K\psi_{Kj}}(\theta_{Kj}; Y_{Kji}, \overline{A}_{Ki}, \overline{X}^*_{Ki}, \overline{C}^*_{Ki}, \overline{Z}_{Ki})\right)^{\mathrm{T}},$$

where the dependence on π_{01}, π_{10} and σ^2_K is reflected through (S3).

S2 Proportion of Optimally Treated Future Patients

Table S1: Proportions of optimally treated individuals

		Scenario 1							
		Stage 1				Stage 2			
$(\pi_{10}, \pi_{01}, \sigma^2_k)$	Setting	Naive	RC	EE-known	EE-estimated	Naive	RC	EE-known	EE-estimated
(0.1,0.1,1)	Regular	0.880	0.979	0.970	0.973	0.870	0.986	0.981	0.983
	Weak non-regular	0.886	0.981	0.976	0.978	0.460	0.768	0.770	0.774
	Non-regular	0.878	0.982	0.978	0.979	0.784	0.721	0.717	0.716
(0.2,0.2,1.5)	Regular	0.820	0.976	0.949	0.954	0.775	0.983	0.968	0.969
	Weak non-regular	0.829	0.978	0.960	0.963	0.445	0.771	0.767	0.772
	Non-regular	0.824	0.980	0.962	0.965	0.813	0.711	0.690	0.697
(0.3,0.3,2)	Regular	0.808	0.973	0.908	0.915	0.732	0.983	0.944	0.947
	Weak non-regular	0.817	0.977	0.924	0.928	0.445	0.768	0.768	0.766
	Non-regular	0.816	0.979	0.934	0.939	0.865	0.712	0.662	0.677
		Scenario 2							
(0.1,0.1,1)	Regular	0.850	0.888	0.812	0.861	0.806	0.860	0.783	0.843
	Weak non-regular	0.855	0.893	0.814	0.864	0.461	0.556	0.891	0.860
	Non-regular	0.855	0.893	0.816	0.865	0.780	0.769	0.663	0.676
(0.2,0.2,1.5)	Regular	0.820	0.867	0.738	0.804	0.763	0.829	0.714	0.790
	Weak non-regular	0.827	0.873	0.741	0.808	0.445	0.543	0.796	0.790
	Non-regular	0.828	0.873	0.745	0.811	0.832	0.810	0.606	0.636
(0.3,0.3,2)	Regular	0.810	0.859	0.671	0.744	0.742	0.814	0.662	0.747
	Weak non-regular	0.818	0.866	0.674	0.751	0.444	0.534	0.696	0.714
	Non-regular	0.818	0.866	0.680	0.757	0.851	0.839	0.558	0.591

S3 Simulation Results for Correction Strategies with Reduced Sample Size

Table S2: Simulation studies for demonstrating biased estimation of the naive method in contrast to the EFLS method with reduced sample size: stages 1-2. Entries in bold are obtained from the setting without mismeasurements

$(\pi_{10}, \pi_{01}, \sigma_k^2)$	Method		regular			weak non-regular			non-regular		
			$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
						Stage 1 ($k = 1$)					
(0,0,0)	EFLS	Bias	**0.009**	**0.008**	**0.002**	**0.012**	**0.014**	**0.002**	**0.001**	**0.003**	**0.000**
		SE	**0.110**	**0.155**	**0.055**	**0.089**	**0.126**	**0.045**	**0.089**	**0.126**	**0.045**
		ESE	**0.125**	**0.158**	**0.058**	**0.105**	**0.126**	**0.045**	**0.117**	**0.132**	**0.044**
		MSE	**0.012**	**0.024**	**0.003**	**0.008**	**0.016**	**0.002**	**0.008**	**0.016**	**0.002**
		PBCR	**0.952**	**0.932**	**0.938**	**0.946**	**0.954**	**0.958**	**0.928**	**0.940**	**0.958**
		DBCR	**0.962**	**0.948**	**0.952**	**0.962**	**0.960**	**0.968**	**0.940**	**0.952**	**0.966**
(0.1,0.1,1)	Naive	Bias	0.129	0.235	0.504	0.152	0.242	0.505	0.134	0.249	0.501
		SE	0.107	0.151	0.038	0.100	0.141	0.035	0.100	0.141	0.035
		ESE	0.148	0.155	0.046	0.123	0.142	0.040	0.123	0.146	0.041
		MSE	0.028	0.078	0.255	0.033	0.078	0.256	0.028	0.082	0.252
		PBCR	0.872	0.748	0.000	0.788	0.664	0.000	0.840	0.690	0.000
		DBCR	0.758	0.488	0.000	0.690	0.432	0.000	0.726	0.412	0.000
(0.2,0.2,1.5)	Naive	Bias	0.268	0.487	0.691	0.303	0.489	0.695	0.255	0.492	0.691
		SE	0.108	0.152	0.030	0.105	0.148	0.029	0.105	0.148	0.029
		ESE	0.148	0.158	0.038	0.132	0.155	0.035	0.132	0.156	0.035
		MSE	0.083	0.260	0.478	0.103	0.261	0.484	0.076	0.264	0.478
		PBCR	0.608	0.020	0.000	0.388	0.034	0.000	0.496	0.014	0.000
		DBCR	0.448	0.046	0.000	0.286	0.052	0.000	0.398	0.050	0.000
(0.3,0.3,2)	Naive	Bias	0.386	0.724	0.799	0.446	0.718	0.799	0.365	0.726	0.800
		SE	0.077	0.108	0.017	0.076	0.108	0.017	0.076	0.108	0.017
		ESE	0.103	0.114	0.021	0.099	0.115	0.021	0.098	0.117	0.022
		MSE	0.155	0.536	0.639	0.205	0.527	0.639	0.139	0.539	0.640
		PBCR	0.018	0.000	0.000	0.002	0.000	0.000	0.004	0.000	0.000
		DBCR	0.052	0.000	0.000	0.008	0.000	0.000	0.034	0.000	0.000
						Stage 2 ($k = 2$)					
(0,0,0)	EFLS	Bias	**0.002**	**0.005**	**0.001**	**0.001**	**0.001**	**0.003**	**0.009**	**0.003**	**0.003**
		SE	**0.086**	**0.116**	**0.041**	**0.086**	**0.116**	**0.041**	**0.086**	**0.116**	**0.041**
		ESE	**0.090**	**0.118**	**0.041**	**0.086**	**0.118**	**0.041**	**0.088**	**0.117**	**0.043**
		MSE	**0.007**	**0.013**	**0.002**	**0.007**	**0.013**	**0.002**	**0.007**	**0.013**	**0.002**
		WTCR	**0.948**	**0.948**	**0.948**	**0.954**	**0.942**	**0.944**	**0.936**	**0.964**	**0.930**
(0.1,0.1,1)	Naive	Bias	0.170	0.284	0.843	0.238	0.394	0.001	0.013	0.035	0.001
		SE	0.121	0.164	0.041	0.100	0.136	0.034	0.088	0.120	0.030
		ESE	0.129	0.174	0.053	0.102	0.135	0.041	0.089	0.119	0.032
		MSE	0.044	0.108	0.712	0.067	0.174	0.001	0.008	0.016	0.001
		WTCR	0.720	0.568	0.000	0.314	0.160	0.902	0.948	0.948	0.926
(0.2,0.2,1.5)	Naive	Bias	0.346	0.595	1.167	0.449	0.778	0.003	0.046	0.073	0.002
		SE	0.134	0.183	0.036	0.109	0.149	0.029	0.089	0.122	0.024
		ESE	0.141	0.190	0.048	0.121	0.156	0.037	0.097	0.131	0.025
		MSE	0.138	0.388	1.363	0.213	0.628	0.001	0.010	0.020	0.001
		WTCR	0.286	0.100	0.000	0.020	0.004	0.874	0.898	0.894	0.946
(0.3,0.3,2)	Naive	Bias	0.499	0.870	1.347	0.676	1.169	0.001	0.066	0.110	0.001
		SE	0.099	0.137	0.022	0.081	0.112	0.018	0.063	0.088	0.014
		ESE	0.102	0.139	0.029	0.090	0.119	0.022	0.065	0.094	0.015
		MSE	0.259	0.776	1.815	0.463	1.379	0.000	0.008	0.020	0.000
		WTCR	0.004	0.000	0.000	0.000	0.000	0.888	0.828	0.728	0.940

Table S3: Simulation studies for assessing the performance of the RC, EE-known, and EE-estimated methods with reduced sample size: stages 1-2 and regular case

