

# Impact of Data Perturbation for Statistical Disclosure Control on the Predictive Performance of Machine Learning Techniques Supplementary Material

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This supplementary material contains (1) detailed descriptions of the predictive machine learning techniques investigated in the paper and (2) additional simulation results.

## 1 Predictive Machine Learning Techniques

In this investigation, we considered six popular machine learning techniques (ML) for predictive modeling as well as stacked ensembles of these techniques. In the following, we describe each of these techniques and their pros and cons.

### 1.1 Linear Regression (LR)

Linear regression is a prominent machine learning technique that can be highly effective while maintaining high interpretability. A major drawback in using linear regression is that linear regression requires more presumptions than would be expected for other machine learning methods ([Hastie et al., 2009](#)), especially in the case of nonparametric methods such as random forest and SVMs. Let  $n$  be the sample size and  $p$  the number of predictors in the data. The coefficients of a linear regression model are obtained via ordinary least squares:

$$\hat{\beta}_{OLS} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}, \quad (1)$$

where  $\mathbf{X}$  is a  $n \times p$  matrix of input data, and  $\mathbf{Y}$  denotes the continuous response of dimension  $n \times 1$ . Depending on the nature of the data in question, the linear regression's predictions can be effective, or highly problematic ([Hastie et al., 2009](#)).

Linear regression serves well for explainability and data that can be described as a linear combination of features or predictors. Existing literature provides implementations of the Akaike Information Criterion and Bayesian Information Criterion for the purpose of model selection. Linear regression can often be a sufficient choice for a baseline in terms of performance before deciding to go with a more complex machine-learning technique. Linear regression becomes problematic when there is high correlation among the predictor variables, outliers, few significant predictors in contrast to the total quantity of predictors, and when the number of data points is less than the number of predictors. Another shortfall of linear regression is that it assumes that the response is described by a linear combination of the predictors, which may or may not be true. These shortfalls can lead to other machine-learning techniques being employed instead of linear regression.

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## 1.2 Least Absolute Shrinkage and Selection Operator (LASSO) Regression

A famous extension for linear regression is the LASSO regression which addresses some shortcomings for linear regression. These shortcomings primarily involve if  $n > p$  and when the number of significant predictors is smaller than the total number of predictors. Another beneficial point of using LASSO is when there are predictors with high correlation, as it can reduce the number of highly correlated predictors. LASSO extends linear regression by applying the  $L_1$  shrinkage to reduce the number of coefficients that are contained in the final predictive model, resolving both issues (Hastie et al., 2009, Ch 3). The LASSO coefficients can be obtained via

$$\hat{\beta}_{\text{LASSO}} = \underset{\beta}{\operatorname{argmin}} \{ (\mathbf{Y} - \mathbf{X}\beta)^T (\mathbf{Y} - \mathbf{X}\beta) + \lambda \cdot \sum_{i=1}^p |\beta_i| \}, \quad (2)$$

where  $\lambda$  is a penalty parameter that can be chosen via cross-validation to minimize the mean squared error (MSE).

The LASSO can be beneficial out of the box for high-dimensional data due to the previously stated variable selection properties and the scenarios when the predictors have a high correlation amongst one another. Additionally, LASSO can provide a means to determine which predictors may be significant. This can be utilized for constructing more informative models downstream. Another benefit of the LASSO regression it shares with linear regression is that the LASSO regression is highly explainable thanks to the response being estimated as a linear combination of predictors and coefficients. The LASSO can be biased if too much shrinkage occurs, and there is no guarantee that LASSO will maintain all of the significant predictors as some of the significant predictors could be removed. This led to the development of many variants of the LASSO to mitigate the shortcomings of the original LASSO. Another shortcoming of LASSO regression is that it, like linear regression, assumes the response is explained by a linear combination of the predictors. Not all data is easily described as linear combinations of the underlying variables, so LASSO can underperform for non-linear and nonparametric data. LASSO's main drawback to linear regression is that it requires hyperparameter tuning to obtain an optimal lambda based on the data.

## 1.3 Support Vector Machine (SVM) Regression

SVMs are nonparametric predictive models that have been used in both regression and classification contexts. SVMs focus on the utilization of hyperplanes in the regression context, to minimize one of a quartet of chosen loss functions to acquire the final model Li et al. (2006). SVMs using other kernels besides the linear kernel can be achieved via alterations to the algorithm for the linear kernel SVM Hastie et al. (2009). The hyperplanes are constructed based on the kernel function used in the regression SVM, such as the linear kernel partitioning data based on linear relationships (Hastie et al., 2009). The SVM can be constructed as

$$g(\mathbf{x}) = \beta_0 + \sum_{i \in V} \kappa_i L(\mathbf{x}, \mathbf{x}_i; \zeta, u), \quad (3)$$

in which  $L$  defines the kernel (Lundell, 2023). We have two tuning hyperparameters  $\zeta$  and  $u$  (Lundell, 2023).  $V$  defines the collection of support vectors (Lundell, 2023).  $\kappa_i$  is calculated based on a cost hyperparameter and the resulting soft margin resulting from the cost hyperparameter (Lundell, 2023). All hyperparameters necessitate positive values. Note that  $u$  is specific for SVM regression, and dictates the distance that any point allowed about the constructed hyperplane Lundell (2023). The radial kernel is noted to be versatile across a multitude of scenarios in preceding research (Lundell, 2023). Another kernel that can be selected for SVMs is the polynomial kernel (Lundell, 2023). Hereafter, we denote a SVM with a radial kernel by Radial\_SVM, a SVM with a linear kernel as Linear\_SVM, and a SVM with a polynomial kernel by Poly\_SVM.

SVMs are not as easily explainable as LASSO and linear regression. LASSO and linear regression can be explained using the calculated coefficients. The sacrifice in explainability for SVMs comes with an advantage in versatility. SVMs' usage of hyperplanes in combination with kernels that define how the hyperplanes are determined allows SVMs to handle a variety of parametric and nonparametric data. There are multiple hyperparameters to consider beyond the choice of kernel to be used with the SVM. For LASSO, the main component to consider is lambda as a hyperparameter. For linear regression, there are no hyperparameters that need to be tuned. SVMs have multiple hyperparameters. This increased number of hyperparameters and versatility characterizing SVMs can lead to SVMs being prone to high variance if used without caution. The best hyperparameters can be chosen from a feasible subset using cross-validation and various search methods such as grid search. Depending on the search method, cross-validation, and the variety of hyperparameters to consider, the tuning of SVMs can be time-consuming and computationally intensive. This added complexity can be problematic for SVMs as there is no guarantee that the final SVM that is chosen is the most optimal SVM beyond the subset of hyperparameters that were considered.

## 1.4 Neural Networks (NNET)

Neural networks are nonparametric predictive models that have become more prominent thanks to the growth and advancement of computing capabilities. Neural networks are constructed from input layers, hidden layers, and output layers as depicted in Fig. 1. The neural network is considered as a nonlinear technique that takes the input data and produces linear combinations of the data after said input data passes through the input layer. Biases and weights are used to assist in transforming and determining which connections are providing more contribution before reaching the activation functions in the hidden nodes for the hidden layer. These outputs are then processed through the hidden layer to be transformed by a final set of activation functions in the output layer that reduces the computations to the final output. The activation functions are chosen based on the aim of the neural network (classification or regression) in combination with the nature of the input data that is to be provided to the model. Neural networks use backpropagation to prioritize only the computations that are directly interacting with a given node in the hidden layer. Precisely, backpropagation is used to calculate derivatives of activation functions in which said derivatives can be used to enhance the neural network over each training iteration or epoch. The weights are tuned via backpropagation and how fast the tuning is performed is manipulated via a hyperparameter known as the *learning rate*.

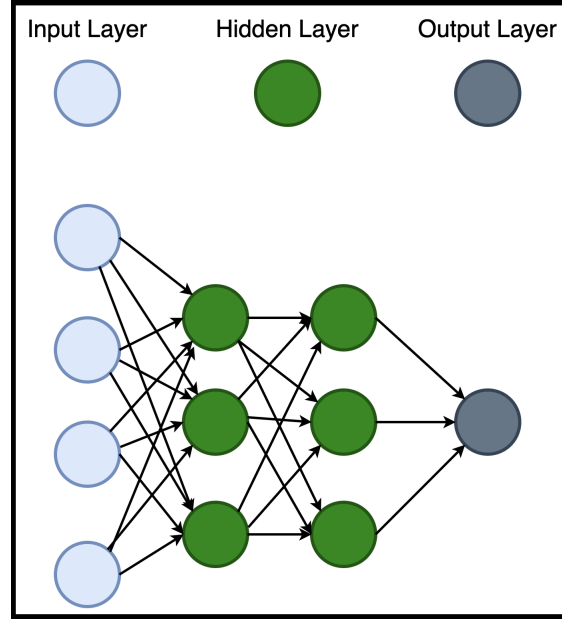


Figure 1: Diagram of a neural network with two hidden layers.

The math behind neural networks stems from projection pursuit regression for obtaining more versatile machine learning techniques (Hastie et al., 2009). To start, new predictors obtained from linear amalgamations of the original predictors with an intercept term,  $\mathbf{X}$  (Hastie et al., 2009). Each new predictor  $\theta_j$ , where  $j = 1, 2, \dots, J$ , is constructed as:

$$\theta_j = \gamma(\kappa_j^T \mathbf{X}), \quad (4)$$

where  $\kappa$ 's denote coefficients for the linear amalgamations contributing to the new predictors (Hastie et al., 2009). Note that  $\gamma(\cdot)$  is an activation function that processes its arguments to transform to a new output that is in turn influenced by weights and biases getting to the succeeding layer in the neural network (Hastie et al., 2009). The  $\theta_j$ 's are used to construct  $U_r$ , where  $r = 1, 2, \dots, R$  via

$$U_r = \zeta_r^T \boldsymbol{\theta}, \quad (5)$$

in preparation for the final transformation (Hastie et al., 2009). The final transformation utilizes a final set of functions to generate an estimate of the response:

$$v_r(\mathbf{X}) = h_r(\mathbf{U}), \quad (6)$$

where  $h_r$  are functions chosen corresponding to the nature of the response such as the identity function corresponding to the continuous response variable (Hastie et al., 2009).

The key advantage of neural networks lies in their versatility in a broad range of scenarios. There is no dependence on the data being described solely by an assumed linear combination of predictors and estimated coefficients. Interactions and higher-order terms can be considered in the neural network as it makes fewer assumptions on the data compared to LASSO and linear regression. This versatility of neural networks has been extended further in the applications demonstrated by deep neural networks. This advantage of neural networks comes with a significant cost. Similar to SVMs, neural networks can suffer from overfitting. This requires hyperparameter tuning to be able to

determine the most optimal set of hyperparameters for the data. Neural networks can rise in complexity rather quickly, leading to more and more methods to try and obtain optimal neural networks with desired performance. The hyperparameters affecting the performance of neural networks include but are not limited to dropout, learning rate, number of hidden layers, choice of activation functions, and weight decay. While neural networks can be excellent for achieving high accuracy or performance, they have much to be desired in terms of explainability.

Neural networks have been expanded upon considerably and their capabilities have been used to address various research problems to the point of becoming one of the most prominent family of machine learning models. For an in-depth introduction to neural networks, we refer the reader to [Hastie et al. \(2009, Ch 11\)](#).

## 1.5 Random Forests (RF)

Random forests are nonparametric predictive models generated via the ensemble method of bagging applied to decision trees. Bagging is a method to take an assortment of predictive models trained via bootstrap samples of the training dataset and compile their predictions into an ensemble model to improve the effectiveness and stability of the predictions. Algorithm 1 summarizes the steps involved in fitting a random forest model.

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**Algorithm 1** Random Forest algorithm ([Hastie et al., 2009](#)).

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Let  $[\mathbf{X} \ \mathbf{Y}]$  denote the data matrix of the training set after a train-test split. Let the total number of decision trees be denoted by  $R$  and  $r$  denote the index of an array of decision trees. Let  $a_n$  denote the lower limit for the size of any terminal node of any  $r$  tree. Note that due to bagging, the trees can be constructed in parallel if resources allow.

1. We build decision (classification or regression) tree  $r$  using the bootstrap sample  $[\tilde{\mathbf{X}}_r \ \tilde{\mathbf{Y}}_r]$  from the training set with dimensions  $n \times z$ . Here,  $z$  denotes the total number of predictors.
  - (a) At each terminal node determine  $q$  predictors from the  $z$  predictors available, and use those  $q$  predictors to evaluate the best way to split the tree into a pair of child nodes that become new terminal nodes. This evaluation is done via minimizing the split criterion. The criterion determining splits for a random forest is typically the Gini index in classification and variance in regression.
  - (b) We halt the splitting process for the terminal nodes that satisfy  $a_n$ .
2. For classification, use the majority vote from all  $R$  trees as the final prediction. For regression, use the average of the predictions from all  $R$  trees as the final prediction:

$$\hat{Y} = \frac{1}{R} \sum_{r=1}^R \hat{f}_r(\tilde{\mathbf{X}}), \quad (7)$$

where  $\hat{f}_r(\tilde{\mathbf{X}})$  is the prediction from the  $r$ -th tree.

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Random forests have several advantages over other machine learning approaches. The first is that they are based on decision trees, which make few assumptions about the relationships and nature of the underlying data. This means that random forests can be used for data that has various amalgamations among predictors. This can include higher-order terms as well as interactions. Additionally, random forests can be set up so that each decision tree considers a subset of features, and the bagging procedure helps

to limit potential overfitting. The overall ensemble approach enables random forests to have increased performance when there is sufficient data and an appropriate setup in the workflow. Methods for analyzing the data via random forest models are available as well. One such method is measuring variable importance. The main advantage of random forests is the great amount of flexibility. The main drawbacks of random forests are the increased complexity of being an ensemble of multiple decision trees as well as the increased computational burden of building the random forest from multiple decision trees. Random forests are not as easily explainable as LASSO or linear regression as a result. Additionally, the hyperparameters to be optimized in a random forest means that hyperparameter tuning will yield different versions of the random forest ensemble. This means hyperparameter tuning will add to the computation time and compute resources needed in training an optimal random forest. Furthermore, random forests are not guaranteed to beat other, simpler methods such as linear regression and LASSO when the data satisfies the assumptions of said simpler methods.