$(\pi_{10}, \pi_{01}, \sigma_k^2)$		RC			EE-known			EE-estimated		
		$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
					Stage 1 ($k = 1$)					
(0.1,0.1,1)	Bias	0.010	0.012	0.001	0.010	0.010	0.008	0.008	0.024	0.011
	SE	0.150	0.190	0.083	0.175	0.271	0.130	0.208	0.265	0.224
	ESE	0.152	0.187	0.083	0.199	0.287	0.142	0.181	0.247	0.140
	MSE	0.023	0.036	0.007	0.031	0.074	0.017	0.043	0.071	0.050
	PBCR	0.954	0.952	0.940	0.942	0.958	0.946	0.936	0.930	0.930
	DBCR	0.948	0.948	0.942	0.948	0.958	0.960	0.938	0.936	0.924
(0.2,0.2,1.5)	Bias	0.011	0.003	0.004	0.035	0.018	0.059	0.013	0.009	0.064
	SE	0.168	0.226	0.095	0.296	0.504	0.269	0.319	0.492	0.434
	ESE	0.164	0.225	0.090	0.316	0.542	0.290	0.291	0.476	0.312
	MSE	0.028	0.051	0.009	0.089	0.254	0.076	0.102	0.242	0.192
	PBCR	0.952	0.930	0.940	0.942	0.948	0.942	0.952	0.946	0.916
	DBCR	0.936	0.922	0.936	0.940	0.952	0.958	0.944	0.944	0.918
(0.3,0.3,2)	Bias	0.019	0.005	0.005	0.013	0.059	0.097	0.010	0.060	0.111
	SE	0.129	0.184	0.072	0.415	0.766	0.382	0.417	0.740	0.488
	ESE	0.124	0.180	0.071	0.436	0.808	0.390	0.398	0.711	0.413
	MSE	0.017	0.034	0.005	0.172	0.590	0.155	0.174	0.551	0.250
	PBCR	0.942	0.946	0.946	0.976	0.974	0.970	0.948	0.950	0.910
	DBCR	0.928	0.930	0.950	0.890	0.916	0.974	0.912	0.895	0.878
					Stage 2 ($k = 2$)					
(0.1,0.1,1)	Bias	0.004	0.001	0.000	0.004	0.020	0.022	0.008	0.004	0.036
	SE	0.138	0.179	0.093	0.170	0.246	0.150	0.188	0.236	0.200
	ESE	0.136	0.175	0.095	0.176	0.249	0.162	0.163	0.227	0.170
	MSE	0.019	0.032	0.009	0.029	0.061	0.023	0.035	0.056	0.041
	WTCR	0.954	0.954	0.934	0.940	0.936	0.934	0.976	0.958	0.990
(0.2,0.2,1.5)	Bias	0.005	0.012	0.002	0.028	0.041	0.108	0.030	0.039	0.118
	SE	0.163	0.217	0.108	0.312	0.483	0.357	0.394	0.562	0.573
	ESE	0.158	0.212	0.106	0.328	0.508	0.359	0.293	0.444	0.399
	MSE	0.027	0.047	0.012	0.098	0.235	0.139	0.156	0.317	0.342
	WTCR	0.960	0.942	0.958	0.954	0.966	0.966	0.984	0.978	0.966
(0.3,0.3,2)	Bias	0.001	0.002	0.004	0.074	0.122	0.151	0.043	0.085	0.104
	SE	0.123	0.165	0.079	0.433	0.715	0.540	0.428	0.710	0.554
	ESE	0.113	0.154	0.078	0.410	0.671	0.514	0.354	0.590	0.464
	MSE	0.015	0.027	0.006	0.193	0.526	0.314	0.185	0.511	0.318
	WTCR	0.974	0.972	0.948	0.976	0.992	0.958	0.992	0.994	0.920

Table S4: Simulation studies for assessing the performance of the RC, EE-known, and EE-estimated methods with reduced sample size: stages 1-2 and weak non-regular case

$(\pi_{10}, \pi_{01}, \sigma_k^2)$		RC			EE-known			EE-estimated		
		$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
					Stage 1 ($k = 1$)					
(0.1,0.1,1)	Bias	0.007	0.000	0.005	0.014	0.005	0.007	0.026	0.010	0.004
	SE	0.133	0.168	0.074	0.134	0.208	0.107	0.162	0.204	0.181
	ESE	0.130	0.159	0.074	0.156	0.222	0.112	0.146	0.199	0.113
	MSE	0.018	0.028	0.006	0.018	0.043	0.011	0.027	0.042	0.033
	PBCR	0.952	0.948	0.928	0.938	0.946	0.934	0.954	0.960	0.920
	DBCR	0.942	0.940	0.938	0.938	0.946	0.946	0.946	0.958	0.898
(0.2,0.2,1.5)	Bias	0.008	0.004	0.002	0.040	0.005	0.040	0.036	0.013	0.054
	SE	0.150	0.200	0.086	0.220	0.375	0.228	0.242	0.373	0.301
	ESE	0.147	0.198	0.087	0.243	0.405	0.231	0.231	0.372	0.261
	MSE	0.023	0.040	0.007	0.050	0.141	0.054	0.060	0.139	0.094
	PBCR	0.948	0.956	0.958	0.950	0.944	0.924	0.934	0.928	0.916
	DBCR	0.936	0.948	0.950	0.958	0.948	0.952	0.946	0.930	0.888
(0.3,0.3,2)	Bias	0.003	0.003	0.003	0.011	0.138	0.132	0.017	0.036	0.098
	SE	0.114	0.158	0.064	0.340	0.624	0.373	0.342	0.598	0.418
	ESE	0.115	0.156	0.061	0.359	0.662	0.394	0.307	0.555	0.346
	MSE	0.013	0.025	0.004	0.116	0.408	0.157	0.117	0.359	0.184
	PBCR	0.950	0.944	0.938	0.978	0.978	0.962	0.936	0.948	0.927
	DBCR	0.948	0.950	0.942	0.938	0.952	0.952	0.927	0.929	0.927
					Stage 2 ($k = 2$)					
(0.1,0.1,1)	Bias	0.004	0.003	0.001	0.005	0.013	0.000	0.007	0.009	0.002
	SE	0.117	0.160	0.061	0.125	0.173	0.085	0.154	0.172	0.160
	ESE	0.113	0.157	0.065	0.123	0.168	0.096	0.133	0.174	0.077
	MSE	0.014	0.026	0.004	0.016	0.030	0.007	0.024	0.030	0.026
	WTCR	0.956	0.954	0.944	0.960	0.968	0.934	0.986	0.958	1.000
(0.2,0.2,1.5)	Bias	0.009	0.000	0.001	0.048	0.065	0.002	0.031	0.052	0.005
	SE	0.138	0.195	0.073	0.191	0.271	0.178	0.222	0.287	0.293
	ESE	0.140	0.202	0.072	0.198	0.276	0.193	0.200	0.270	0.173
	MSE	0.019	0.038	0.005	0.039	0.078	0.032	0.050	0.085	0.086
	WTCR	0.958	0.950	0.946	0.938	0.942	0.956	0.974	0.944	0.994
(0.3,0.3,2)	Bias	0.005	0.006	0.000	0.061	0.114	0.016	0.060	0.100	0.009
	SE	0.105	0.152	0.056	0.257	0.414	0.398	0.268	0.381	0.348
	ESE	0.109	0.154	0.057	0.238	0.341	0.308	0.244	0.332	0.225
	MSE	0.011	0.023	0.003	0.070	0.184	0.159	0.075	0.155	0.121
	WTCR	0.941	0.951	0.939	0.963	0.963	0.996	0.959	0.959	0.996

Table S5: Simulation studies for assessing the performance of the RC, EE-known, and EE-estimated methods with reduced sample size: stages 1-2 and non-regular case