## 1.6 XGB Trees (XGBTrees)

Extreme gradient boosting (XGB) trees are nonparametric predictive models generated via the ensemble method of boosting being applied to decision trees. XGB trees utilize the method of boosting, where tree models are constructed sequentially one after another. Succeeding trees reweight training data so that observations with a higher error are given more priority to predict correctly.

XGBoosting starts with the concept of additive models (Chen et al., 2023). The prediction of a given  $y_j$  is based on aggregating the results of multiple decision tree models together (Chen and Guestrin, 2016):

$$\hat{y}_j = \sum_{h=1}^H g_h(\mathbf{x}_j), \text{ where } g_h \in G, \quad (8)$$

in which  $G = \{g(\mathbf{x}) = a_{b(\mathbf{x})}\}$  ( $b : \mathbb{R}^p \rightarrow M$ , and  $a \in \mathbb{R}^M$ ) constitutes the expanse of the decision trees,  $b$  is the formation of a decision tree concerning the leaves that the data points fall into,  $a$  denotes the weight for the associated leaf within tree  $g_h$  that will be used in the prediction,  $M$  is the quantity of leaves per decision tree, and  $g_h$  denote the individual decision trees. With this notation established, the XGB algorithm can be summarized as in Algorithm 2. Additional details on optimizations, approximations, and scenario-specific implementations or technicalities are available in (Chen and Guestrin, 2016).

XGB trees have the benefits of capturing nonparametric relationships among variables in the data as well as utilizing boosting to improve the performance of the final model. XGB trees can offer better performance than random forests and other machine learning techniques thanks to these two attributes. There are two major downsides. The first drawback is the lack of explainability since XGB trees are ensembles of decision trees built successively one after another, which in turn means each decision tree is influenced by the others. The second drawback is that XGB trees can be prone to overfitting. This can lead to high variance from less-than-optimal models being built. Hyperparameter tuning can mitigate this, but that same hyperparameter tuning incurs a heavier computational toll and greater amounts of time expended to attempt to build an optimal XGB tree model. The hyperparameters considered in XGB trees include the number



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**Algorithm 2** XGBTrees algorithm ([Chen and Guestrin, 2016](#)).

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Let  $[\mathbf{X} \ \mathbf{Y}]$  denote the data matrix of the training set after a train-test split and any necessary preprocessing. Let the total number of decision trees be denoted by  $R$  and  $r$  denote the index of an array of decision trees. Each decision tree is trained successively, one after another. Let  $n$  denote the number of rows in  $[\mathbf{X} \ \mathbf{Y}]$ .

1. Build each decision tree using the the matrix  $[\mathbf{X} \ \mathbf{Y}]$ . For every  $r$  decision tree out of  $R$  decision trees in the complete ensemble:
  - (a) Let  $z = \hat{\partial}_{y^{r-1}}(q(y_j, \hat{y}_j^{r-1}))$  denote the first derivative and  $t = \hat{\partial}_{y^{r-1}}^2(q(y_j, \hat{y}_j^{r-1}))$  denote the second derivative of the convex loss. Let  $r - 1$  denote the index of the preceding tree in the ensemble.
  - (b) For the current decision tree  $r$ , the minimization of

$$Q(\theta)^r \approx \sum_{j=1}^n \left[ q(y_j, \hat{y}_j^{r-1}) + z_j g_r(\mathbf{x}_j) + \frac{1}{2} t_j g_r^2(\mathbf{x}_j) \right] + \sum_r^R \zeta M + \frac{1}{2} \nu ||a||, \quad (9)$$

or its approximation given in ([Chen and Guestrin, 2016](#)) for each  $r$  would provide the most optimal ensemble for predicting  $y$  for data point  $j$ .

- (c) Repeat (a) and (b) until  $R$  decision trees are obtained.
2. Once all the decision trees have been obtained, we can make our predictions using the ensemble. For any prediction(s) for  $y$ , we input  $\mathbf{X}_{\text{new}}$  which has the same features as the original  $\mathbf{X}$ .
3. For each row or data point,  $j$ , in  $\mathbf{X}_{\text{new}}$ , we calculate the summation of the resulting predictions from the decision trees within the ensemble as our final prediction:

$$\hat{y}_j = \sum_r^R g_r(\mathbf{x}_{\text{new},j}), \text{ where } g_r \in G. \quad (10)$$


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1 of trees, the size of the terminal nodes, the shrinkage applied to the weighting for the  
 2 decision trees, and the portion of columns to be subsampled. For more details on XGB  
 3 tree models, we refer the reader to [Hastie et al. \(2009\)](#) and [Chen and Guestrin \(2016\)](#).

## 4 1.7 Stacked Ensembles

5 In addition to the above predictive modeling techniques, we also considered two types  
 6 of stacked ensemble regressions. The first stacked ensemble is defined as the mean of  
 7 the above eight predictive models' predictions with equal weights. It is identified as  
 8 SE\_equal. The second stacked ensemble is defined as a weighted mean of the above  
 9 eight predictive models' predictions where the weights are inversely proportionate to the  
 10 training mean squared error of each model. This implementation of the stacked ensemble  
 11 is identified as SE\_prop.

## 2 Additional Simulation Results

### 2.1 Average Mean Square Error Ratio (AMSER) Visualizations

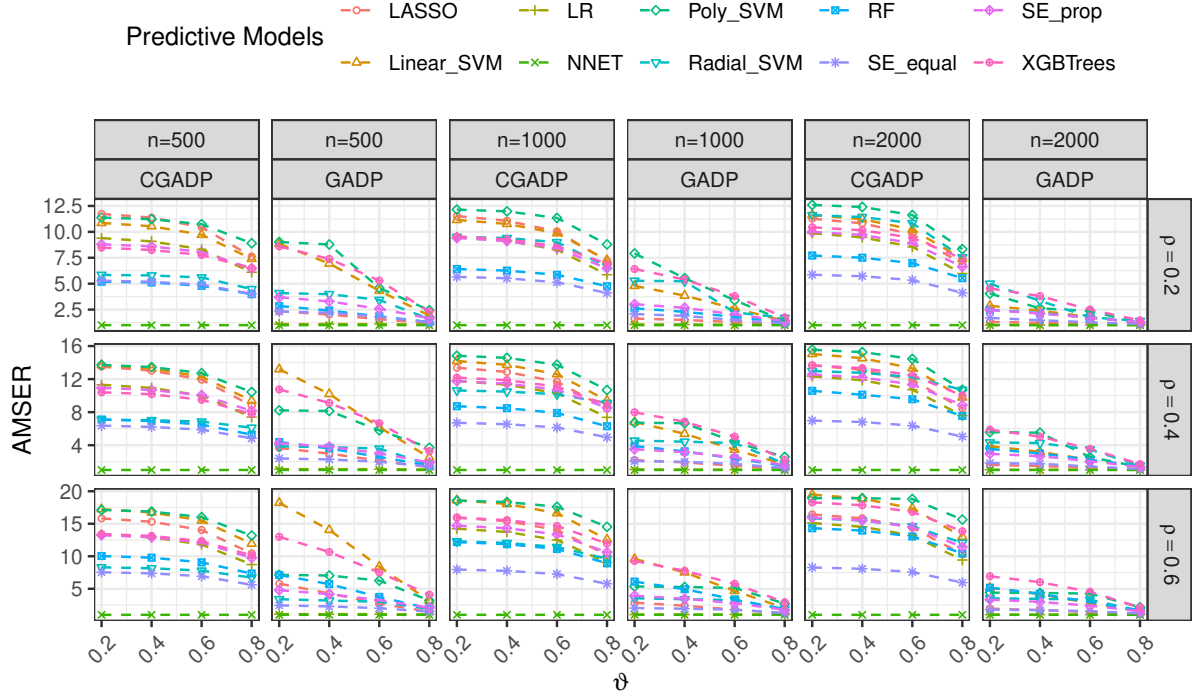


Figure 2: AMSER for ten ML techniques under **Model I** with  $p = 50$  and  $\sigma_\varepsilon = 0.2$ .