$(\pi_{10}, \pi_{01}, \sigma_k^2)$		RC			EE-known			EE-estimated		
		$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
		Stage 1 ($k = 1$)								
(0.1,0.1,1)	Bias	0.005	0.003	0.001	0.007	0.004	0.019	0.005	0.009	0.018
	SE	0.129	0.167	0.074	0.130	0.201	0.106	0.160	0.198	0.181
	ESE	0.131	0.167	0.072	0.150	0.211	0.114	0.149	0.193	0.120
	MSE	0.017	0.028	0.005	0.017	0.040	0.012	0.026	0.039	0.033
	PBCR	0.940	0.942	0.956	0.942	0.944	0.950	0.940	0.948	0.936
	DBCR	0.934	0.926	0.938	0.946	0.946	0.954	0.928	0.934	0.906
(0.2,0.2,1.5)	Bias	0.001	0.000	0.003	0.007	0.020	0.060	0.002	0.018	0.034
	SE	0.144	0.198	0.085	0.207	0.349	0.224	0.222	0.340	0.294
	ESE	0.150	0.201	0.083	0.222	0.363	0.247	0.209	0.343	0.245
	MSE	0.021	0.039	0.007	0.043	0.122	0.054	0.049	0.116	0.088
	PBCR	0.972	0.944	0.956	0.948	0.956	0.938	0.958	0.936	0.944
	DBCR	0.950	0.940	0.946	0.958	0.958	0.962	0.936	0.930	0.902
(0.3,0.3,2)	Bias	0.005	0.004	0.002	0.037	0.064	0.091	0.017	0.026	0.082
	SE	0.109	0.157	0.064	0.285	0.515	0.352	0.293	0.521	0.404
	ESE	0.108	0.152	0.061	0.306	0.545	0.359	0.271	0.480	0.323
	MSE	0.012	0.025	0.004	0.083	0.269	0.132	0.086	0.272	0.170
	PBCR	0.962	0.962	0.944	0.978	0.974	0.970	0.952	0.944	0.897
	DBCR	0.946	0.938	0.948	0.954	0.950	0.952	0.929	0.948	0.893
		Stage 2 ($k = 2$)								
(0.1,0.1,1)	Bias	0.010	0.001	0.001	0.013	0.008	0.000	0.002	0.008	0.003
	SE	0.097	0.138	0.053	0.104	0.151	0.064	0.141	0.154	0.150
	ESE	0.098	0.136	0.054	0.102	0.146	0.068	0.101	0.142	0.058
	MSE	0.010	0.019	0.003	0.011	0.023	0.004	0.020	0.024	0.023
	WTCR	0.932	0.948	0.936	0.946	0.960	0.934	0.992	0.962	1.000
(0.2,0.2,1.5)	Bias	0.008	0.016	0.001	0.009	0.021	0.004	0.001	0.005	0.007
	SE	0.109	0.161	0.059	0.139	0.212	0.098	0.173	0.220	0.260
	ESE	0.113	0.164	0.060	0.149	0.222	0.099	0.131	0.195	0.093
	MSE	0.012	0.026	0.003	0.019	0.045	0.010	0.030	0.048	0.068
	WTCR	0.944	0.956	0.958	0.930	0.942	0.956	0.994	0.974	1.000
(0.3,0.3,2)	Bias	0.002	0.004	0.005	0.024	0.045	0.007	0.010	0.020	0.002
	SE	0.085	0.130	0.045	0.152	0.241	0.121	0.183	0.271	0.252
	ESE	0.078	0.124	0.046	0.151	0.243	0.115	0.132	0.204	0.116
	MSE	0.007	0.017	0.002	0.024	0.060	0.015	0.034	0.074	0.064
	WTCR	0.965	0.971	0.951	0.955	0.937	0.982	0.982	0.980	1.000

S4 Simulation Results for Correction Strategies with Reduced Validation Subsample Size

Table S6: Simulation studies for assessing the performance of the RC, EE-known, and EE-estimated methods with reduced validation subsample size: stages 1-2 and regular case

$(\pi_{10}, \pi_{01}, \sigma_k^2)$		RC			EE-known			EE-estimated		
		$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
		Stage 1 ($k = 1$)								
(0.1,0.1,1)	Bias	0.019	0.005	0.012	0.005	0.015	0.010	0.008	0.022	0.069
	SE	0.183	0.202	0.171	0.122	0.191	0.090	0.162	0.232	0.179
	ESE	0.168	0.180	0.159	0.135	0.200	0.099	0.192	0.325	0.339
	MSE	0.034	0.041	0.029	0.015	0.037	0.008	0.026	0.054	0.037
	PBCR	0.971	0.936	0.931	0.948	0.942	0.946	0.927	0.905	0.652
	DBCR	0.943	0.943	0.945	0.954	0.954	0.944	0.932	0.915	0.677
(0.2,0.2,1.5)	Bias	0.044	0.044	0.054	0.008	0.044	0.036	0.037	0.062	0.031
	SE	0.265	0.352	0.285	0.199	0.340	0.173	0.234	0.359	0.232
	ESE	0.237	0.315	0.259	0.223	0.368	0.177	0.349	0.476	0.388
	MSE	0.072	0.126	0.084	0.040	0.118	0.031	0.056	0.133	0.055
	PBCR	0.920	0.932	0.932	0.962	0.956	0.956	0.858	0.895	0.557
	DBCR	0.920	0.929	0.934	0.972	0.972	0.974	0.872	0.918	0.571
(0.3,0.3,2)	Bias	0.048	0.072	0.016	0.011	0.018	0.065	0.041	0.152	0.127
	SE	0.348	0.547	0.328	0.393	0.723	0.350	0.555	0.919	0.625
	ESE	0.314	0.503	0.282	0.399	0.735	0.337	0.461	0.864	0.694
	MSE	0.123	0.304	0.108	0.155	0.523	0.127	0.310	0.868	0.407
	PBCR	0.960	0.940	0.962	0.967	0.965	0.974	0.910	0.934	0.632
	DBCR	0.904	0.896	0.874	0.965	0.959	0.976	0.889	0.920	0.656
		Stage 2 ($k = 2$)								
(0.1,0.1,1)	Bias	0.008	0.010	0.040	0.013	0.023	0.025	0.012	0.015	0.083
	SE	0.241	0.213	0.289	0.121	0.173	0.105	0.147	0.192	0.175
	ESE	0.227	0.214	0.268	0.134	0.184	0.112	0.179	0.256	0.442
	MSE	0.058	0.045	0.085	0.015	0.030	0.012	0.022	0.037	0.038
	WTCR	0.958	0.938	0.958	0.928	0.934	0.926	0.900	0.874	0.558
(0.2,0.2,1.5)	Bias	0.005	0.011	0.053	0.028	0.043	0.045	0.044	0.031	0.183
	SE	0.334	0.360	0.453	0.206	0.318	0.221	0.200	0.292	0.221
	ESE	0.292	0.334	0.393	0.198	0.319	0.240	0.254	0.348	0.423
	MSE	0.112	0.130	0.208	0.043	0.103	0.051	0.042	0.086	0.082
	WTCR	0.974	0.944	0.944	0.970	0.952	0.944	0.848	0.882	0.517
(0.3,0.3,2)	Bias	0.042	0.075	0.121	0.021	0.032	0.110	0.093	0.096	0.363
	SE	0.413	0.525	0.580	0.402	0.661	0.455	0.303	0.494	0.304
	ESE	0.358	0.497	0.553	0.386	0.620	0.420	0.328	0.495	0.467
	MSE	0.172	0.281	0.351	0.162	0.438	0.219	0.100	0.253	0.224
	WTCR	0.954	0.892	0.946	0.982	0.973	0.937	0.915	0.949	0.525

Table S7: Simulation studies for assessing the performance of the RC, EE-known, and EE-estimated methods with reduced validation subsample size: stages 1-2 and weak non-regular case