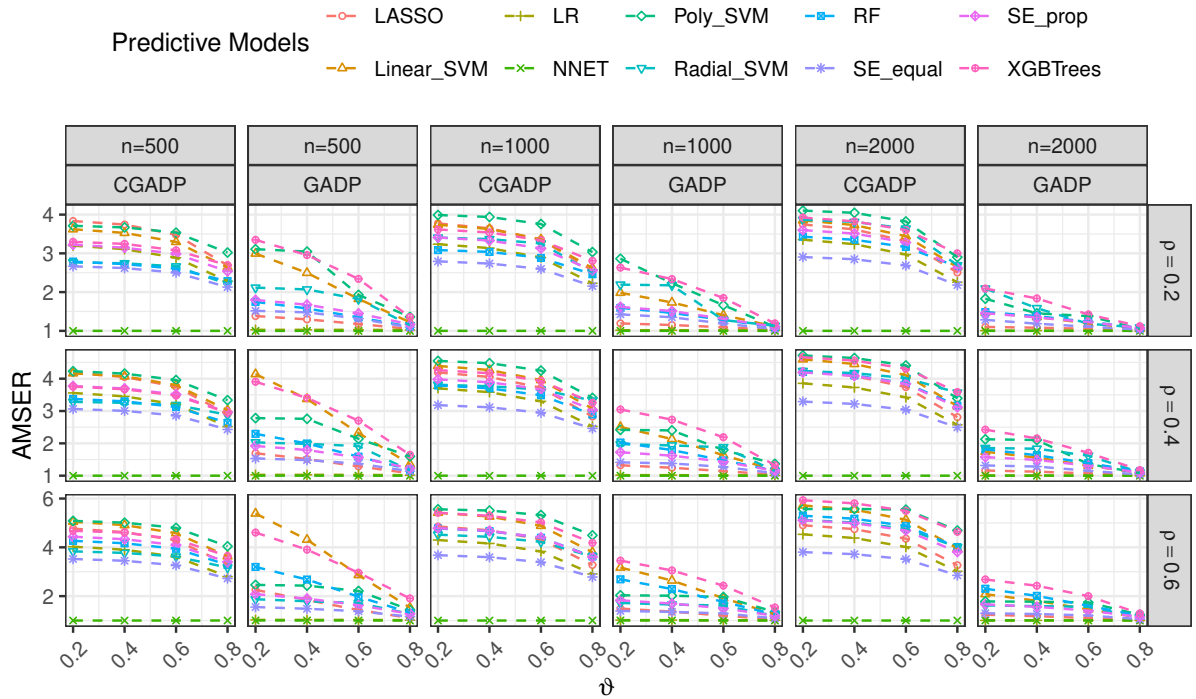
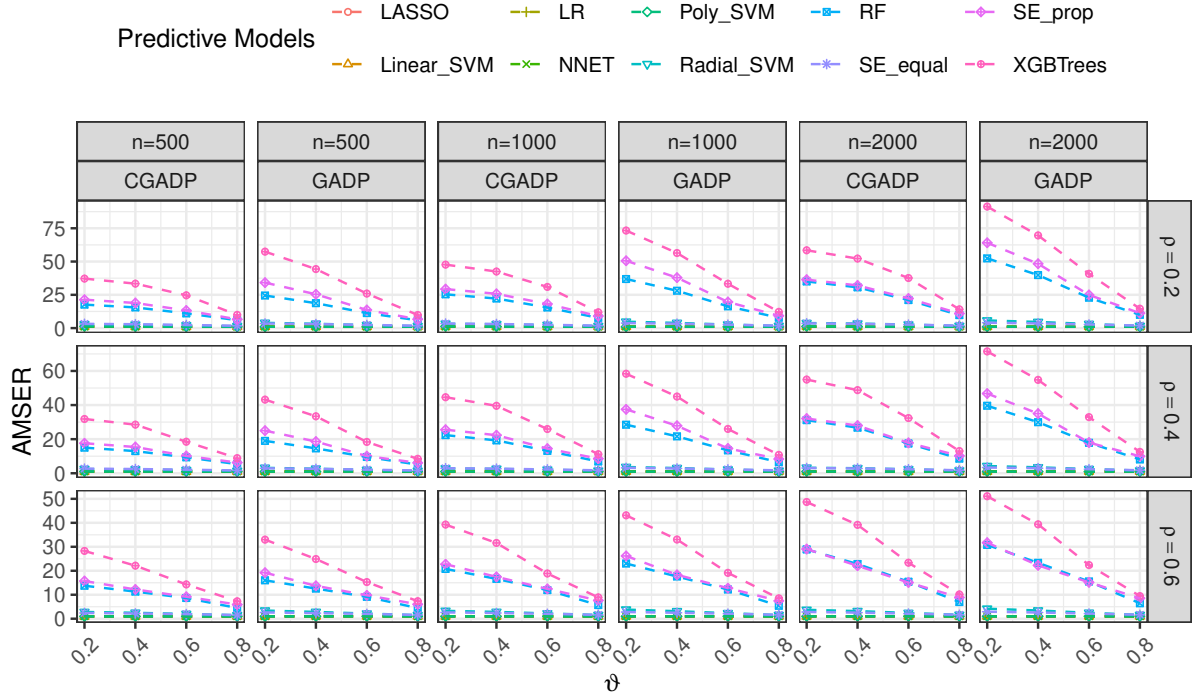
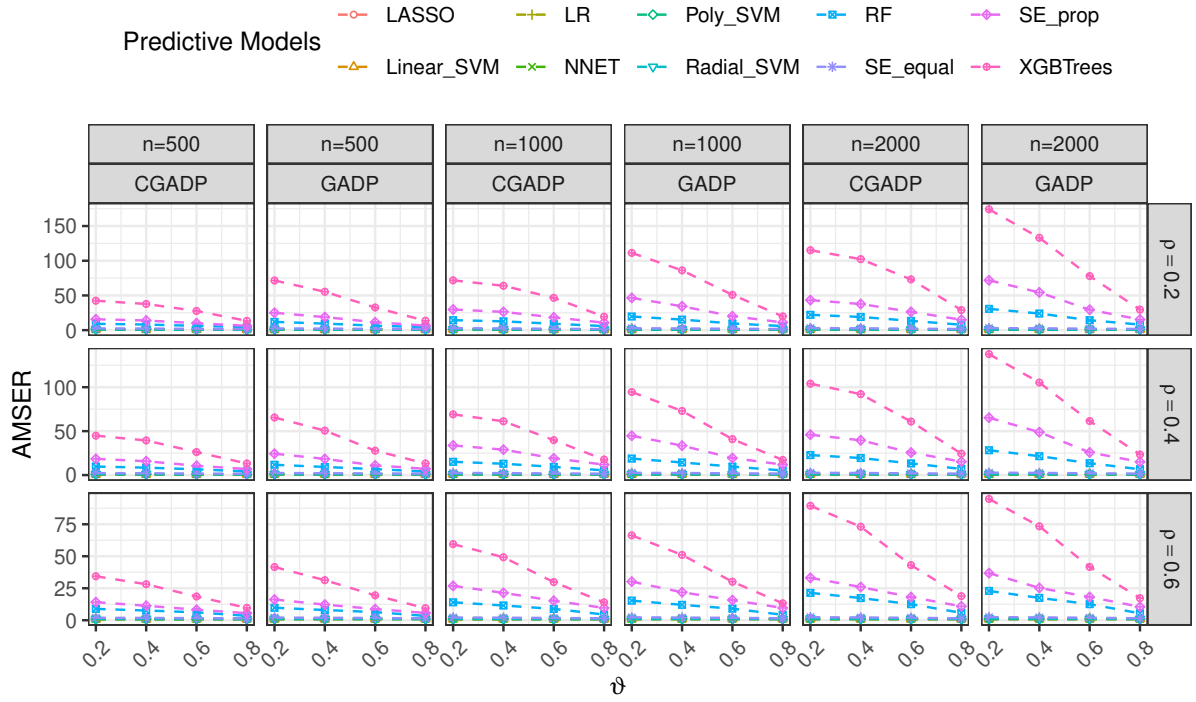
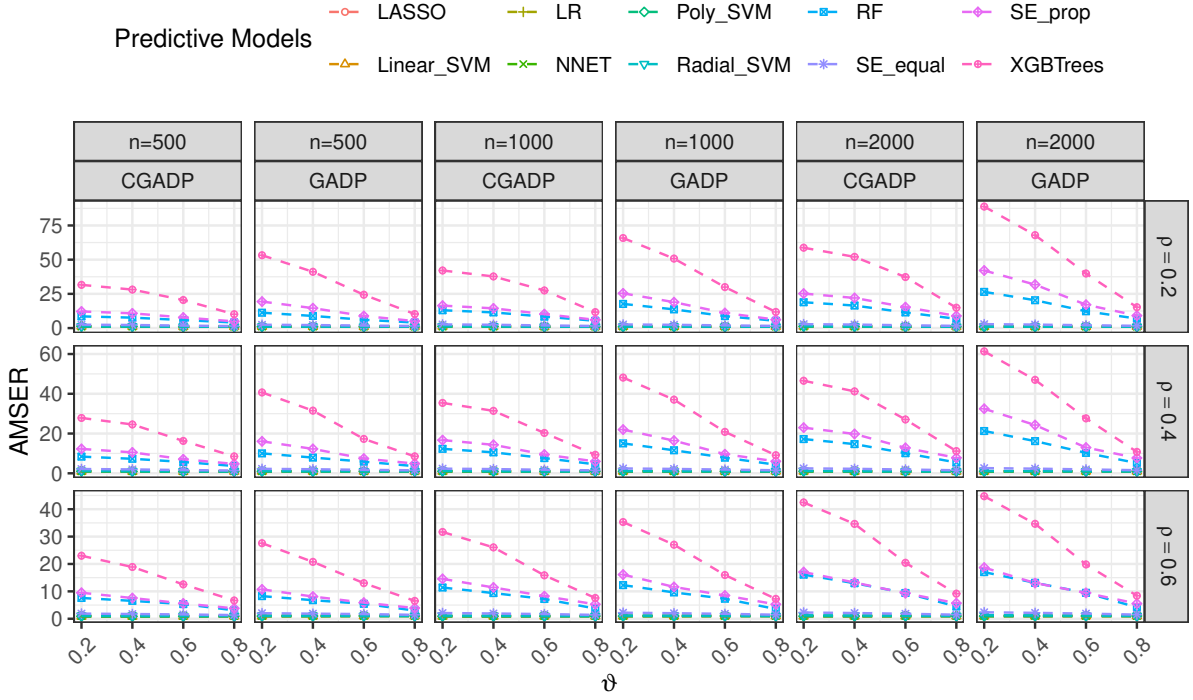
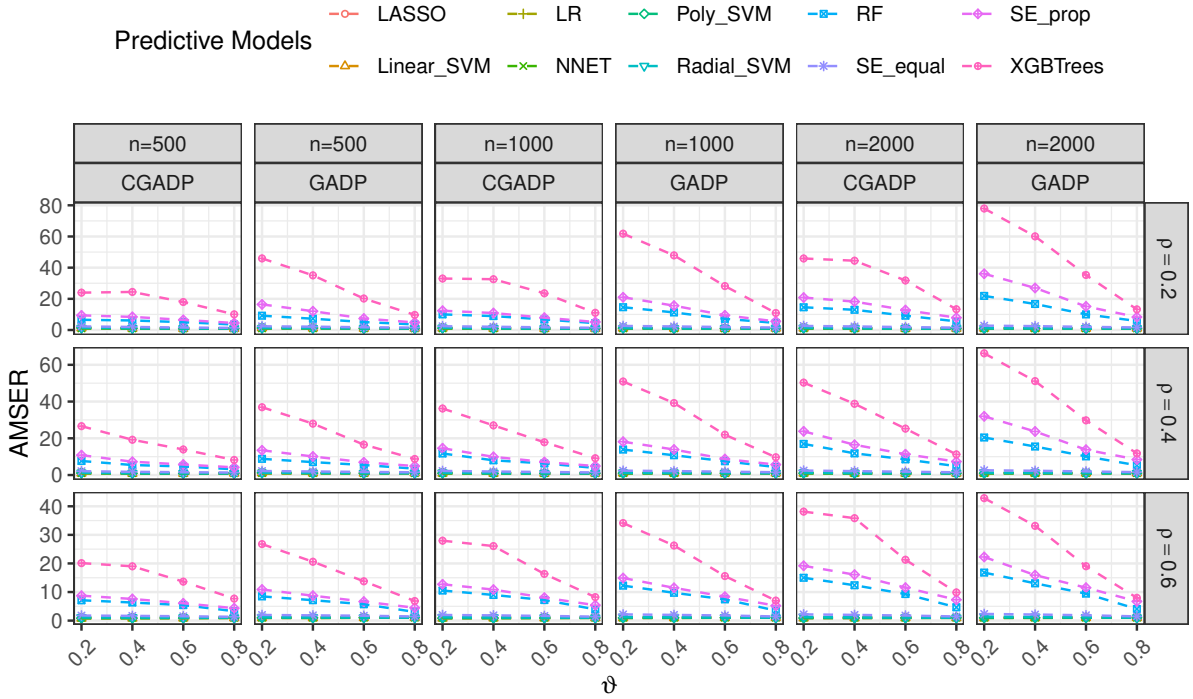
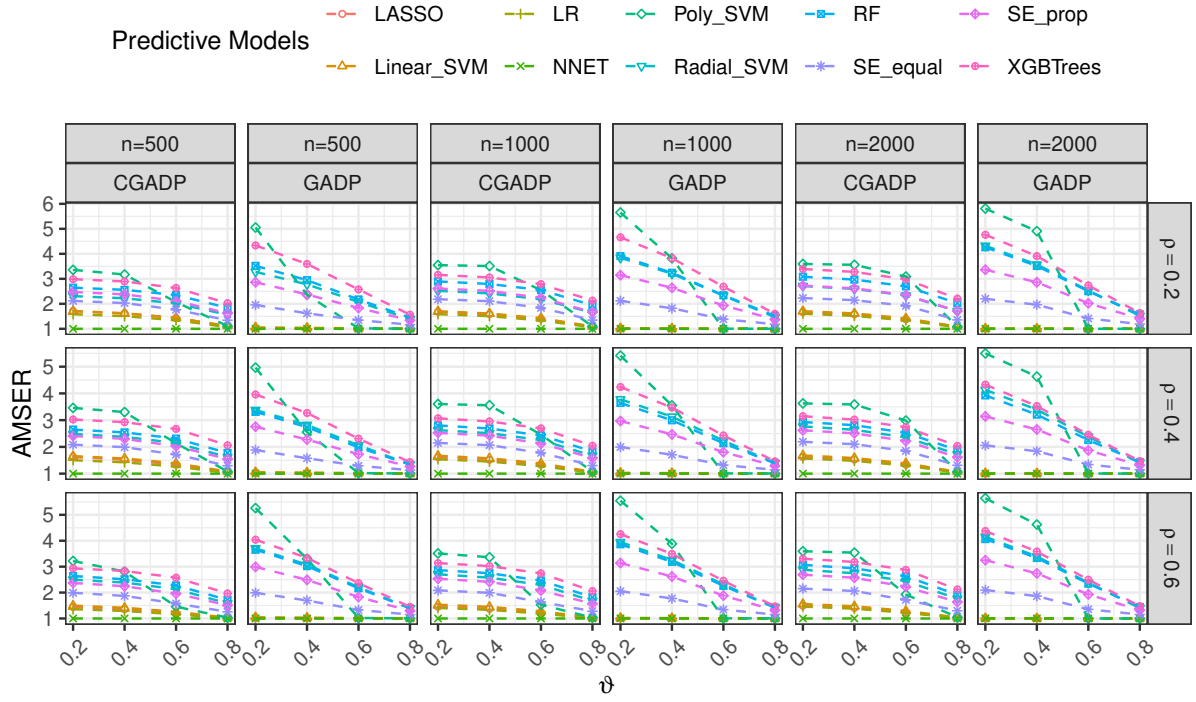
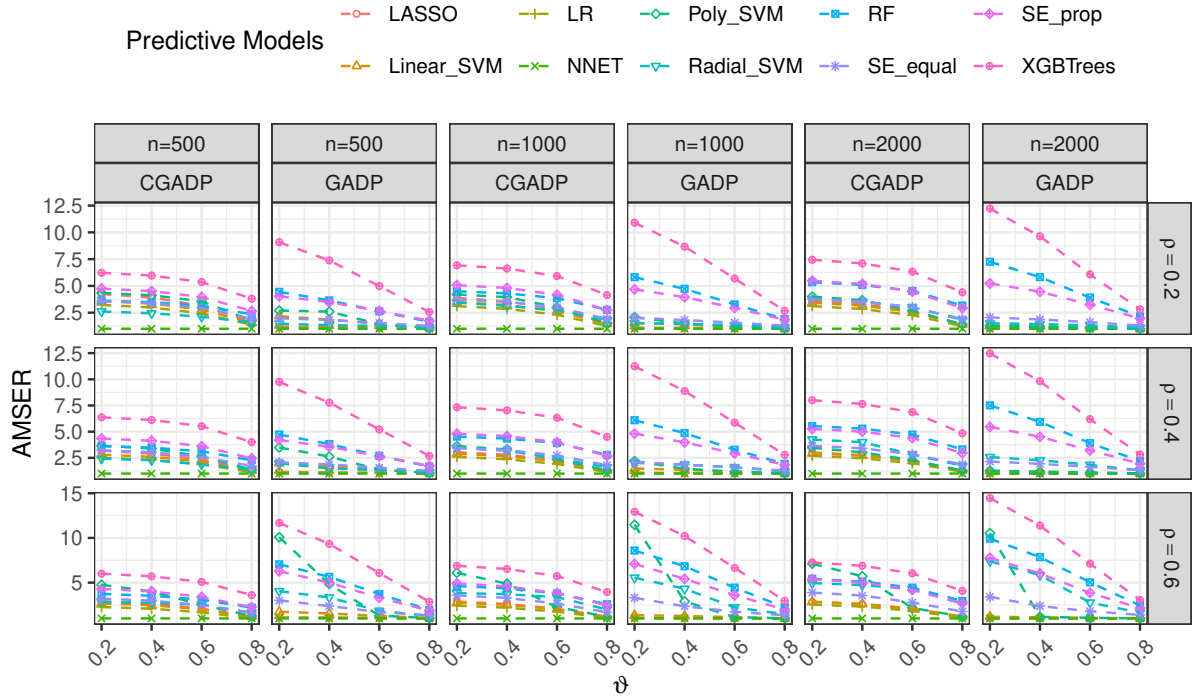