$(\pi_{10}, \pi_{01}, \sigma_k^2)$		RC			EE-known			EE-estimated		
		$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
					Stage 1 ($k = 1$)					
(0.1,0.1,1)	Bias	0.005	0.022	0.011	0.023	0.003	0.001	0.005	0.042	0.066
	SE	0.189	0.200	0.173	0.094	0.145	0.074	0.126	0.171	0.167
	ESE	0.178	0.186	0.160	0.115	0.150	0.081	0.161	0.230	0.347
	MSE	0.036	0.040	0.030	0.009	0.021	0.005	0.016	0.031	0.032
	PBCR	0.945	0.933	0.942	0.938	0.952	0.954	0.871	0.866	0.584
	DBCR	0.942	0.930	0.952	0.946	0.962	0.966	0.894	0.889	0.597
(0.2,0.2,1.5)	Bias	0.030	0.069	0.044	0.040	0.006	0.013	0.034	0.065	0.041
	SE	0.276	0.350	0.281	0.151	0.255	0.145	0.183	0.289	0.234
	ESE	0.255	0.341	0.256	0.177	0.291	0.155	0.262	0.427	0.368
	MSE	0.077	0.127	0.081	0.024	0.065	0.021	0.035	0.088	0.056
	PBCR	0.949	0.965	0.961	0.950	0.938	0.936	0.851	0.861	0.587
	DBCR	0.919	0.919	0.928	0.954	0.950	0.958	0.856	0.861	0.606
(0.3,0.3,2)	Bias	0.007	0.046	0.042	0.019	0.066	0.039	0.080	0.246	0.244
	SE	0.340	0.512	0.350	0.311	0.567	0.299	0.825	1.312	1.959
	ESE	0.301	0.457	0.307	0.318	0.598	0.295	0.675	0.966	1.217
	MSE	0.116	0.264	0.124	0.097	0.326	0.091	0.687	1.782	3.897
	PBCR	0.990	0.972	0.968	0.962	0.966	0.958	0.894	0.897	0.656
	DBCR	0.924	0.904	0.914	0.964	0.972	0.958	0.875	0.916	0.667
					Stage 2 ($k = 2$)					
(0.1,0.1,1)	Bias	0.008	0.022	0.004	0.008	0.010	0.004	0.022	0.045	0.003
	SE	0.177	0.238	0.059	0.088	0.121	0.060	0.117	0.138	0.113
	ESE	0.175	0.232	0.054	0.087	0.119	0.067	0.215	0.278	0.073
	MSE	0.031	0.057	0.003	0.008	0.015	0.004	0.014	0.021	0.013
	WTCR	0.900	0.890	0.966	0.944	0.942	0.926	0.792	0.708	0.996
(0.2,0.2,1.5)	Bias	0.004	0.003	0.002	0.016	0.030	0.002	0.083	0.044	0.023
	SE	0.280	0.426	0.095	0.131	0.184	0.117	0.143	0.189	0.189
	ESE	0.282	0.421	0.082	0.127	0.181	0.124	0.270	0.314	0.135
	MSE	0.078	0.181	0.009	0.017	0.035	0.014	0.027	0.038	0.036
	WTCR	0.912	0.914	0.988	0.958	0.948	0.958	0.623	0.779	0.984
(0.3,0.3,2)	Bias	0.090	0.156	0.004	0.061	0.098	0.019	0.110	0.115	0.008
	SE	0.369	0.598	0.131	0.231	0.339	0.258	0.255	0.358	0.467
	ESE	0.359	0.581	0.116	0.228	0.320	0.236	0.365	0.445	0.252
	MSE	0.144	0.382	0.017	0.057	0.125	0.067	0.077	0.141	0.218
	WTCR	0.886	0.882	0.982	0.956	0.954	0.993	0.685	0.804	1.000

Table S8: Simulation studies for assessing the performance of the RC, EE-known, and EE-estimated methods with reduced validation subsample size: stages 1-2 and non-regular case

$(\pi_{10}, \pi_{01}, \sigma_k^2)$		RC			EE-known			EE-estimated		
		$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$	$\psi_{0k\delta}$	$\psi_{1k\delta}$	$\psi_{2k\delta}$
		Stage 1 ($k = 1$)								
(0.1,0.1,1)	Bias	0.003	0.017	0.013	0.001	0.013	0.007	0.003	0.024	0.065
	SE	0.175	0.203	0.176	0.091	0.141	0.073	0.120	0.162	0.166
	ESE	0.155	0.181	0.162	0.109	0.145	0.084	0.158	0.215	0.336
	MSE	0.031	0.041	0.031	0.008	0.020	0.005	0.014	0.027	0.032
	PBCR	0.957	0.938	0.945	0.960	0.954	0.958	0.874	0.919	0.605
	DBCR	0.952	0.959	0.928	0.960	0.956	0.964	0.910	0.928	0.592
(0.2,0.2,1.5)	Bias	0.006	0.015	0.045	0.016	0.010	0.018	0.046	0.144	0.092
	SE	0.247	0.330	0.281	0.139	0.234	0.140	0.193	0.317	0.300
	ESE	0.233	0.301	0.248	0.149	0.247	0.158	0.278	0.475	0.521
	MSE	0.061	0.109	0.081	0.020	0.055	0.020	0.039	0.121	0.098
	PBCR	0.958	0.922	0.946	0.962	0.956	0.950	0.861	0.844	0.532
	DBCR	0.918	0.920	0.939	0.968	0.964	0.974	0.886	0.878	0.549
(0.3,0.3,2)	Bias	0.039	0.078	0.073	0.005	0.000	0.031	0.101	0.262	0.029
	SE	0.336	0.534	0.356	0.255	0.461	0.267	0.505	0.956	0.661
	ESE	0.306	0.493	0.344	0.264	0.473	0.236	0.573	0.986	0.668
	MSE	0.114	0.291	0.132	0.065	0.213	0.072	0.265	0.983	0.438
	PBCR	0.978	0.976	0.952	0.958	0.970	0.944	0.898	0.877	0.602
	DBCR	0.878	0.906	0.904	0.946	0.966	0.942	0.877	0.866	0.609
		Stage 2 ($k = 2$)								
(0.1,0.1,1)	Bias	0.003	0.004	0.001	0.000	0.000	0.002	0.008	0.009	0.003
	SE	0.077	0.110	0.046	0.074	0.106	0.044	0.104	0.122	0.139
	ESE	0.079	0.114	0.046	0.071	0.100	0.045	0.083	0.114	0.066
	MSE	0.006	0.012	0.002	0.005	0.011	0.002	0.011	0.015	0.019
	WTCR	0.954	0.948	0.960	0.956	0.964	0.952	0.984	0.956	1.000
(0.2,0.2,1.5)	Bias	0.001	0.002	0.002	0.003	0.000	0.002	0.012	0.028	0.006
	SE	0.105	0.163	0.064	0.097	0.148	0.066	0.128	0.170	0.182
	ESE	0.102	0.154	0.059	0.099	0.146	0.063	0.109	0.163	0.111
	MSE	0.011	0.027	0.004	0.009	0.022	0.004	0.017	0.030	0.033
	WTCR	0.962	0.974	0.990	0.952	0.950	0.964	0.980	0.959	0.997
(0.3,0.3,2)	Bias	0.009	0.015	0.002	0.016	0.019	0.006	0.001	0.037	0.012
	SE	0.145	0.237	0.089	0.150	0.238	0.114	0.167	0.253	0.172
	ESE	0.129	0.209	0.077	0.145	0.228	0.113	0.147	0.222	0.118
	MSE	0.021	0.056	0.008	0.023	0.057	0.013	0.028	0.065	0.030
	WTCR	0.976	0.970	0.990	0.940	0.954	0.977	0.961	0.961	1.000

S5 Data Analysis

The data include publicly available COVID-19 information for 164 countries, covering a period of approximately ten months with time windows beginning from the date of the first confirmed COVID-19 case in each country; the study periods of all those 164 countries span from April 1, 2020 to September 30, 2020. The data contain information about containment and closure policies, including workplace closure and international travel control, as well as health system policies, including testing and contact tracing policies. In addition, the data include country-level information over the study period, such as the number of COVID-19 cases per million people, the total number of COVID-19 deaths, the economic growth percent change in quarterly real gross domestic product, the care system quality score, obesity prevalence, smoking prevalence, substance use prevalence, and socioeconomic factors.

Information regarding containment and closure policies, as well as health system policies, is collected from the OxCGRT (Hale et al., 2021). Following Hale et al. (2021), the strictness score of implementing each of the preventive policies on day t, denoted $I_{j,t}$, is calculated by:

$$I_{j,t} = 100\left\{ \frac{v_{j,t} - 0.5(F_j - f_{j,t})}{N_j} \right\} \qquad \text{for } j = 1, \cdots, 7,$$

where $v_{j,t}$ denotes the ordinal variable for policy j on day t, N_j represents the maximum ordinal value of $v_{j,t}$, F_j denotes a binary value from $\{0, 1\}$ to indicate whether the jth policy has a time-dependent flag variable, denoted $f_{j,t}$, for a geographic scope, with 1 representing "yes" and 0 otherwise; and $f_{j,t}$ is the binary flag variable for the jth policy on day t, taking value 0 if the policy is "geographically targeted" (i.e., being applied only to a sub-region of a jurisdiction) or 1 if the policy is "general" (i.e., being applied throughout that jurisdiction).

Similar to Khadem Charvadeh and Yi (2024), we let $\mathcal{E}_1$ denote the set of labels for workplace closure and international travel control policies, and let $\mathcal{E}_2$ denote the set of labels for testing and contact tracing policies. Then the overall strictness score for policies of the same nature on a given day, denoted Index_{lt}, is calculated by:

$$\text{Index}_{lt} = \frac{1}{|\mathcal{E}_l|} \sum_{j \in \mathcal{E}_l} I_{j,t},$$

where $l = 1, 2$.

Data on the total number of COVID-19 cases per million people and the total number of COVID-19 deaths are extracted from the website "Ourworldindata" (Ritchie et al., 2020). Data on economic growth percent change in the quarterly real gross domestic product, denoted *eco-growth*, are extracted from the website "The Global Economy" (The Global Economy, 2020). Care system quality score (care-score), obesity prevalence (obesity-prev), smoking prevalence (smoking-prev), and substance use prevalence (substance-prev) for 2019 are obtained from the Legatum Institute (The Legatum Institute, 2019).

As in Khadem Charvadeh and Yi (2024), we consider the following socioeconomic factors: the most recent population weighted geometric mean density (popu-density) (Edwards et al., 2021), the population proportion of people aged 65 and above (senior-prop) for 2019 (The World Bank, 2019a), gross domestic product per capita based on purchasing power parity (GDP) for 2019 (The Global Economy, 2019), government effectiveness score (government-eff) for 2019 (The World Bank, 2019b), and infrastructure and market access score (infra-market) for 2019 (The Legatum Institute, 2019).

The inclusion of these factors is driven by the following considerations. The "care-score" assesses the ability of a health system to treat and cure diseases and illnesses in the population. It is measured based on a number of indicators, including healthcare coverage, health facilities, health practitioners and staff, satisfaction with healthcare, etc., and takes values from 0 (worst) to 100 (best). As most people and economic agents live much more concentrated in space, the "popu-density" is regarded as a more meaningful measure than the simple population density. The "GDP" is the most commonly used measure of economic activity and represents the total monetary value of the produced goods and services in a country during a specific period. A country with a larger "GDP" tends to have a higher standard of living. The "government-eff" measures perceptions of the quality of public and civil services, the quality of policy formulation and implementation, and the credibility of the government's commitment to such policies. Its value ranges from -2.5 to 2.5, with a higher value indicating better governance. The "infra-market," with values ranging from 0 (worst) to 100 (best), measures the quality of the infrastructure that enables trade and distortions in the market for goods and services.

Although strict mitigation policies, such as international travel control policy, have been successful in slowing the spread of the virus (Wells et al., 2020), their inevitable impacts on economic performance can be immense (Ozili and Arun, 2023). To examine the trade-off between the economic fallout and health costs of the pandemic, we use the Q-learning approach with bivariate outcomes. This approach allows us to model the interdependence between economic and health outcomes and estimate the optimal policy decisions that minimize the overall costs. Since mismeasurement is ubiquitous in applications, it is interesting to investigate how mismeasurement in covariates can affect estimation results.

Now we take the first outcome as the number of COVID-19 deaths per hundred COVID-19 cases, denoted *CFR*, and the second outcome as the negative of *eco-growth*, denoted −*eco-growth*. We examine how different degrees of covariate mismeasurement can affect the parameter estimates associated with policy decisions.

The study period is defined as six months, starting from April 1, 2020, to September 30, 2020. Since the information on the *eco-growth* is available only on a quarterly basis, this period is divided into two stages ($K = 2$) to align with the quarterly reporting of *eco-growth*: stage 1 from April 1 to June 30, 2020, and stage 2 from July 1 to September 30, 2020. The information about the total number of COVID-19 cases per million people, COVID-19 *CFR*, and the first quarter *eco-growth* of 2020 gathered at the end of March, 2020 are taken as the baseline features.

Following the same procedure described in Khadem Charvadeh and Yi (2024), we obtain *CFR* and stringency score of preventive policies for each stage. Furthermore, we log-transform the *CFR* at each stage to remove the nonnegativity constraint of *CFR*. The *eco-growth* for stages 1 and 2 represents the percent change in the real gross domestic product in the second and third quarters of 2020 compared to the second and third quarters of 2019, respectively.

For $k = 1, 2$, let A_k denote a binary action at stage k, which is defined as follows. For $l = 1, 2$ and $k = 1, 2$, let average-Index$_{l(k)}$ represent the average of the Index$_{lt}$ with t indexing the days in the period of stage k. Let $A_k = 1$ if average-Index$_{1(k)}$ is greater than average-Index$_{2(k)}$ and $A_k = 0$ otherwise. Let "cases-enter$_k$" represent the total number of COVID-19 cases per million people at the start of stage k, let "*CFR*-enter$_k$" represent the recorded *CFR* at the start of the stage k, and let "*eco-growth*-enter$_k$" represent the recorded *eco-growth* at the start of the stage k. As the government-eff value ranges from -2.5 to 2.5, we convert it into a binary variable, taking the value 1 or 0, corresponding to "high government effectiveness" if its original value is no smaller than the threshold value 0, and "low government effectiveness" otherwise.

In the following analyses, for sensible comparisons, we normalize all the non-binary covariates by subtracting their means from the values and dividing them by their standard deviations. For the first outcome, *CFR*, we take the covariates senior-prop, GDP, government-eff, obesity-prev, smoking-prev, substance-prev, popu-density, and care-score as confounders, and treat GDP, government-eff, and popu-density, together with *CFR*-enter as prescriptive variables; for the second outcome, −*eco-growth*, we take GDP, government-eff, infra-market, and cases-enter as confounders, and consider GDP, government-eff, infra-market, together with *eco-growth*-enter$_k$ as prescriptive variables. For covariates that are considered for one outcome but not the other, we assume zero effect on the non-associated outcome. This assumption applies to all effects, including both main and interaction effects.

We employ linear regression models to describe the Q-functions for $k = 1, 2$:

$$\begin{aligned}
Q_2(H_2, A_2, \delta) = \delta\Big\{&\beta_{02,1} + \beta_{12,1} \times \text{senior-prop} + \beta_{22,1} \times \text{GDP} + \beta_{32,1} \times \text{government-eff} \\
&+ \beta_{42,1} \times \text{obesity-prev} + \beta_{52,1} \times \text{smoking-prev} + \beta_{62,1} \times \text{substance-prev} \\
&+ \beta_{72,1} \times \text{popu-density} + \beta_{82,1} \times \text{care-score} + (\psi_{02,1} + \psi_{12,1} \times \text{GDP} \\
&+ \psi_{22,1} \times \text{government-eff} + \psi_{32,1} \times \text{popu-density} + \psi_{42,1} \textit{CFR}\text{-enter}_2) A_2 \Big\} \\
&+ (1-\delta)\Big\{\beta_{02,2} + \beta_{12,2} \times \text{GDP} + \beta_{22,2} \times \text{government-eff} + \beta_{32,2} \times \text{infra-market} \\
&+ \beta_{42,2} \times \text{cases-enter}_2 + (\psi_{02,2} + \psi_{12,2} \times \text{GDP} + \psi_{22,2} \times \text{government-eff} \\
&+ \psi_{32,2} \times \text{infra-market} + \psi_{42,2} \times \textit{eco-growth}\text{-enter}_2) A_2 \Big\},
\end{aligned}$$

and

$$\begin{aligned}
Q_1(H_1, A_1, \delta) = \delta\Big\{&\beta_{01,1} + \beta_{11,1} \times \text{senior-prop} + \beta_{21,1} \times \text{GDP} + \beta_{31,1} \times \text{government-eff} \\
&+ \beta_{41,1} \times \text{obesity-prev} + \beta_{51,1} \times \text{smoking-prev} + \beta_{61,1} \times \text{substance-prev} \\
&+ \beta_{71,1} \times \text{popu-density} + \beta_{81,1} \times \text{care-score} + (\psi_{01,1} + \psi_{11,1} \times \text{GDP} \\
&+ \psi_{21,1} \times \text{government-eff} + \psi_{31,1} \times \text{popu-density} + \psi_{41,1} \textit{CFR}\text{-enter}_1) A_1 \Big\} \\
&+ (1-\delta)\Big\{\beta_{01,2} + \beta_{11,2} \times \text{GDP} + \beta_{21,2} \times \text{government-eff} + \beta_{31,2} \times \text{infra-market} \\
&+ \beta_{41,2} \times \text{cases-enter}_1 + (\psi_{01,2} + \psi_{11,2} \times \text{GDP} + \psi_{21,2} \times \text{government-eff} \\
&+ \psi_{31,2} \times \text{infra-market} + \psi_{41,2} \times \textit{eco-growth}\text{-enter}_1) A_1 \Big\}.
\end{aligned}$$

We conduct three analyses here by setting $\delta = 0.9$ or $\delta = 0.1$. In Analysis 1, we treat all the variables to be error-free and implement the procedure in Section 2.1. The estimation results for the model parameters are reported in Tables S9-S10 under the heading $(\pi_{10}, \pi_{01}, \sigma^2) = (0, 0, 0)$ for stages 1 and 2, respectively.