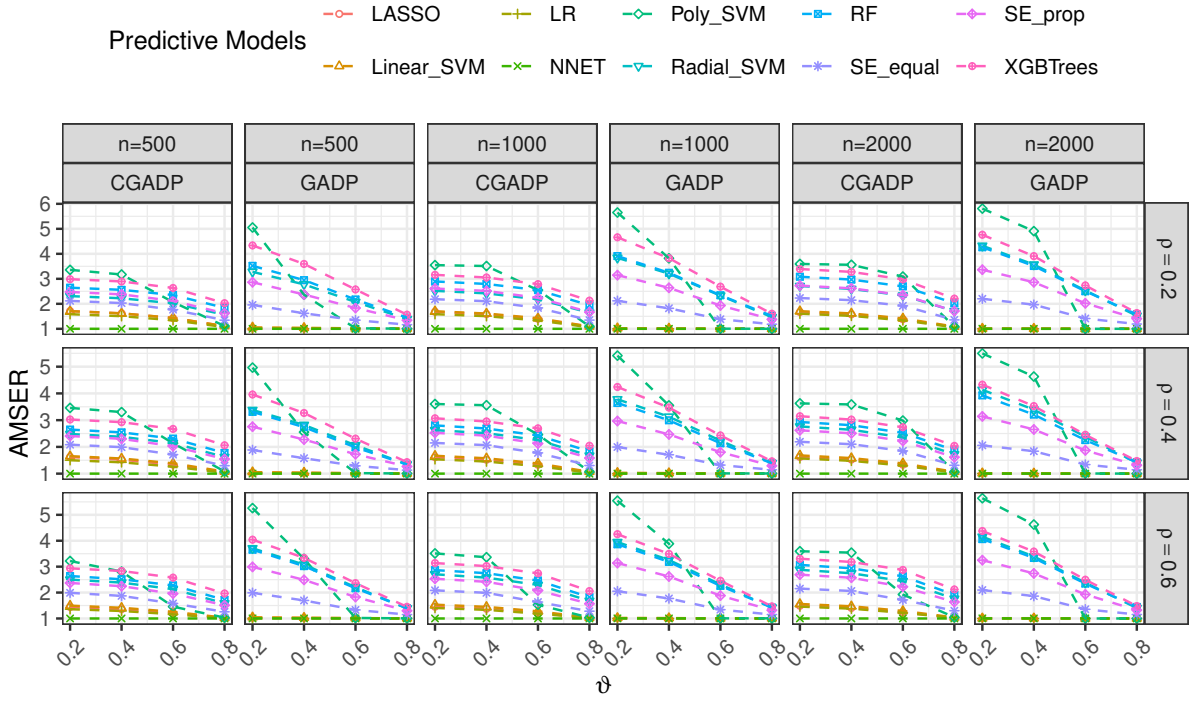
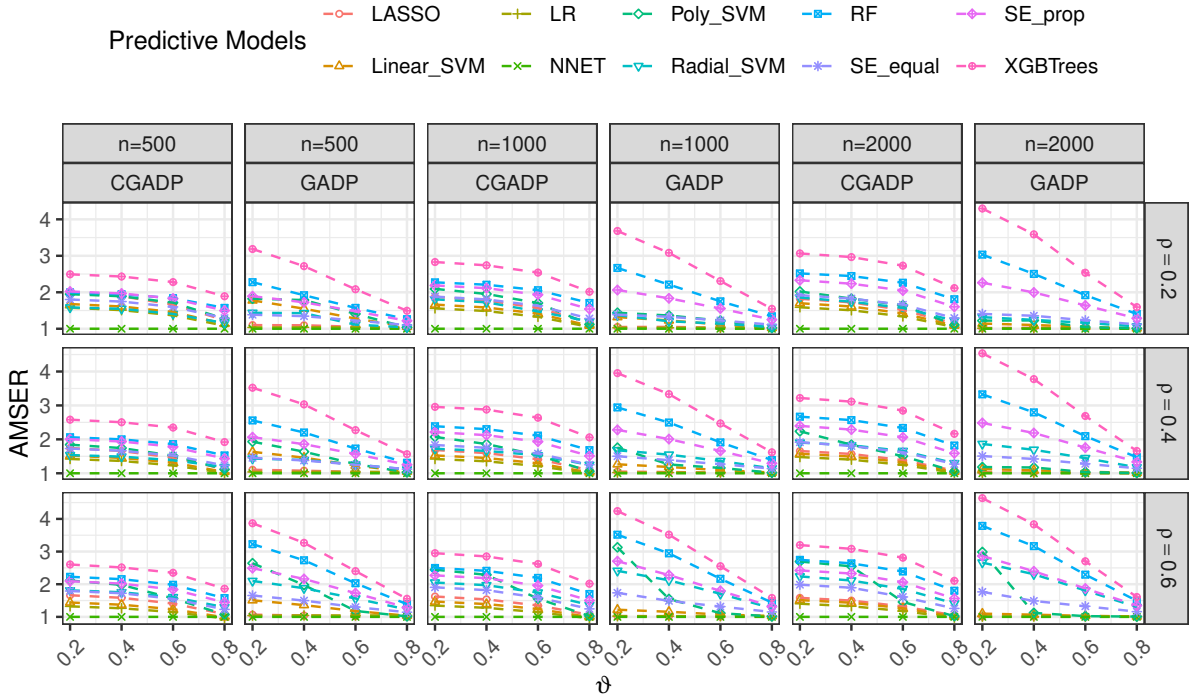
Figure 3: AMSER for ten ML techniques under **Model I** with  $p = 50$  and  $\sigma_\varepsilon = 0.4$ .



Figure 4: AMSER for ten ML techniques under **Model II** with  $p = 10$  and  $\sigma_\varepsilon = 0.4$ .Figure 5: AMSER for ten ML techniques under **Model II** with  $p = 50$  and  $\sigma_\varepsilon = 0.2$ .

Figure 6: AMSER for ten ML techniques under **Model II** with  $p = 50$  and  $\sigma_\varepsilon = 0.4$ .Figure 7: AMSER for ten ML techniques under **Model II** with  $p = 100$  and  $\sigma_\varepsilon = 0.4$ .

Figure 8: AMSER for ten ML techniques under **Model III** with  $p = 10$  and  $\sigma_\varepsilon = 0.4$ .Figure 9: AMSER for ten ML techniques under **Model III** with  $p = 50$  and  $\sigma_\varepsilon = 0.2$ .

Figure 10: AMSER for ten ML techniques under **Model III** with  $p = 50$  and  $\sigma_\varepsilon = 0.4$ .Figure 11: AMSER for ten ML techniques under **Model III** with  $p = 100$  and  $\sigma_\varepsilon = 0.4$ .

## 2.2 Best AMSER and AMSE Tables

Table 1: Best performing ML model in terms of AMSER and AMSE under Model I for GADP.

p	n	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
				rho											
10	500	0.2	0.2	LR	0.040	LR	1.026	LR	0.040	LR	1.026	LR	0.040	LR	1.026
			0.4	LR	0.178	LR	1.006	LR	0.178	LR	1.006	LR	0.178	LR	1.006
		0.4	0.2	LR	0.040	LR	1.026	LR	0.040	LR	1.026	LR	0.040	LR	1.026
			0.4	LR	0.178	LR	1.006	LR	0.178	LR	1.006	LR	0.178	LR	1.006
		0.6	0.2	LR	0.040	LR	1.026	LR	0.040	LR	1.026	LR	0.040	LR	1.026
			0.4	LR	0.178	LR	1.006	LR	0.178	LR	1.006	LR	0.177	LR	1.000
		0.8	0.2	LR	0.039	LR	1.000	LR	0.039	LR	1.000	LR	0.039	LR	1.000
			0.4	LR	0.177	LR	1.000	LASSO/LR	0.177	LR	1.000	LR	0.177	LR	1.000
	1000	0.2	0.2	LR	0.039	LR	1.026	LR	0.039	LR	1.026	LR	0.039	LR	1.026
			0.4	LR	0.175	LR	1.000	LR	0.175	LR	1.000	LR	0.175	LR	1.000
		0.4	0.2	LR	0.039	LR	1.026	LR	0.039	LR	1.026	LR	0.039	LR	1.026
			0.4	LR	0.175	LR	1.000	LR	0.175	LR	1.000	LR	0.175	LR	1.000
		0.6	0.2	LR	0.039	LR	1.026	LR	0.039	LR	1.026	LR	0.039	LR	1.026
			0.4	LR	0.175	LR	1.000	LR	0.175	LR	1.000	LR	0.175	LR	1.000
		0.8	0.2	LASSO/LR	0.039	LASSO/LR	1.026	LR	0.039	LR	1.026	LR	0.039	LR	1.026
			0.4	LR	0.175	LR	1.000	LASSO/LR	0.175	LR	1.000	LR	0.175	LR	1.000
	2000	0.2	0.2	LR	0.038	LR	1.000	LR	0.038	LR	1.000	LR	0.038	LR	1.000
			0.4	LR	0.174	LR	1.000	LR	0.174	LR	1.000	LR	0.174	LR	1.000
		0.4	0.2	LR	0.038	LR	1.000	LR	0.038	LR	1.000	LR	0.038	LR	1.000
			0.4	LR	0.174	LR	1.000	LR	0.174	LR	1.000	LR	0.174	LR	1.000
		0.6	0.2	LR	0.038	LR	1.000	LR	0.038	LR	1.000	LR	0.038	LR	1.000
			0.4	LR	0.174	LR	1.000	LR	0.174	LR	1.000	LR	0.174	LR	1.000
		0.8	0.2	LASSO/LR	0.038	LASSO/LR	1.000	LR	0.038	LR	1.000	LR	0.038	LR	1.000
			0.4	LR	0.174	LR	1.000	LASSO/LR	0.174	LR	1.000	LR	0.174	LR	1.000
50	500	0.2	0.2	LR	0.053	LR	1.104	LR	0.053	LR	1.104	LR	0.053	LR	1.104
			0.4	LR	0.191	LR	1.027	LR	0.191	LR	1.027	LR	0.191	LR	1.027
		0.4	0.2	LR	0.053	LR	1.104	LR	0.053	LR	1.104	LR	0.053	LR	1.104
			0.4	LR	0.191	LR	1.027	LR	0.191	LR	1.027	LR	0.191	LR	1.027
		0.6	0.2	LR	0.053	LR	1.104	LR	0.053	LR	1.104	LR	0.053	LR	1.104
			0.4	LR	0.191	LR	1.027	LR	0.191	LR	1.027	LR	0.191	LR	1.027
		0.8	0.2	LR	0.051	LR	1.062	LR	0.051	LR	1.062	LR	0.051	LR	1.062
			0.4	LASSO	0.184	LR	1.022	LASSO	0.186	LR	1.016	LR	0.189	LR	1.016
	1000	0.2	0.2	LR	0.049	LR	1.065	LR	0.049	LR	1.065	LR	0.048	LR	1.043
			0.4	LR	0.180	LR	1.017	LR	0.180	LR	1.017	LR	0.180	LR	1.017
		0.4	0.2	LR	0.049	LR	1.065	LR	0.048	LR	1.043	LR	0.048	LR	1.043
			0.4	LR	0.180	LR	1.017	LR	0.180	LR	1.017	LR	0.180	LR	1.017
		0.6	0.2	LR	0.048	LR	1.043	LR	0.048	LR	1.043	LR	0.048	LR	1.043
			0.4	LR	0.180	LR	1.017	LR	0.179	LR	1.011	LR	0.179	LR	1.011
		0.8	0.2	LASSO/LR	0.048	LR	1.043	LR	0.048	LR	1.043	LR	0.047	LR	1.022
			0.4	LASSO	0.177	LR	1.011	LASSO	0.177	LR	1.011	LASSO	0.178	LR	1.011
	2000	0.2	0.2	LR	0.046	LR	1.022	LR	0.046	LR	1.022	LR	0.046	LR	1.022
			0.4	LR	0.174	LR	1.006	LR	0.174	LR	1.006	LR	0.174	LR	1.006
		0.4	0.2	LR	0.046	LR	1.022	LR	0.046	LR	1.022	LR	0.046	LR	1.022
			0.4	LR	0.174	LR	1.006	LR	0.174	LR	1.006	LR	0.174	LR	1.006
		0.6	0.2	LR	0.046	LR	1.022	LR	0.046	LR	1.022	LR	0.046	LR	1.022
			0.4	LR	0.174	LR	1.006	LR	0.174	LR	1.006	LR	0.174	LR	1.006
		0.8	0.2	LASSO/LR	0.046	LR	1.022	LR	0.046	LR	1.022	LR	0.046	LR	1.022
			0.4	LASSO/LR	0.173	LR	1.000	LASSO/LR	0.173	LR	1.000	LASSO/LR	0.173	LR	1.000

Table 2: Best performing ML model in terms of AMSER and AMSE under Model I for GADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
100	500	0.2	0.2	LR	0.055	LR	1.222	LR	0.054	LR	1.200	LR	0.054	LR	1.200
			0.4	LR	0.229	LR	1.041	LR	0.229	LR	1.046	LR	0.228	LR	1.041
		0.4	0.2	LR	0.055	LR	1.222	LR	0.055	LR	1.222	LR	0.055	LR	1.222
			0.4	LR	0.230	LR	1.045	LR	0.229	LR	1.046	LR	0.229	LR	1.046
		0.6	0.2	LR	0.055	LR	1.222	LR	0.054	LR	1.200	LR	0.054	LR	1.200
			0.4	LASSO	0.224	LR	1.041	LR	0.229	LR	1.046	LR	0.228	LR	1.041
		0.8	0.2	LASSO	0.047	LR	1.156	LR	0.052	LR	1.156	LR	0.052	LR	1.156
			0.4	LASSO	0.198	LR	1.032	LASSO	0.216	LR	1.032	LASSO	0.220	LR	1.032
	1000	0.2	0.2	LR	0.045	LR	1.125	LR	0.044	LR	1.128	LR	0.044	LR	1.128
			0.4	LR	0.202	LR	1.025	LR	0.202	LR	1.025	LR	0.201	LR	1.020
		0.4	0.2	LR	0.045	LR	1.125	LR	0.044	LR	1.128	LR	0.044	LR	1.128
			0.4	LR	0.202	LR	1.025	LR	0.202	LR	1.025	LR	0.201	LR	1.020
		0.6	0.2	LR	0.044	LR	1.100	LR	0.044	LR	1.128	LR	0.044	LR	1.128
			0.4	LASSO	0.200	LR	1.025	LR	0.201	LR	1.020	LR	0.201	LR	1.020
		0.8	0.2	LASSO	0.040	LR	1.075	LR	0.043	LR	1.103	LR	0.043	LR	1.103
			0.4	LASSO	0.188	LR	1.020	LASSO	0.197	LR	1.015	LASSO	0.197	LR	1.015
2000	2000	0.2	0.2	LR	0.040	LR	1.081	LR	0.040	LR	1.081	LR	0.040	LR	1.081
			0.4	LR	0.190	LR	1.011	LR	0.190	LR	1.011	LR	0.190	LR	1.011
		0.4	0.2	LR	0.040	LR	1.081	LR	0.040	LR	1.081	LR	0.040	LR	1.081
			0.4	LR	0.190	LR	1.011	LR	0.190	LR	1.011	LR	0.190	LR	1.011
		0.6	0.2	LR	0.039	LR	1.054	LR	0.039	LR	1.054	LR	0.039	LR	1.054
			0.4	LASSO/LR	0.190	LR	1.011	LR	0.190	LR	1.011	LR	0.190	LR	1.011
		0.8	0.2	LASSO	0.038	LR	1.054	LR	0.039	LR	1.054	LR	0.039	LR	1.054
			0.4	LASSO	0.184	LR	1.011	LASSO/LR	0.189	LR	1.005	LASSO	0.188	LR	1.005