The next two analyses are to assess the effects of possibly mismeasured covariates on each stage parameter estimates. In particular, for the first and second outcomes, we take the government-eff as an error-prone binary covariate. We further take the popu-density as an error-prone continuous covariate for the first outcome, and the infra-market as the error-prone continuous covariate for the second outcome. We carry out sensitivity analyses using the two correction methods described in Sections 4 and 5, and respectively call them Analysis 3 and Analysis 2. For Analysis 2, we consider three sets of misclassification probabilities as well as measurement error degrees $(\pi_{10}, \pi_{01}, \sigma^2)^{\mathrm{T}} = (0.02, 0.02, 0.02)^{\mathrm{T}}, (0.03, 0.03, 0.05)^{\mathrm{T}}, (0.04, 0.04, 0.07)^{\mathrm{T}}$. For Analysis 3, we consider models (20) and (21) with

$$m\,(\text{government-eff};\zeta) = \zeta_0 + \zeta_1 \times \text{government-eff},$$

$$m\left(\text{popu-density};\xi_1\right) = \xi_{10} + \xi_{11} \times \text{popu-density},$$

and

$$m\left(\text{infra-market};\xi_2\right) = \xi_{20} + \xi_{21} \times \text{infra-market},$$

where we consider three sets of values for the model parameters $\zeta = \left(\zeta_0, \zeta_1\right)^{\mathrm{T}}$, $\xi_1 = \left(\xi_{10}, \xi_{11}\right)^{\mathrm{T}}$, and $\xi_2 = \left(\xi_{20}, \xi_{21}\right)^{\mathrm{T}}$ that are listed in Table S11. Numerical results of Analysis 2 and Analysis 3 are respectively reported in Tables S9-S10 and S12-S13.

The numerical results of Analyses 1 and 2 reveal different evidence for the significance of some parameters. Consider the case when $\delta = 0.9$. For stage 1 and when the mismeasurement degree is set to be $(0, 0, 0)$, $(0.02, 0.02, 0.02)$, or $(0.04, 0.04, 0.07)$, there is no evidence to support the significance of $\psi_{21,1}$. In contrast, if the mismeasurement degree is $(0.03, 0.03, 0.05)$, $\psi_{21,1}$ is statistically significant. When $\delta = 0.1$ and the mismeasurement degree is set to be $(0, 0, 0)$, there exists no evidence suggesting the significance of $\psi_{21,1}$, $\psi_{01,2}$, and $\psi_{21,2}$, while for all the other mismeasurement degrees, $\psi_{21,1}$, $\psi_{01,2}$, and $\psi_{21,2}$ are found to be statistically significant. These findings suggest that if the government-eff is subject to misclassification with sensitivity and specificity of 0.98, 0.97, or 0.96, naively estimating parameters may lower statistical power.

On the other hand, the three analyses do find some common evidence. Analyses 1, 2, and 3 collectively underscore the substantive importance of covariates associated with $\psi_{41,1}$ and $\psi_{41,2}$, regardless of δ being 0.9 or 0.1, and the extent of mismeasurement considered here. All three analyses find the evidence to support that $\psi_{41,1}$ and $\psi_{41,2}$ are statistically significant.

Similar to the case for stage 1, Analyses 1 and 2 reveal different findings for stage 2. In particular, when the mismeasurement degree is set to be $(0, 0, 0)$, $\psi_{42,1}$ is statistically insignificant, while for all the other mismeasurement degrees, $\psi_{42,1}$ is significantly different from zero. Furthermore, $\psi_{42,2}$ is statistically significant when the mismeasurement degree is set to be $(0, 0, 0)$, while for all the other mismeasurement degrees, $\psi_{42,2}$ is statistically insignificant. Regarding Analysis 3, it is evident that $\psi_{12,1}$, $\psi_{42,1}$, and $\psi_{42,2}$ are statistically significant for all three sets of mismeasurement degrees.

In Tables S14-S15, we report the estimated optimal actions at stages 1 and 2 derived from Analyses 1-3 for some selected countries with δ set to be 0.9 or 0.1, respectively. The countries included in Tables S14-S15 are divided into two groups based on the disparities observed in Analyses 1-3. We consider Italy and the UK from the first group and the UAE, the USA, and Canada from the second group.

Examining Italy and the UK within the first group, it is evident that all conducted analyses support the precedence of health system policies over containment and closure policies in both stage 1 and stage 2. This holds true irrespective of the weight parameter $\delta = 0.9$ or 0.1. These findings underscore the significance of affording greater prominence to health system policies compared to containment and closure measures when prioritizing health outcomes. Moreover, even when accentuating economic considerations, the preeminence of health system policies over containment and closure strategies remains the same.

Now, we consider the UAE within the second group. If reducing the first outcome *CFR* is more important than reducing the second outcome $-$*eco-growth* (i.e., when $\delta = 0.9$), Analysis 1 indicates the advantage of emphasizing containment and closure policies over health system policies in both stages 1 and 2; Analysis 2 mirrors these findings, except for the scenario with the most pronounced degree of mismeasurement, where the recommendation deviates by suggesting prioritization of health system policies in stage 1 and the opposite priority in stage 2. Conversely, the implications of Analysis 3 point toward the precedence of containment and closure policies in stage 1, which flips in favor of health system policies in stage 2. Shifting the perspective to $\delta = 0.1$, Analyses 1 and 2 both advocate for the prioritization of containment and closure policies in stage 1, followed by a shift in favor of health system policies in stage 2. This consensus remains true for Analysis 3 except with the highest degree of mismeasurement, which recommends a focus on health system policies across both stages 1 and 2.

Regarding the USA, Analyses 1 and 2 suggest giving precedence to health system policies in stage 1, and favoring containment and closure policies in stage 2 when $\delta = 0.9$. However, Analysis 3 suggests the opposite. When we change our perspective to $\delta = 0.1$, both Analyses 1 and 2 reach a common recommendation: prioritizing containment and closure policies in both stages 1 and 2. This consensus is robust, with the exception of Analysis 2 with the highest mismeasurement degree. In this specific case, the counsel pivots towards favoring health system policies in stage 1, and containment and closure policies in stage 2. Analysis 3 maintains its stance irrespective of the shift in δ, advocating the same prioritization strategy as in the scenario where $\delta = 0.9$.

Regarding Canada with δ set to 0.9, all three analyses imply a preference for prioritizing health system policies over containment and closure measures in both stages 1 and 2. This unanimity holds true except for Analysis 3 with the highest degree of mismeasurement, which recommends prioritizing health system policies over containment and closure measures solely in stage 1. As the parameter δ is adjusted to $\delta = 0.1$, Analyses 1 and 2 retain their alignment by suggesting precedence of health system policies for both stages 1 and 2. However, Analysis 3 introduces a nuanced perspective. When the mismeasurement degree is specified by Set 1, it maintains the stance of prioritizing health system policies over containment and closure measures. Yet, when the mismeasurement degree is specified by Set 2 or 3, a distinctive strategy emerges: favoring containment and closure policies over health system policies in stage 1 and vice versa in stage 2.

The variations in the results obtained from different analyses underscore the substantial adverse consequences stemming from covariate mismeasurement. These discrepancies highlight the potential implications of erroneous measurement of relevant factors in determining optimal strategies. With the uncertainty of quantifying the potential influence of covariate mismeasurement, it is crucial to recognize the associated uncertainties and limitations in policy recommendations derived from such analyses.

Finally, we emphasize that the data analysis serves primarily to illustrate the utility of the method in Section 2, rather than to provide guidance for decision-making. The results should not be over-interpreted due to the limitations of the analysis. First, the continuous "government-eff" variable was dichotomized into a binary indicator for "high government effectiveness" and "low government effectiveness" to represent a misclassification-prone binary variable. While this approach is common in practice and often simplifies interpretation, it can lead to a loss of information and potential estimation bias. Second, the dataset includes only 164 countries, and this small sample size limits the reliability of the analysis.

Table S9: Analyses 1 and 2 results for stage 1 parameters with their W-type 95% CIs

$(\pi_{10}, \pi_{01}, \sigma^2)$		$\psi_{01,1}$	$\psi_{11,1}$	$\psi_{21,1}$	$\psi_{31,1}$	$\psi_{41,1}$	$\psi_{01,2}$	$\psi_{11,2}$	$\psi_{21,2}$	$\psi_{31,2}$	$\psi_{41,2}$
$\delta = \mathbf{0.9}$											
(0,0,0)	Estimate	0.14	0.24	0.02	0.35	0.48	5.47	−6.13	−0.48	1.72	−1.21
	SE	0.34	0.21	0.48	0.43	0.11	3.41	2.43	5.47	3.92	0.33
	95% CI	(−0.54, 0.82)	(−0.18, 0.66)	(−0.93, 0.97)	(−0.49, 1.20)	(0.26, 0.69)	(−1.30, 12.24)	(−10.95, −1.30)	(−11.34, 10.38)	(−6.05, 9.48)	(−1.88, −0.55)
(0.02,0.02, 0.02)	Estimate	0.15	0.24	0.01	0.38	0.47	4.98	−6.62	0.21	2.03	−1.17
	SE	0.17	0.14	0.02	0.26	0.08	2.51	5.47	1.01	8.91	0.32
	95% CI	(−0.18, 0.48)	(−0.02, 0.51)	(−0.03, 0.06)	(−0.14, 0.90)	(0.32, 0.62)	(0.05, 9.91)	(−17.34, 4.11)	(−1.78, 2.19)	(−15.43, 19.49)	(−1.79, −0.55)
(0.03, 0.03, 0.05)	Estimate	0.09	0.24	0.08	0.35	0.47	7.01	−4.47	−1.21	−0.38	−1.26
	SE	0.17	0.14	0.04	0.28	0.08	2.70	5.98	1.56	9.34	0.34
	95% CI	(−0.25, 0.42)	(−0.03, 0.51)	(0.00, 0.16)	(−0.19, 0.90)	(0.32, 0.63)	(1.72, 12.29)	(−16.19, 7.26)	(−4.26, 1.83)	(−18.68, 17.92)	(−1.92, −0.60)
(0.04,0.04, 0.07)	Estimate	0.14	0.25	−0.04	0.48	0.45	8.66	−4.45	−1.46	−1.03	−1.46
	SE	0.17	0.14	0.03	0.30	0.08	3.13	7.13	2.68	10.28	0.44
	95% CI	(−0.20, 0.48)	(−0.03, 0.53)	(−0.09, 0.01)	(−0.12, 1.07)	(0.29, 0.61)	(2.53, 14.78)	(−18.42, 9.53)	(−6.72, 3.80)	(−21.18, 19.13)	(−2.32, −0.60)
$\delta = \mathbf{0.1}$											
(0,0,0)	Estimate	−0.28	−0.01	0.68	0.33	0.47	5.69	−4.38	−0.47	0.03	−1.28
	SE	0.41	0.25	0.57	0.50	0.13	3.49	2.49	5.60	4.01	0.34
	95% CI	(−1.08, 0.53)	(−0.50, 0.49)	(−0.44, 1.81)	(−0.67, 1.32)	(0.22, 0.71)	(−1.23, 12.62)	(−9.32, 0.56)	(−11.58, 10.64)	(−7.92, 7.98)	(−1.96, −0.60)
(0.02,0.02, 0.02)	Estimate	−0.31	−0.15	0.82	0.50	0.48	7.32	−4.68	−2.74	1.27	−1.26
	SE	0.20	0.20	0.11	0.32	0.09	2.61	5.44	1.32	8.63	0.32
	95% CI	(−0.71, 0.09)	(−0.54, 0.24)	(0.59, 1.04)	(−0.13, 1.13)	(0.31, 0.66)	(2.21, 12.43)	(−15.34, 5.98)	(−5.32, −0.16)	(−15.65, 18.18)	(−1.88, −0.64)
(0.03, 0.03, 0.05)	Estimate	−0.47	−0.15	0.96	0.13	0.50	7.58	−6.24	−4.71	4.46	−1.24
	SE	0.21	0.20	0.14	0.36	0.09	2.68	5.52	1.44	8.59	0.31
	95% CI	(−0.88, −0.05)	(−0.53, 0.24)	(0.67, 1.24)	(−0.58, 0.83)	(0.33, 0.67)	(2.34, 12.83)	(−17.05, 4.58)	(−7.54, −1.87)	(−12.38, 21.30)	(−1.85, −0.63)
(0.04,0.04, 0.07)	Estimate	−0.69	−0.39	1.38	0.52	0.48	16.66	−1.09	−20.44	5.25	−1.83
	SE	0.22	0.23	0.16	0.40	0.10	4.62	10.37	5.80	12.60	0.55
	95% CI	(−1.13, −0.25)	(−0.84, 0.06)	(1.06, 1.70)	(−0.26, 1.30)	(0.29, 0.68)	(7.61, 25.71)	(−21.41, 19.24)	(−31.82, −9.07)	(−19.46, 29.95)	(−2.92, −0.75)

Table S10: Analyses 1 and 2 results for stage 2 parameters with their W-type 95% CIs

$(\pi_{10}, \pi_{01}, \sigma^2)$		$\psi_{02,1}$	$\psi_{12,1}$	$\psi_{22,1}$	$\psi_{32,1}$	$\psi_{42,1}$	$\psi_{02,2}$	$\psi_{12,2}$	$\psi_{22,2}$	$\psi_{32,2}$	$\psi_{42,2}$
(0,0,0)	Estimate	0.09	−0.80	0.56	0.45	0.28	−2.27	3.36	−2.32	−4.13	−0.48
	SE	0.32	0.38	0.43	0.84	0.15	2.02	2.18	2.55	2.09	0.08
	95% CI	(−0.54, 0.72)	(−1.56, −0.04)	(−0.30, 1.42)	(−1.21, 2.10)	(−0.02, 0.57)	(−6.27, 1.73)	(−0.97, 7.70)	(−7.38, 2.74)	(−8.27, 0.01)	(−0.64, −0.31)
(0.02,0.02, 0.02)	Estimate	0.07	−0.85	0.63	0.46	0.28	−2.09	3.35	−2.63	−3.98	−0.47
	SE	0.27	0.39	0.48	0.58	0.12	8.44	9.12	10.93	9.50	0.53
	95% CI	(−0.46, 0.59)	(−1.62, −0.08)	(−0.30, 1.56)	(−0.67, 1.59)	(0.05, 0.51)	(−18.63, 14.45)	(−14.52, 21.22)	(−24.05, 18.79)	(−22.59, 14.64)	(−1.52, 0.57)
(0.03, 0.03, 0.05)	Estimate	0.05	−0.82	0.64	0.40	0.30	−2.29	3.39	−2.41	−4.11	−0.48
	SE	0.29	0.44	0.53	0.65	0.12	8.88	9.22	12.12	10.01	0.54
	95% CI	(−0.52, 0.63)	(−1.67, 0.03)	(−0.40, 1.67)	(−0.88, 1.67)	(0.06, 0.53)	(−19.69, 15.11)	(−14.68, 21.46)	(−26.16, 21.35)	(−23.74, 15.51)	(−1.53, 0.57)
(0.04,0.04, 0.07)	Estimate	−0.04	−1.07	0.83	0.75	0.27	−2.23	3.22	−2.45	−3.87	−0.47
	SE	0.34	0.49	0.61	0.75	0.13	9.41	9.26	13.62	10.66	0.53
	95% CI	(−0.71, 0.63)	(−2.04, −0.11)	(−0.37, 2.03)	(−0.73, 2.22)	(0.01, 0.53)	(−20.68, 16.21)	(−14.92, 21.37)	(−29.15, 24.25)	(−24.76, 17.03)	(−1.52, 0.57)

Table S11: Values of regression parameters of calibration functions for Analysis 3

	ζ_0	ζ_1	ξ_{10}	ξ_{11}	ξ_{20}	ξ_{21}
Set 1	0.05	0.95	0.05	−0.85	0.05	−0.85
Set 2	0.05	0.85	0.05	−0.75	0.05	−0.75
Set 3	0.05	0.75	0.05	−0.65	0.05	−0.65

Table S12: Analysis 3 results for stage 1 parameters with their Normal bootstrap 95% CIs