Table 3: Best performing ML model in terms of AMSER and AMSE under Model I for CGADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
10	500	0.2	0.2	LR	0.398	SE_equal	5.824	LR	0.532	SE_equal	7.169	LR	0.596	SE_equal	7.978
			0.4	LR	0.546	SE_equal	2.754	LR	0.648	SE_equal	3.181	LR	0.722	SE_equal	3.591
		0.4	0.2	LR	0.381	SE_equal	5.659	LR	0.512	SE_equal	6.966	LR	0.573	SE_equal	7.758
			0.4	LR	0.528	SE_equal	2.692	LR	0.627	SE_equal	3.105	LR	0.698	SE_equal	3.504
		0.6	0.2	LR	0.342	SE_equal	5.259	LR	0.464	SE_equal	6.494	LR	0.520	SE_equal	7.220
			0.4	LR	0.485	SE_equal	2.538	LR	0.577	SE_equal	2.920	LR	0.642	SE_equal	3.293
		0.8	0.2	LR	0.236	SE_equal	4.165	LR	0.330	SE_equal	5.146	LR	0.370	SE_equal	5.703
			0.4	LR	0.370	LR	2.090	LR	0.441	SE_equal	2.399	LR	0.488	SE_equal	2.702
	1000	0.2	0.2	LR	0.404	SE_equal	5.765	LR	0.542	SE_equal	7.273	LR	0.643	SE_equal	8.378
			0.4	LR	0.553	SE_equal	2.765	LR	0.657	SE_equal	3.234	LR	0.771	SE_equal	3.750
		0.4	0.2	LR	0.387	SE_equal	5.588	LR	0.521	SE_equal	7.068	LR	0.618	SE_equal	8.133
			0.4	LR	0.534	SE_equal	2.693	LR	0.635	SE_equal	3.149	LR	0.745	SE_equal	3.654
		0.6	0.2	LR	0.347	SE_equal	5.176	LR	0.470	SE_equal	6.557	LR	0.559	SE_equal	7.556
			0.4	LR	0.490	SE_equal	2.529	LR	0.583	SE_equal	2.949	LR	0.683	SE_equal	3.425
		0.8	0.2	LR	0.237	SE_equal	4.024	LR	0.330	SE_equal	5.125	LR	0.396	SE_equal	5.911
			0.4	LR	0.371	SE_equal	2.080	LR	0.440	SE_equal	2.396	LR	0.514	SE_equal	2.783
	2000	0.2	0.2	LR	0.416	SE_equal	5.892	LR	0.551	SE_equal	7.414	LR	0.680	SE_equal	8.753
			0.4	LR	0.567	SE_equal	2.821	LR	0.667	SE_equal	3.302	LR	0.810	SE_equal	3.907
		0.4	0.2	LR	0.399	SE_equal	5.711	LR	0.529	SE_equal	7.195	LR	0.654	SE_equal	8.494
			0.4	LR	0.547	SE_equal	2.748	LR	0.644	SE_equal	3.216	LR	0.782	SE_equal	3.806
		0.6	0.2	LR	0.357	SE_equal	5.265	LR	0.477	SE_equal	6.667	LR	0.591	SE_equal	7.876
			0.4	LR	0.501	SE_equal	2.573	LR	0.589	SE_equal	3.004	LR	0.716	SE_equal	3.561
		0.8	0.2	LR	0.244	SE_equal	4.024	LR	0.332	SE_equal	5.172	LR	0.416	SE_equal	6.112
			0.4	LR	0.378	SE_equal	2.085	LR	0.441	SE_equal	2.422	LR	0.535	SE_equal	2.873
50	500	0.2	0.2	LR	0.451	RF	5.183	LR	0.543	SE_equal	6.366	LR	0.641	SE_equal	7.540
			0.4	LR	0.595	SE_equal	2.665	LR	0.662	SE_equal	3.064	LR	0.749	SE_equal	3.522
		0.4	0.2	LR	0.436	RF	5.120	LR	0.525	SE_equal	6.218	LR	0.619	SE_equal	7.370
			0.4	LR	0.578	SE_equal	2.620	LR	0.642	SE_equal	3.004	LR	0.726	SE_equal	3.449
		0.6	0.2	LR	0.399	RF	4.785	LR	0.480	SE_equal	5.871	LR	0.567	SE_equal	6.930
			0.4	LR	0.537	SE_equal	2.502	LR	0.596	SE_equal	2.861	LR	0.672	SE_equal	3.263
		0.8	0.2	LR	0.293	RF	3.989	LR	0.355	SE_equal	4.832	LR	0.419	SE_equal	5.610
			0.4	LR	0.425	SE_equal	2.129	LR	0.468	SE_equal	2.430	LR	0.521	SE_equal	2.725
	1000	0.2	0.2	LR	0.437	SE_equal	5.656	LR	0.541	SE_equal	6.708	LR	0.655	SE_equal	7.958
			0.4	LR	0.573	SE_equal	2.793	LR	0.655	SE_equal	3.181	LR	0.761	SE_equal	3.679
		0.4	0.2	LR	0.420	SE_equal	5.521	LR	0.521	SE_equal	6.542	LR	0.632	SE_equal	7.760
			0.4	LR	0.555	SE_equal	2.737	LR	0.634	SE_equal	3.115	LR	0.736	SE_equal	3.596
		0.6	0.2	LR	0.381	SE_equal	5.134	LR	0.473	SE_equal	6.146	LR	0.575	SE_equal	7.260
			0.4	LR	0.512	SE_equal	2.598	LR	0.584	SE_equal	2.947	LR	0.678	SE_equal	3.392
		0.8	0.2	LR	0.270	SE_equal	4.104	LR	0.339	SE_equal	4.958	LR	0.416	SE_equal	5.781
			0.4	LR	0.394	SE_equal	2.155	LR	0.447	SE_equal	2.461	LR	0.515	SE_equal	2.783
	2000	0.2	0.2	LR	0.444	SE_equal	5.860	LR	0.555	SE_equal	6.978	LR	0.680	SE_equal	8.277
			0.4	LR	0.580	SE_equal	2.905	LR	0.667	SE_equal	3.291	LR	0.784	SE_equal	3.805
		0.4	0.2	LR	0.426	SE_equal	5.720	LR	0.534	SE_equal	6.806	LR	0.654	SE_equal	8.074
			0.4	LR	0.560	SE_equal	2.848	LR	0.645	SE_equal	3.219	LR	0.758	SE_equal	3.720
		0.6	0.2	LR	0.385	SE_equal	5.344	LR	0.483	SE_equal	6.366	LR	0.594	SE_equal	7.574
			0.4	LR	0.514	SE_equal	2.687	LR	0.592	SE_equal	3.038	LR	0.695	SE_equal	3.513
		0.8	0.2	LR	0.269	SE_equal	4.118	LR	0.341	SE_equal	5.043	LR	0.424	SE_equal	5.957
			0.4	LR	0.390	SE_equal	2.181	LR	0.445	SE_equal	2.494	LR	0.522	SE_equal	2.852

Table 4: Best performing ML model in terms of AMSER and AMSE under Model I for CGADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
100	500	0.2	0.2	RF	0.467	Radial_SVM	5.135	RF	0.599	SE_equal	6.833	LR	0.667	SE_equal	7.724
			0.4	RF	0.609	Radial_SVM	2.531	RF	0.772	SE_equal	3.070	LR	0.811	SE_equal	3.386
		0.4	0.2	RF	0.457	Radial_SVM	5.115	RF	0.585	SE_equal	6.708	LR	0.647	SE_equal	7.582
			0.4	RF	0.598	RF	2.492	RF	0.757	SE_equal	3.022	LR	0.790	SE_equal	3.330
		0.6	0.2	RF	0.434	Radial_SVM	5.029	LR	0.549	SE_equal	6.385	LR	0.598	SE_equal	7.235
			0.4	RF	0.572	RF	2.383	RF	0.721	SE_equal	2.900	LR	0.739	SE_equal	3.193
		0.8	0.2	RF	0.369	RF	4.341	LR	0.426	SE_equal	5.333	LR	0.459	SE_equal	6.173
			0.4	RF	0.501	RF	2.087	LR	0.591	SE_equal	2.498	LR	0.596	LR	2.721
	1000	0.2	0.2	LR	0.457	SE_equal	6.000	LR	0.555	SE_equal	7.337	LR	0.666	SE_equal	8.402
			0.4	LR	0.603	SE_equal	2.659	LR	0.718	SE_equal	3.217	LR	0.803	SE_equal	3.587
		0.4	0.2	LR	0.442	SE_equal	5.884	LR	0.536	SE_equal	7.191	LR	0.643	SE_equal	8.228
			0.4	LR	0.587	SE_equal	2.622	LR	0.698	SE_equal	3.163	LR	0.779	SE_equal	3.520
		0.6	0.2	LR	0.406	SE_equal	5.581	LR	0.490	SE_equal	6.809	LR	0.588	SE_equal	7.804
			0.4	LR	0.547	SE_equal	2.506	LR	0.648	SE_equal	3.019	LR	0.721	SE_equal	3.354
		0.8	0.2	LR	0.301	SE_equal	4.640	LR	0.360	SE_equal	5.539	LR	0.433	SE_equal	6.543
			0.4	LR	0.436	RF	2.157	LR	0.509	SE_equal	2.550	LR	0.561	LR	2.848
	2000	0.2	0.2	LR	0.434	SE_equal	6.108	LR	0.554	SE_equal	7.593	LR	0.700	SE_equal	8.888
			0.4	LR	0.578	SE_equal	2.712	LR	0.716	SE_equal	3.324	LR	0.839	SE_equal	3.754
		0.4	0.2	LR	0.418	SE_equal	5.964	LR	0.533	SE_equal	7.430	LR	0.675	SE_equal	8.685
			0.4	LR	0.560	SE_equal	2.658	LR	0.693	SE_equal	3.264	LR	0.812	SE_equal	3.677
		0.6	0.2	LR	0.380	SE_equal	5.602	LR	0.484	SE_equal	7.023	LR	0.614	SE_equal	8.191
			0.4	LR	0.518	SE_equal	2.519	LR	0.639	SE_equal	3.112	LR	0.748	SE_equal	3.488
		0.8	0.2	LR	0.272	SE_equal	4.506	LR	0.344	SE_equal	5.663	LR	0.441	SE_equal	6.730
			0.4	LR	0.403	SE_equal	2.115	LR	0.490	LR	2.606	LR	0.568	SE_equal	2.931

Table 5: Best performing ML model in terms of AMSER and AMSE under Model II for GADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
10	500	0.2	0.2	LASSO	3.377	Linear_SVM	0.982	LASSO	3.713	Lin/Poly SVM	0.958	Linear_SVM	3.871	Lin/Poly SVM	0.946
			0.4	LASSO	3.378	Linear_SVM	0.988	LASSO	3.920	Linear_SVM	0.964	Poly_SVM	4.077	Lin/Poly SVM	0.953
		0.4	0.2	LASSO	3.369	Linear_SVM	0.965	LASSO	3.721	Lin/Poly SVM	0.952	LASSO	3.920	Poly_SVM	0.960
			0.4	LASSO	3.371	Lin/Poly SVM	0.971	LASSO	3.928	Lin/Poly SVM	0.959	LASSO	4.125	Lin/Poly SVM	0.966
		0.6	0.2	LASSO/Poly_SVM	3.381	Lin/Poly SVM	0.958	LASSO	3.775	Poly_SVM	0.972	LASSO	3.954	Poly_SVM	0.974
			0.4	LASSO	3.386	Lin/Poly SVM	0.963	LASSO	3.981	Poly_SVM	0.978	LASSO	4.158	Poly_SVM	0.980
		0.8	0.2	LASSO	3.442	Poly_SVM	0.985	SE_prop	3.460	Poly_SVM	0.984	XGBTrees	2.752	Poly_SVM	0.975
			0.4	LASSO	3.447	Lin/Poly SVM	0.989	SE_prop	3.667	Poly_SVM	0.988	RF	2.940	Poly_SVM	0.981
	1000	0.2	0.2	LASSO	3.335	Linear_SVM	0.984	LASSO	3.689	Lin/Poly SVM	0.966	Linear_SVM	3.818	Lin/Poly SVM	0.953
			0.4	LASSO	3.335	Lin/Poly SVM	0.990	LASSO	3.894	Lin/Poly SVM	0.971	Lin/Poly SVM	4.028	Linear_SVM	0.959
		0.4	0.2	LASSO	3.331	Lin/Poly SVM	0.974	Poly_SVM	3.690	Lin/Poly SVM	0.962	Poly_SVM	3.860	Lin/Poly SVM	0.964
			0.4	LASSO	3.332	Linear_SVM	0.978	Poly_SVM	3.898	Lin/Poly SVM	0.968	Poly_SVM	4.067	Lin/Poly SVM	0.969
		0.6	0.2	Poly_SVM	3.332	Lin/Poly SVM	0.969	LASSO	3.721	Lin/Poly SVM	0.975	LASSO	3.877	Poly_SVM	0.972
			0.4	Poly_SVM	3.341	Lin/Poly SVM	0.973	LASSO	3.926	Poly_SVM	0.979	LASSO	4.083	Lin/Poly SVM	0.977
		0.8	0.2	LASSO	3.367	Lin/Poly SVM	0.985	SE_prop	3.314	Poly_SVM	0.980	RF/XGBTrees	2.455	Lin/Poly SVM	0.973
			0.4	LASSO	3.371	Poly_SVM	0.987	SE_prop	3.524	Lin/Poly SVM	0.985	RF	2.643	Lin/Poly SVM	0.978
	2000	0.2	0.2	LASSO	3.315	Lin/Poly SVM	0.984	LASSO	3.674	Lin/Poly SVM	0.972	Lin/Poly SVM	3.797	Lin/Poly SVM	0.959
			0.4	LASSO	3.316	Linear_SVM	0.988	LASSO	3.878	Lin/Poly SVM	0.977	Poly_SVM	4.004	Lin/Poly SVM	0.964
		0.4	0.2	LASSO	3.314	Linear_SVM	0.976	Poly_SVM	3.670	Lin/Poly SVM	0.970	Poly_SVM	3.828	Lin/Poly SVM	0.967
			0.4	LASSO	3.316	Linear_SVM	0.981	Poly_SVM	3.876	Linear_SVM	0.974	Poly_SVM	4.034	Lin/Poly SVM	0.972
		0.6	0.2	Poly_SVM	3.310	Lin/Poly SVM	0.974	LASSO	3.694	Lin/Poly SVM	0.979	LASSO	3.843	Poly_SVM	0.973
			0.4	Poly_SVM	3.318	Linear_SVM	0.977	LASSO	3.897	Lin/Poly SVM	0.982	LASSO	4.048	Lin/Poly SVM	0.977
		0.8	0.2	LASSO	3.334	Linear_SVM	0.984	RF	3.176	Lin/Poly SVM	0.982	RF	2.270	Lin/Poly SVM	0.974
			0.4	LASSO	3.338	Lin/Poly SVM	0.987	RF	3.383	Linear_SVM	0.985	RF	2.461	Lin/Poly SVM	0.978
	50	0.2	0.2	LASSO	3.984	LR	1.001	LASSO	3.418	Linear_SVM	0.947	LASSO	3.762	Linear_SVM	0.902
			0.4	LASSO	4.036	LR	1.001	LASSO	3.512	Linear_SVM	0.958	LASSO	3.731	Linear_SVM	0.908
		0.4	0.2	LASSO	3.908	Linear_SVM	0.979	LASSO	3.392	Linear_SVM	0.912	LASSO	3.804	Linear_SVM	0.921
			0.4	LASSO	3.962	Linear_SVM	0.983	LASSO	3.484	Linear_SVM	0.922	LASSO	3.778	Linear_SVM	0.930
		0.6	0.2	LASSO	3.793	Linear_SVM	0.915	LASSO	3.470	Linear_SVM	0.943	LASSO	3.969	Linear_SVM	0.985
			0.4	LASSO	3.860	Linear_SVM	0.922	LASSO	3.565	Linear_SVM	0.954	LASSO	3.944	Linear_SVM	0.992
		0.8	0.2	LASSO	3.995	Radial_SVM	0.949	SE_prop	3.473	Radial_SVM	1.003	XGBTrees	2.939	Linear_SVM	0.999
			0.4	LASSO	4.074	Radial_SVM	0.958	SE_prop	3.592	Radial_SVM	1.003	XGBTrees	3.000	Radial_SVM	1.003
	1000	0.2	0.2	LR	3.773	LR/Linear_SVM	1.001	LASSO	3.337	Linear_SVM	0.944	LASSO	3.699	Linear_SVM	0.919
			0.4	LASSO	3.847	LR	1.001	LASSO	3.425	Linear_SVM	0.953	LASSO	3.663	Linear_SVM	0.929
		0.4	0.2	LASSO	3.748	Linear_SVM	0.968	LASSO	3.330	Linear_SVM	0.927	LASSO	3.741	Linear_SVM	0.944
			0.4	LASSO	3.811	Linear_SVM	0.973	LASSO	3.416	Linear_SVM	0.935	LASSO	3.704	Linear_SVM	0.954
		0.6	0.2	LASSO	3.692	Lin/Poly SVM	0.941	LASSO	3.396	Linear_SVM	0.961	LASSO	3.814	Linear_SVM	0.978
			0.4	LASSO	3.765	Linear_SVM	0.947	LASSO	3.484	Linear_SVM	0.969	LASSO	3.776	Linear_SVM	0.987
		0.8	0.2	LASSO	3.789	Radial_SVM	0.944	XGBTrees	3.170	Radial_SVM	0.977	XGBTrees	2.517	Linear_SVM	0.983
			0.4	LASSO	3.868	Radial_SVM	0.955	XGBTrees	3.297	Radial_SVM	0.978	XGBTrees	2.573	Radial_SVM	0.988
	2000	0.2	0.2	LR	3.672	Linear_SVM	0.992	LASSO	3.306	Linear_SVM	0.954	LASSO	3.674	Linear_SVM	0.936
			0.4	LR	3.756	Linear_SVM	0.998	LASSO	3.390	Linear_SVM	0.961	LASSO	3.633	Linear_SVM	0.945
		0.4	0.2	LASSO/LR	3.672	Linear_SVM	0.973	LASSO	3.305	Linear_SVM	0.944	LASSO	3.697	Linear_SVM	0.956
			0.4	LASSO	3.740	Linear_SVM	0.979	LASSO	3.389	Linear_SVM	0.952	LASSO	3.656	Linear_SVM	0.963
		0.6	0.2	LASSO	3.643	Poly_SVM	0.959	LASSO	3.346	Linear_SVM	0.969	LASSO	3.731	Linear_SVM	0.974
			0.4	LASSO	3.719	Lin/Poly SVM	0.967	LASSO	3.429	Linear_SVM	0.975	LASSO	3.689	Linear_SVM	0.980
		0.8	0.2	LASSO	3.690	Radial_SVM	0.960	XGBTrees	2.926	Radial_SVM	0.962	XGBTrees	2.276	Linear_SVM	0.975
			0.4	LASSO	3.768	Radial_SVM	0.969	XGBTrees	3.038	Radial_SVM	0.966	XGBTrees	2.313	Linear_SVM	0.981

				$\rho=0.2$				$\rho=0.4$				$\rho=0.6$				
$p$	$n$	$\vartheta$	$\sigma$	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	
100	500	0.2	0.2	LASSO	3.687	LR	1.002	LASSO	3.923	LR	1.002	LASSO	3.515	Linear_SVM	0.913	
			0.4	LASSO	3.750	LR	1.003	LASSO	3.949	LR	1.002	LASSO	3.767	Linear_SVM	0.920	
		0.4	0.2	LASSO	3.617	LR	1.002	LASSO	3.874	Linear_SVM	0.967	LASSO	3.521	Linear_SVM	0.882	
			0.4	LASSO	3.684	LR	1.003	LASSO	3.901	Linear_SVM	0.969	LASSO	3.781	Linear_SVM	0.893	
		0.6	0.2	LASSO	3.517	Linear_SVM	0.927	LASSO	3.843	Linear_SVM	0.903	LASSO	3.792	Linear_SVM	0.972	
			0.4	LASSO	3.590	Linear_SVM	0.932	LASSO	3.882	Linear_SVM	0.911	LASSO	4.076	Linear_SVM	0.984	
		0.8	0.2	LASSO	3.913	Radial_SVM	0.982	XGBTrees	3.972	Radial_SVM	1.021	XGBTrees	2.751	Radial_SVM	1.003	
			0.4	LASSO	4.009	Radial_SVM	0.982	SE_prop	4.066	Radial_SVM	1.012	XGBTrees	2.996	Radial_SVM	0.993	
	1000	0.2	0.2	LASSO	3.488	LR	1.001	LASSO	3.777	Linear_SVM	0.963	LASSO	3.464	Linear_SVM	0.888	
			0.4	LASSO	3.558	LR	1.002	LASSO	3.806	Linear_SVM	0.966	LASSO	3.724	Linear_SVM	0.899	
		0.4	0.2	LASSO	3.461	Linear_SVM	0.980	LASSO	3.759	Linear_SVM	0.927	LASSO	3.512	Linear_SVM	0.910	
			0.4	LASSO	3.534	Linear_SVM	0.985	LASSO	3.789	Linear_SVM	0.931	LASSO	3.781	Linear_SVM	0.922	
		0.6	0.2	LASSO	3.427	Linear_SVM	0.911	LASSO	3.813	Linear_SVM	0.950	LASSO	3.674	Linear_SVM	0.979	
			0.4	LASSO	3.507	Linear_SVM	0.918	LASSO	3.853	Linear_SVM	0.956	LASSO	3.947	Linear_SVM	0.987	
		0.8	0.2	LASSO	3.660	Radial_SVM	0.984	XGBTrees	3.525	Linear_SVM	1.007	XGBTrees	2.299	Linear_SVM	0.993	
			0.4	LASSO	3.748	Radial_SVM	0.984	XGBTrees	3.589	Radial_SVM	1.011	XGBTrees	2.541	Lin/Poly SVM	1.000	
		2000	0.2	0.2	LASSO	3.418	LR	1.001	LASSO	3.725	Linear_SVM	0.957	LASSO	3.459	Linear_SVM	0.907
				0.4	LASSO	3.493	LR	1.001	LASSO	3.756	Linear_SVM	0.962	LASSO	3.723	Linear_SVM	0.919
			0.4	0.2	LASSO	3.406	Linear_SVM	0.979	LASSO	3.718	Linear_SVM	0.940	LASSO	3.501	Linear_SVM	0.938
				0.4	LASSO	3.483	Linear_SVM	0.983	LASSO	3.750	Linear_SVM	0.945	LASSO	3.769	Linear_SVM	0.948
0.6	0.2		LASSO	3.400	Linear_SVM	0.946	LASSO	3.767	Linear_SVM	0.969	LASSO	3.579	Linear_SVM	0.973		
	0.4		LASSO	3.483	Linear_SVM	0.951	LASSO	3.807	Linear_SVM	0.976	LASSO	3.849	Linear_SVM	0.980		
0.8	0.2		LASSO	3.516	Radial_SVM	0.999	XGBTrees	3.242	Linear_SVM	0.995	XGBTrees	2.053	Linear_SVM	0.978		
	0.4		LASSO	3.604	Radial_SVM	0.997	XGBTrees	3.310	Linear_SVM	1.000	XGBTrees	2.287	Linear_SVM	0.984		