		$\psi_{01,1}$	$\psi_{11,1}$	$\psi_{21,1}$	$\psi_{31,1}$	$\psi_{41,1}$	$\psi_{01,2}$	$\psi_{11,2}$	$\psi_{21,2}$	$\psi_{31,2}$	$\psi_{41,2}$
$\delta = \mathbf{0.9}$											
Set 1	Estimate	0.12	0.24	0.09	−0.41	0.48	6.70	−6.13	−2.20	−2.02	−1.21
	Bootstrap SE	2.07	0.29	3.09	0.69	0.11	15.44	3.53	24.80	5.32	0.31
	95% CI	(−3.94, 4.17)	(−0.34, 0.82)	(−5.97, 6.16)	(−1.77, 0.94)	(0.25, 0.70)	(−23.55, 36.96)	(−13.04, 0.79)	(−50.81, 46.41)	(−12.44, 8.41)	(−1.81, −0.62)
Set 2	Estimate	0.11	0.24	0.10	−0.47	0.48	6.83	−6.13	−2.43	−2.29	−1.21
	Bootstrap SE	2.25	0.31	3.44	0.88	0.11	17.05	3.54	27.84	6.25	0.31
	95% CI	(−4.30, 4.53)	(−0.37, 0.85)	(−6.65, 6.85)	(−2.19, 1.25)	(0.26, 0.69)	(−26.58, 40.24)	(−13.07, 0.82)	(−57.00, 52.14)	(−14.54, 9.96)	(−1.83, −0.60)
Set 3	Estimate	0.11	0.24	0.12	−0.54	0.48	7.00	−6.13	−2.71	−2.64	−1.21
	Bootstrap SE	2.30	0.28	3.59	0.97	0.11	18.13	3.65	30.19	7.37	0.31
	95% CI	(−4.39, 4.61)	(−0.32, 0.80)	(−6.93, 7.16)	(−2.44, 1.36)	(0.26, 0.69)	(−28.53, 42.53)	(−13.28, 1.02)	(−61.89, 56.46)	(−17.08, 11.80)	(−1.83, −0.60)
$\delta = \mathbf{0.1}$											
Set 1	Estimate	−1.86	−0.01	3.13	−0.38	0.47	6.80	−4.38	−2.16	−0.03	−1.28
	Bootstrap SE	2.27	0.37	3.36	0.87	0.14	15.36	3.42	24.85	5.32	0.31
	95% CI	(−6.30, 2.58)	(−0.72, 0.71)	(−3.46, 9.71)	(−2.09, 1.32)	(0.20, 0.73)	(−23.31, 36.91)	(−11.09, 2.32)	(−50.86, 46.54)	(−10.46, 10.39)	(−1.89, −0.68)
Set 2	Estimate	−2.02	−0.01	3.44	−0.43	0.47	6.91	−4.38	−2.38	−0.04	−1.28
	Bootstrap SE	2.52	0.37	3.80	0.97	0.13	16.37	3.41	26.90	5.98	0.30
	95% CI	(−6.96, 2.92)	(−0.74, 0.73)	(−4.00, 10.89)	(−2.34, 1.47)	(0.20, 0.73)	(−25.18, 39.00)	(−11.07, 2.31)	(−55.11, 50.36)	(−11.75, 11.67)	(−1.88, −0.69)
Set 3	Estimate	−2.22	−0.01	3.85	−0.50	0.47	7.06	−4.38	−2.66	−0.05	−1.28
	Bootstrap SE	2.82	0.38	4.38	1.16	0.13	17.73	3.48	29.49	6.90	0.30
	95% CI	(−7.75, 3.30)	(−0.74, 0.73)	(−4.74, 12.45)	(−2.78, 1.78)	(0.21, 0.72)	(−27.69, 41.80)	(−11.19, 2.43)	(−60.46, 55.15)	(−13.58, 13.49)	(−1.88, −0.69)

Table S13: Analysis 3 results for stage 2 parameters with their Normal bootstrap 95% CIs

		$\psi_{02,1}$	$\psi_{12,1}$	$\psi_{22,1}$	$\psi_{32,1}$	$\psi_{42,1}$	$\psi_{02,2}$	$\psi_{12,2}$	$\psi_{22,2}$	$\psi_{32,2}$	$\psi_{42,2}$
Set 1	Estimate	−1.20	−0.80	2.57	−0.53	0.28	2.92	3.36	−10.60	4.86	−0.48
	Bootstrap SE	1.21	0.39	1.82	1.38	0.14	7.49	2.19	11.67	2.63	0.09
	95% CI	(−3.57, 1.17)	(−1.56, −0.04)	(−1.00, 6.14)	(−3.24, 2.19)	(0.00, 0.56)	(−11.76, 17.59)	(−0.92, 7.65)	(−33.47, 12.26)	(−0.30, 10.02)	(−0.65, −0.30)
Set 2	Estimate	−1.33	−0.80	2.83	−0.60	0.28	3.44	3.37	−11.68	5.51	−0.48
	Bootstrap SE	1.33	0.40	2.06	1.51	0.14	7.64	2.18	12.15	3.16	0.09
	95% CI	(−3.94, 1.27)	(−1.58, −0.02)	(−1.20, 6.86)	(−3.55, 2.36)	(0.01, 0.55)	(−11.54, 18.41)	(−0.91, 7.64)	(−35.50, 12.14)	(−0.69, 11.71)	(−0.65, −0.31)
Set 3	Estimate	−1.50	−0.80	3.17	−0.69	0.28	4.10	3.37	−13.06	6.36	−0.48
	Bootstrap SE	1.41	0.39	2.21	1.71	0.13	8.92	2.30	14.33	3.62	0.09
	95% CI	(−4.26, 1.26)	(−1.56, −0.04)	(−1.16, 7.49)	(−4.04, 2.67)	(0.02, 0.54)	(−13.38, 21.59)	(−1.15, 7.88)	(−41.14, 15.03)	(−0.73, 13.44)	(−0.65, −0.31)

Table S14: Estimated optimal actions for selected countries from Analyses 1, 2 and 3: $\delta = 0.9$

Country	Analysis 1		Analysis 2						Analysis 3					
	(0,0,0)		(0.02,0.02,0.02)		(0.03,0.03,0.05)		(0.04,0.04,0.07)		Set 1		Set 2		Set 3	
	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2
	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$
Greece	0	0	0	0	0	0	0	0	0	0	0	0	0	0
India	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Italy	0	0	0	0	0	0	0	0	0	0	0	0	0	0
United Kingdom	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sweden	0	1	0	1	0	1	0	1	0	0	0	0	0	0
China	0	1	0	1	0	1	0	1	0	0	0	0	0	0
France	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Egypt	0	0	0	0	0	0	0	0	0	1	0	1	0	1
South Korea	0	1	0	1	0	1	0	1	1	0	1	0	1	0
Hong Kong	0	0	0	0	0	0	0	0	1	1	1	1	1	1
USA	0	1	0	1	0	1	0	1	1	0	1	0	1	0
United Arab Emirates	1	1	1	1	1	1	0	1	1	0	1	0	1	0
Canada	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Israel	0	1	0	1	0	1	0	1	1	0	1	0	1	0
Finland	0	1	0	1	0	1	0	1	1	0	1	0	1	0
Germany	0	1	0	1	0	1	0	1	1	0	1	0	1	0

Table S15: Estimated optimal actions for selected countries from Analyses 1, 2 and 3: $\delta = 0.1$

Country	Analysis 1		Analysis 2						Analysis 3					
	(0,0,0)		(0.02,0.02,0.02)		(0.03,0.03,0.05)		(0.04,0.04,0.07)		Set 1		Set 2		Set 3	
	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2
	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$	$\hat{A}_1^{\text{opt}}$	$\hat{A}_2^{\text{opt}}$
Greece	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Italy	0	0	0	0	0	0	0	0	0	0	0	0	0	0
France	0	0	0	0	0	0	0	0	0	0	0	0	0	0
United Kingdom	0	0	0	0	0	0	0	0	0	0	0	0	0	0
China	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Egypt	0	1	0	1	0	1	0	1	0	0	0	0	0	0
India	0	0	0	0	0	0	1	0	0	0	0	0	0	0
South Korea	1	1	1	1	0	1	1	1	1	0	1	0	1	0
Sweden	1	1	1	1	0	1	0	1	1	0	1	0	1	0
Hong Kong	0	1	0	1	0	1	0	1	0	0	0	0	0	0
USA	1	1	1	1	1	1	0	1	1	0	1	0	1	0
United Arab Emirates	1	0	1	0	1	0	0	0	1	0	1	0	1	0
Canada	0	0	0	0	0	0	0	0	0	0	1	0	1	0
Israel	0	1	0	1	0	1	1	1	0	0	0	0	0	0
Finland	0	1	0	1	0	1	0	1	1	0	1	0	1	0
Germany	0	1	0	1	0	1	0	1	0	0	0	0	1	0

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