Table 7: Best performing ML model in terms of AMSER and AMSE under Model II for CGADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
10	500	0.2	0.2	LR	3.919	Linear_SVM	1.152	LR	3.812	Linear_SVM	0.979	LASSO	3.752	Lin/Poly SVM	0.929
			0.4	LR	3.906	Linear_SVM	1.148	LR	4.011	Linear_SVM	0.983	LASSO/LR	3.959	Linear_SVM	0.936
		0.4	0.2	LR	3.618	Linear_SVM	1.064	LR	3.681	Linear_SVM	0.947	LASSO	3.773	Poly_SVM	0.933
			0.4	LR	3.610	Linear_SVM	1.062	LR	3.888	Linear_SVM	0.954	LASSO	3.981	Poly_SVM	0.941
		0.6	0.2	LR	3.322	Linear_SVM	0.970	LASSO	3.656	Poly_SVM	0.941	LASSO	3.809	Poly_SVM	0.943
			0.4	LR	3.328	Linear_SVM	0.972	LASSO	3.871	Lin/Poly SVM	0.951	LASSO	4.017	Poly_SVM	0.950
		0.8	0.2	LASSO	3.354	Poly_SVM	0.961	SE_prop	3.503	Poly_SVM	0.963	XGBTrees	2.735	Poly_SVM	0.956
			0.4	LASSO	3.360	Poly_SVM	0.965	SE_prop	3.726	Poly_SVM	0.970	XGBTrees	2.954	Poly_SVM	0.962
	1000	0.2	0.2	LR	3.851	Linear_SVM	1.149	LR	3.784	Linear_SVM	0.986	LR	3.753	Lin/Poly SVM	0.947
			0.4	LR	3.836	Linear_SVM	1.144	LR	3.980	Linear_SVM	0.988	LR	3.958	Lin/Poly SVM	0.953
		0.4	0.2	LR	3.554	Linear_SVM	1.062	LR	3.662	Linear_SVM	0.958	LASSO/LR	3.771	Lin/Poly SVM	0.953
			0.4	LR	3.544	LR	1.059	LR	3.865	Linear_SVM	0.963	LASSO/LR	3.977	Poly_SVM	0.958
		0.6	0.2	LR	3.297	Linear_SVM	0.981	LASSO	3.652	Lin/Poly SVM	0.960	LASSO	3.796	Poly_SVM	0.961
			0.4	LR	3.303	Linear_SVM	0.984	LASSO	3.860	Lin/Poly SVM	0.966	LASSO	4.002	Lin/Poly SVM	0.966
		0.8	0.2	LASSO	3.322	Lin/Poly SVM	0.976	SE_prop	3.402	Poly_SVM	0.979	XGBTrees	2.578	Poly_SVM	0.970
			0.4	LASSO	3.327	Lin/Poly SVM	0.978	SE_prop	3.616	Lin/Poly SVM	0.983	XGBTrees	2.778	Lin/Poly SVM	0.974
	2000	0.2	0.2	LR	3.848	Linear_SVM	1.151	Linear_SVM	3.791	Linear_SVM	1.001	LR	3.779	Linear_SVM	0.965
			0.4	LR	3.828	Linear_SVM	1.146	LR	3.985	Linear_SVM	1.002	LR	3.982	Linear_SVM	0.969
		0.4	0.2	LR	3.544	Linear_SVM	1.065	LR	3.670	Linear_SVM	0.972	LASSO/LR	3.789	Lin/Poly SVM	0.971
			0.4	LR	3.531	LR/Linear_SVM	1.062	LR	3.869	Linear_SVM	0.975	LR	3.993	Lin/Poly SVM	0.975
		0.6	0.2	LR	3.293	Linear_SVM	0.989	LASSO	3.662	Lin/Poly SVM	0.976	LASSO	3.803	Lin/Poly SVM	0.976
			0.4	LR	3.298	Linear_SVM	0.991	LASSO	3.865	Lin/Poly SVM	0.980	LASSO	4.009	Lin/Poly SVM	0.980
		0.8	0.2	LASSO	3.308	Linear_SVM	0.982	XGBTrees	3.335	Lin/Poly SVM	0.992	RF	2.468	Lin/Poly SVM	0.980
			0.4	LASSO	3.311	Linear_SVM	0.984	XGBTrees	3.545	Lin/Poly SVM	0.993	RF	2.665	Lin/Poly SVM	0.983
50	500	0.2	0.2	LR	4.925	Linear_SVM	1.218	LR	3.524	Linear_SVM	0.944	LR	3.588	Linear_SVM	0.841
			0.4	LR	4.970	Linear_SVM	1.207	LR	3.602	Linear_SVM	0.947	LR	3.558	Linear_SVM	0.844
		0.4	0.2	LR	4.488	Linear_SVM	1.102	LR	3.331	Linear_SVM	0.875	LASSO	3.558	Linear_SVM	0.830
			0.4	LR	4.547	Linear_SVM	1.095	LR	3.412	Linear_SVM	0.880	LASSO	3.533	Linear_SVM	0.833
		0.6	0.2	LASSO	3.849	Linear_SVM	0.924	LASSO	3.256	Linear_SVM	0.831	LASSO	3.617	Linear_SVM	0.860
			0.4	LASSO	3.918	Linear_SVM	0.930	LASSO	3.358	Linear_SVM	0.842	LASSO	3.588	Linear_SVM	0.862
		0.8	0.2	LASSO	3.662	Poly_SVM	0.903	SE_prop	3.351	Linear_SVM	0.893	XGBTrees	2.992	Linear_SVM	0.908
			0.4	LASSO	3.765	Poly_SVM	0.920	LASSO	3.450	Linear_SVM	0.901	XGBTrees	3.053	Linear_SVM	0.910
	1000	0.2	0.2	LR	4.622	Linear_SVM	1.194	LR	3.391	Linear_SVM	0.942	LR	3.517	Linear_SVM	0.872
			0.4	LR	4.654	Linear_SVM	1.180	LR	3.468	Linear_SVM	0.945	LR	3.485	Linear_SVM	0.878
		0.4	0.2	LR	4.204	Linear_SVM	1.084	LR	3.238	Linear_SVM	0.892	LR	3.551	Linear_SVM	0.878
			0.4	LR	4.253	Linear_SVM	1.076	LR	3.318	Linear_SVM	0.897	LR	3.518	Linear_SVM	0.884
		0.6	0.2	LR	3.675	Linear_SVM	0.942	LASSO	3.245	Linear_SVM	0.886	LASSO	3.605	Linear_SVM	0.908
			0.4	LR	3.768	Linear_SVM	0.946	LASSO	3.335	Linear_SVM	0.893	LASSO	3.569	Linear_SVM	0.912
		0.8	0.2	LASSO	3.634	Radial_SVM	0.909	SE_prop	3.203	Linear_SVM	0.935	XGBTrees	2.667	Linear_SVM	0.938
			0.4	LASSO	3.729	Radial_SVM	0.924	SE_prop	3.317	Linear_SVM	0.941	XGBTrees	2.702	Linear_SVM	0.943
	2000	0.2	0.2	LR	4.556	Linear_SVM	1.216	LR	3.396	Linear_SVM	0.968	LR	3.552	Linear_SVM	0.913
			0.4	LR	4.576	Linear_SVM	1.197	LR	3.473	Linear_SVM	0.971	LR	3.519	Linear_SVM	0.920
		0.4	0.2	LR	4.133	Linear_SVM	1.103	LR	3.256	Linear_SVM	0.929	LR	3.584	Linear_SVM	0.923
			0.4	LR	4.172	Linear_SVM	1.092	LR	3.335	Linear_SVM	0.932	LR	3.546	Linear_SVM	0.928
		0.6	0.2	LR	3.646	Poly_SVM	0.972	LASSO	3.256	Linear_SVM	0.931	LASSO	3.624	Linear_SVM	0.944
			0.4	LR	3.732	Linear_SVM	0.976	LASSO	3.341	Linear_SVM	0.936	LASSO	3.584	Linear_SVM	0.947
		0.8	0.2	LASSO	3.629	Radial_SVM	0.943	XGBTrees	3.049	Radial_SVM	0.946	XGBTrees	2.485	Linear_SVM	0.961
			0.4	LASSO	3.716	Radial_SVM	0.954	XGBTrees	3.154	Radial_SVM	0.947	XGBTrees	2.516	Linear_SVM	0.964

Table 8: Best performing ML model in terms of AMSER and AMSE under Model II for CGADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
100	500	0.2	0.2	LASSO	5.007	LR	1.203	LR	4.304	LR	0.940	LR	3.456	Linear_SVM	0.759
			0.4	LASSO	5.024	LR	1.179	LR	4.388	LR	0.945	LASSO	3.704	Linear_SVM	0.773
		0.4	0.2	LR	4.297	LR	0.981	LR	4.134	Linear_SVM	0.851	LASSO	3.657	Linear_SVM	0.797
			0.4	LR	4.437	LR	0.962	LR	4.257	Linear_SVM	0.840	LASSO	3.748	Linear_SVM	0.806
		0.6	0.2	LASSO	3.565	Linear_SVM	0.820	LASSO	3.789	Linear_SVM	0.805	LASSO	3.614	Linear_SVM	0.821
			0.4	LASSO	3.734	Linear_SVM	0.816	LASSO	3.964	Linear_SVM	0.801	LASSO	3.733	Linear_SVM	0.834
		0.8	0.2	LASSO	3.540	Linear_SVM	0.843	SE_prop	3.828	Linear_SVM	0.875	XGBTrees	3.244	Linear_SVM	0.892
			0.4	LASSO	3.730	Linear_SVM	0.846	SE_prop	3.953	Linear_SVM	0.873	XGBTrees	3.386	Linear_SVM	0.902
	1000	0.2	0.2	LR	4.258	Linear_SVM	1.126	LR	3.809	Linear_SVM	0.914	LR	3.263	Linear_SVM	0.790
			0.4	LR	4.306	LR	1.112	LR	3.868	Linear_SVM	0.920	LR	3.537	Linear_SVM	0.806
		0.4	0.2	LR	3.701	Linear_SVM	0.939	LR	3.742	Linear_SVM	0.840	LASSO	3.562	Linear_SVM	0.831
			0.4	LR	3.856	Linear_SVM	0.932	LR	3.889	Linear_SVM	0.838	LASSO	3.662	Linear_SVM	0.842
		0.6	0.2	LR	3.405	Linear_SVM	0.828	LASSO	3.712	Linear_SVM	0.842	LASSO	3.580	Linear_SVM	0.876
			0.4	LR	3.587	Linear_SVM	0.833	LASSO	3.897	Linear_SVM	0.846	LASSO	3.692	Linear_SVM	0.887
		0.8	0.2	LASSO	3.487	Linear_SVM	0.889	SE_prop	3.594	Linear_SVM	0.904	XGBTrees	2.813	Linear_SVM	0.928
			0.4	LASSO	3.683	Linear_SVM	0.893	SE_prop	3.741	Linear_SVM	0.909	XGBTrees	2.937	Linear_SVM	0.937
2000	0.2	0.2	0.2	LR	4.021	Linear_SVM	1.109	LR	3.757	Linear_SVM	0.934	LR	3.298	Linear_SVM	0.847
			0.4	LR	4.068	Linear_SVM	1.099	LR	3.796	Linear_SVM	0.937	LR	3.566	Linear_SVM	0.860
		0.4	0.2	LR	3.624	Linear_SVM	0.967	LR	3.701	Linear_SVM	0.880	LASSO	3.548	Linear_SVM	0.888
			0.4	LR	3.790	Linear_SVM	0.961	LR	3.865	Linear_SVM	0.880	LASSO	3.651	Linear_SVM	0.897
		0.6	0.2	LR	3.374	Linear_SVM	0.884	LASSO	3.723	Linear_SVM	0.895	LASSO	3.579	Linear_SVM	0.924
			0.4	LR	3.566	Linear_SVM	0.889	LASSO	3.912	Linear_SVM	0.899	LASSO	3.686	Linear_SVM	0.932
		0.8	0.2	LASSO	3.474	Linear_SVM	0.924	XGBTrees	3.367	Linear_SVM	0.933	XGBTrees	2.554	Poly_SVM	0.952
			0.4	LASSO	3.673	Linear_SVM	0.929	XGBTrees	3.528	Linear_SVM	0.937	XGBTrees	2.662	Lin/Poly SVM	0.959
	0.4	0.2	0.2	LR	4.021	Linear_SVM	1.109	LR	3.757	Linear_SVM	0.934	LR	3.298	Linear_SVM	0.847
			0.4	LR	4.068	Linear_SVM	1.099	LR	3.796	Linear_SVM	0.937	LR	3.566	Linear_SVM	0.860
		0.4	0.2	LR	3.624	Linear_SVM	0.967	LR	3.701	Linear_SVM	0.880	LASSO	3.548	Linear_SVM	0.888
			0.4	LR	3.790	Linear_SVM	0.961	LR	3.865	Linear_SVM	0.880	LASSO	3.651	Linear_SVM	0.897



Table 9: Best performing ML model in terms of AMSER and AMSE under Model III for GADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
10	500	0.2	0.2	LR	0.063	LR	1.016	LR	0.062	LR	1.016
			0.4	LR	0.209	LR	1.005	LR	0.199	LR	1.010
		0.4	0.2	LR	0.063	LR	1.016	LR	0.062	LR	1.016
			0.4	LASSO/LR	0.209	LR	1.005	LR	0.199	LR	1.010
		0.6	0.2	LR	0.063	LR	1.016	LASSO/LR	0.062	LASSO/LR	1.016
			0.4	LASSO	0.208	LASSO/LR	1.005	LASSO/LR	0.199	LASSO/LR	1.010
		0.8	0.2	LASSO/LR	0.062	LASSO/LR/Lin/Poly SVM	1.000	LASSO	0.061	LASSO/Lin/Poly SVM	1.000
			0.4	LASSO	0.208	LASSO/LR/Poly SVM	1.005	LASSO/LR	0.199	Lin/Poly SVM	1.005
	1000	0.2	0.2	LR	0.062	LR	1.016	LR	0.061	LR	1.000
			0.4	LR	0.206	LR	1.000	LR	0.196	LR	1.005
		0.4	0.2	LR	0.062	LR	1.016	LR	0.061	LR	1.000
			0.4	LASSO/LR	0.206	LR	1.000	LR	0.196	LR	1.005
		0.6	0.2	LASSO/LR	0.062	LASSO/LR	1.016	LASSO/LR	0.061	LASSO/LR	1.000
			0.4	LASSO/LR	0.206	LR	1.000	LR	0.196	LASSO/LR	1.005
		0.8	0.2	LASSO	0.061	LASSO/Lin/Poly SVM	1.000	LASSO/LR/Lin/Poly SVM	0.061	LASSO/LR/Lin/Poly SVM	1.000
			0.4	LASSO	0.206	Lin/Poly SVM	1.000	LASSO/LR	0.196	LASSO	1.000
	2000	0.2	0.2	LR	0.061	LR	1.000	LASSO/LR	0.061	LASSO/LR	1.017
			0.4	LASSO/LR	0.205	LR	1.000	LR	0.195	LASSO/LR	1.005
		0.4	0.2	LR	0.061	LR	1.000	LASSO/LR	0.061	LASSO/LR	1.017
			0.4	LASSO/LR	0.205	LR	1.000	LR	0.195	LASSO/LR	1.005
		0.6	0.2	LASSO/LR	0.061	LASSO/LR	1.000	LASSO/LR	0.061	Lin/Poly SVM	1.016
			0.4	LASSO/LR/Lin/Poly SVM	0.205	LR/Lin/Poly SVM	1.000	LASSO/LR	0.195	LASSO	1.000
		0.8	0.2	LASSO/LR/Lin/Poly SVM	0.061	LASSO/LR/Lin/Poly SVM	1.000	LASSO	0.060	LASSO/Lin/Poly SVM	1.000
			0.4	LASSO/LR/Lin/Poly SVM	0.205	LR/Lin/Poly SVM	1.000	LASSO/LR/Lin/Poly SVM	0.195	LASSO/Lin/Poly SVM	1.000
50	500	0.2	0.2	LR	0.073	LR	1.074	LR	0.075	LR	1.071
			0.4	LASSO	0.215	LR	1.024	LASSO	0.208	LR	1.024
		0.4	0.2	LASSO/LR	0.073	LR	1.074	LASSO	0.073	LR	1.071
			0.4	LASSO	0.211	LR	1.024	LASSO	0.205	LR	1.024
		0.6	0.2	LASSO	0.068	LR	1.059	LASSO	0.069	LR	1.057
			0.4	LASSO	0.205	LR	1.024	LASSO	0.201	LASSO	1.015
		0.8	0.2	LASSO	0.064	LASSO	1.016	LASSO	0.065	LASSO	1.000
			0.4	LASSO	0.200	LASSO	1.005	LASSO	0.200	LASSO	1.010
	1000	0.2	0.2	LR	0.067	LR	1.031	LASSO/LR	0.069	LR	1.045
			0.4	LASSO	0.203	LR	1.015	LASSO	0.199	LR	1.015
		0.4	0.2	LASSO/LR	0.067	LR	1.031	LASSO	0.067	LR	1.045
			0.4	LASSO	0.201	LR	1.015	LASSO	0.198	LASSO/LR	1.015
		0.6	0.2	LASSO	0.065	LR	1.031	LASSO	0.066	LR	1.030
			0.4	LASSO	0.199	LR	1.015	LASSO	0.196	LASSO	1.005
		0.8	0.2	LASSO	0.063	LR	1.000	LASSO	0.064	LASSO	1.000
			0.4	LASSO	0.196	LASSO	1.005	LASSO	0.196	LASSO	1.005
	2000	0.2	0.2	LASSO/LR	0.064	LR	1.016	LASSO/LR	0.066	LR	1.015
			0.4	LASSO/LR	0.198	LR	1.010	LASSO	0.195	LR	1.005
		0.4	0.2	LASSO/LR	0.064	LR	1.016	LASSO	0.065	LR	1.015
			0.4	LASSO	0.197	LR	1.010	LASSO	0.195	LR	1.005
		0.6	0.2	LASSO	0.063	LASSO/LR	1.016	LASSO	0.064	LASSO	1.000
			0.4	LASSO	0.195	LASSO	1.005	LASSO	0.194	LASSO/LR	1.005
		0.8	0.2	LASSO	0.062	LASSO/LR/Lin/Poly SVM	1.000	LASSO	0.064	LASSO/LR	1.000
			0.4	LASSO	0.194	LASSO	1.000	LASSO	0.194	LASSO/LR/Poly SVM	1.005

Table 10: Best performing ML model in terms of AMSER and AMSE under Model III for GADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
10	500	0.2	0.2	LR	0.062	LR	1.016	LR	0.061	LR	1.017
			0.4	LR	0.199	LR	1.010	LR	0.200	LR	1.005
		0.4	0.2	LR	0.062	LR	1.016	LR	0.061	LR	1.017
			0.4	LR	0.199	LR	1.010	LR	0.200	LR	1.005
		0.6	0.2	LASSO/LR	0.062	LASSO/LR	1.016	LASSO/LR	0.061	LASSO/LR	1.017
			0.4	LASSO/LR	0.199	LASSO/LR	1.010	LR	0.200	LR	1.005
		0.8	0.2	LASSO	0.061	LASSO/Lin/Poly SVM	1.000	LASSO/LR	0.060	LASSO/LR	1.000
			0.4	LASSO/LR	0.199	Lin/Poly SVM	1.005	LR	0.200	LASSO/LR/Lin/Poly SVM	1.005
	1000	0.2	0.2	LR	0.061	LR	1.000	LR	0.060	LR	1.017
			0.4	LR	0.196	LR	1.005	LR	0.198	LR	1.005
		0.4	0.2	LR	0.061	LR	1.000	LASSO/LR	0.060	LASSO/LR	1.017
			0.4	LR	0.196	LR	1.005	LR	0.198	LR	1.005
		0.6	0.2	LASSO/LR	0.061	LASSO/LR	1.000	LASSO/LR	0.060	LASSO/LR	1.017
			0.4	LR	0.196	LASSO/LR	1.005	LR	0.197	LR	1.000
		0.8	0.2	LASSO/LR/Lin/Poly SVM	0.061	LASSO/LR/Lin/Poly SVM	1.000	LASSO/LR/Lin/Poly SVM	0.059	LASSO/LR/Lin/Poly SVM	1.000
			0.4	LASSO/LR	0.196	LASSO	1.000	LASSO/LR	0.198	LASSO	1.000
	2000	0.2	0.2	LASSO/LR	0.061	LASSO/LR	1.017	LASSO/LR	0.059	LASSO/LR	1.000
			0.4	LR	0.195	LASSO/LR	1.005	LR	0.196	LR	1.000
		0.4	0.2	LASSO/LR	0.061	LASSO/LR	1.017	LASSO/LR	0.059	LASSO/LR	1.000
			0.4	LR	0.195	LASSO/LR	1.005	LR	0.196	LR	1.000
		0.6	0.2	LASSO/LR	0.061	Lin/Poly SVM	1.016	LASSO/LR	0.059	LASSO/LR	1.000
			0.4	LASSO/LR	0.195	LASSO	1.000	LR	0.196	LR/Lin/Poly SVM	1.000
		0.8	0.2	LASSO	0.060	LASSO/Lin/Poly SVM	1.000	LASSO/LR/Lin/Poly SVM	0.059	LASSO/LR/Lin/Poly SVM	1.000
			0.4	LASSO/LR/Lin/Poly SVM	0.195	LASSO/Lin/Poly SVM	1.000	LR	0.196	LR/Lin/Poly SVM	1.000
50	500	0.2	0.2	LR	0.075	LR	1.071	LASSO	0.080	LR	1.066
			0.4	LASSO	0.208	LR	1.024	LASSO	0.211	LR	1.023
		0.4	0.2	LASSO	0.073	LR	1.071	LASSO	0.078	LR	1.066
			0.4	LASSO	0.205	LR	1.024	LASSO	0.209	LASSO	1.015
		0.6	0.2	LASSO	0.069	LR	1.057	LASSO	0.075	LASSO	1.042
			0.4	LASSO	0.201	LASSO	1.015	LASSO	0.208	LASSO	1.010
		0.8	0.2	LASSO	0.065	LASSO	1.000	LASSO	0.072	LASSO	1.000
			0.4	LASSO	0.200	LASSO	1.010	LASSO	0.209	LASSO	1.015
	1000	0.2	0.2	LASSO/LR	0.069	LR	1.045	LASSO	0.074	LR	1.027
			0.4	LASSO	0.199	LR	1.015	LASSO	0.205	LASSO	1.005
		0.4	0.2	LASSO	0.067	LR	1.045	LASSO	0.073	LR	1.027
			0.4	LASSO	0.198	LASSO/LR	1.015	LASSO	0.205	LASSO	1.005
		0.6	0.2	LASSO	0.066	LR	1.030	LASSO	0.072	LR	1.027
			0.4	LASSO	0.196	LASSO	1.005	LASSO	0.204	LASSO	1.000
		0.8	0.2	LASSO	0.064	LASSO	1.000	LASSO	0.071	LASSO/LR/Lin SVM	1.000
			0.4	LASSO	0.196	LASSO	1.005	LASSO	0.206	LASSO	1.010
	2000	0.2	0.2	LASSO/LR	0.066	LR	1.015	LASSO/LR	0.072	LR	1.014
			0.4	LASSO	0.195	LR	1.005	LASSO	0.203	LASSO	1.000
		0.4	0.2	LASSO	0.065	LR	1.015	LASSO	0.071	LASSO/LR	1.014
			0.4	LASSO	0.195	LR	1.005	LASSO	0.203	LASSO	1.000
		0.6	0.2	LASSO	0.064	LASSO	1.000	LASSO	0.071	LASSO/LR	1.014
			0.4	LASSO	0.194	LASSO/LR	1.005	LASSO	0.203	LASSO	1.000
		0.8	0.2	LASSO	0.064	LASSO/LR	1.000	LASSO	0.070	LASSO/LR/Lin/Poly SVM	1.000
			0.4	LASSO	0.194	LASSO/LR/Poly SVM	1.005	LASSO	0.204	LASSO/Linear_SVM	1.005

Table 11: Best performing ML model in terms of AMSER and AMSE under Model III for GADP.

$p$	$n$	$\theta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
100	500	0.2	0.2	LASSO	0.077	LR	1.127	LASSO	0.081	LR	1.135	LASSO	0.075	LR	1.122
			0.4	LASSO	0.234	LR	1.036	LASSO	0.247	LR	1.039	LASSO	0.234	LR	1.039
		0.4	0.2	LASSO	0.079	LR	1.120	LASSO	0.076	LR	1.135	LASSO	0.071	LR	1.122
			0.4	LASSO	0.242	LR	1.039	LASSO	0.241	LR	1.042	LASSO	0.228	LASSO	1.041
		0.6	0.2	LASSO	0.072	LR	1.120	LASSO	0.070	LR	1.122	LASSO	0.066	LASSO	1.082
			0.4	LASSO	0.233	LR	1.039	LASSO	0.233	LASSO	1.040	LASSO	0.223	LASSO	1.018
		0.8	0.2	LASSO	0.065	LASSO	1.048	LASSO	0.064	LASSO	1.032	LASSO	0.062	LASSO	1.016
			0.4	LASSO	0.223	LASSO	1.009	LASSO	0.226	LASSO	1.009	LASSO	0.221	LASSO	1.009
	1000	0.2	0.2	LASSO	0.065	LR	1.081	LASSO	0.069	LR	1.076	LASSO	0.065	LR	1.077
			0.4	LASSO	0.219	LR	1.027	LASSO	0.231	LR	1.021	LASSO	0.220	LR	1.022
		0.4	0.2	LASSO	0.068	LR	1.076	LASSO	0.067	LR	1.076	LASSO	0.064	LASSO	1.067
			0.4	LASSO	0.227	LR	1.026	LASSO	0.228	LR	1.021	LASSO	0.219	LASSO	1.019
		0.6	0.2	LASSO	0.065	LASSO	1.066	LASSO	0.064	LASSO	1.049	LASSO	0.062	LASSO	1.033
			0.4	LASSO	0.222	LR	1.026	LASSO	0.224	LASSO	1.018	LASSO	0.217	LASSO	1.009
		0.8	0.2	LASSO	0.062	LASSO	1.016	LASSO	0.061	LASSO	1.000	LASSO	0.060	LASSO	1.000
			0.4	LASSO	0.217	LASSO	1.005	LASSO	0.221	LASSO	1.005	LASSO	0.217	LASSO	1.009
	2000	0.2	0.2	LASSO	0.060	LR	1.034	LASSO	0.064	LR	1.032	LASSO	0.062	LASSO	1.033
			0.4	LASSO	0.211	LR	1.014	LASSO	0.224	LR	1.013	LASSO	0.215	LASSO/LR	1.009
		0.4	0.2	LASSO	0.064	LR	1.032	LASSO	0.063	LR	1.032	LASSO	0.061	LASSO	1.017
			0.4	LASSO	0.219	LR	1.014	LASSO	0.222	LR	1.013	LASSO	0.215	LASSO/LR	1.009
		0.6	0.2	LASSO	0.062	LR	1.032	LASSO	0.062	LR	1.032	LASSO	0.060	LASSO	1.000
			0.4	LASSO	0.217	LR	1.014	LASSO	0.220	LASSO	1.009	LASSO	0.214	LASSO	1.005
		0.8	0.2	LASSO	0.061	LR	1.016	LASSO	0.061	LR	1.000	LASSO	0.060	LASSO/Linear_SVM	1.000
			0.4	LASSO	0.214	LASSO	1.005	LASSO	0.218	LASSO	1.000	LASSO	0.214	LASSO	1.005

Table 12: Best performing ML model in terms of AMSER and AMSE under Model III for CGADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
10	500	0.2	0.2	LR	0.197	LR	3.339	LR	0.182	LR	2.984	LR	0.150	LR	2.500
			0.4	LR	0.320	LR	1.584	LR	0.305	LR	1.502	LR	0.277	LR	1.358
		0.4	0.2	LR	0.182	LR	3.085	LR	0.167	LR	2.738	LR	0.136	LR	2.267
			0.4	LR	0.304	LR	1.505	LR	0.289	LR	1.424	LR	0.263	LR	1.289
		0.6	0.2	LR	0.149	LR	2.525	LR	0.133	LR	2.180	LR	0.109	LR	1.817
			0.4	LR	0.271	LR	1.342	LR	0.256	LR	1.261	LR	0.237	LR	1.162
		0.8	0.2	LR	0.085	LR	1.441	LR	0.077	LR	1.262	LR	0.067	LR	1.117
			0.4	LR	0.216	LR	1.069	LR	0.210	LR	1.034	LR	0.205	LR	1.005
	1000	0.2	0.2	LR	0.195	LR	3.305	LR	0.186	LR	3.049	LR	0.156	LR	2.644
			0.4	LR	0.317	LR	1.585	LR	0.309	LR	1.537	LR	0.284	LR	1.406
		0.4	0.2	LR	0.179	LR	3.034	LR	0.170	LR	2.787	LR	0.142	LR	2.407
			0.4	LR	0.301	LR	1.505	LR	0.292	LR	1.453	LR	0.269	LR	1.332
		0.6	0.2	LR	0.145	LR	2.458	LR	0.135	LR	2.213	LR	0.112	LR	1.898
			0.4	LR	0.267	LR	1.335	LR	0.257	LR	1.279	LR	0.240	LR	1.188
		0.8	0.2	LR	0.081	LR	1.373	LR	0.076	LR	1.246	LR	0.067	LR	1.136
			0.4	LR	0.212	LR	1.060	LR	0.208	LR	1.035	LR	0.205	LR	1.015
	2000	0.2	0.2	LR	0.196	LR	3.379	LR	0.190	LR	3.167	LR	0.163	LR	2.763
			0.4	LR	0.319	LR	1.603	LR	0.314	LR	1.570	LR	0.293	LR	1.458
		0.4	0.2	LR	0.180	LR	3.103	LR	0.173	LR	2.883	LR	0.148	LR	2.508
			0.4	LR	0.302	LR	1.518	LR	0.296	LR	1.480	LR	0.277	LR	1.378
		0.6	0.2	LR	0.145	LR	2.500	LR	0.138	LR	2.300	LR	0.117	LR	1.983
			0.4	LR	0.266	LR	1.337	LR	0.260	LR	1.300	LR	0.245	LR	1.219
		0.8	0.2	LR	0.080	LR	1.379	LR	0.076	LR	1.267	LR	0.067	LR	1.136
			0.4	LR	0.211	LR	1.060	LR	0.208	LR	1.040	LR	0.206	LASSO/LR	1.025
50	500	0.2	0.2	LR	0.216	Radial_SVM	2.617	LR	0.188	Radial_SVM	2.426	LR	0.166	LR	2.274
			0.4	LR	0.354	Radial_SVM	1.621	LR	0.288	LR	1.309	LR	0.284	LR	1.279
		0.4	0.2	LR	0.200	Radial_SVM	2.443	LR	0.173	Radial_SVM	2.279	LR	0.152	LR	2.082
			0.4	LR	0.335	Radial_SVM	1.539	LR	0.273	LR	1.241	LR	0.270	LR	1.216
		0.6	0.2	LR	0.163	Radial_SVM	2.122	LR	0.141	Radial_SVM	1.869	LR	0.122	LR	1.671
			0.4	LR	0.295	LR	1.372	LR	0.246	LR	1.118	LR	0.243	LR	1.095
		0.8	0.2	LR	0.092	LR	1.373	LR	0.086	LR	1.147	LR	0.076	LR	1.041
			0.4	LR	0.229	LR	1.065	LASSO	0.208	LR	0.964	LASSO	0.212	LR	0.964
	1000	0.2	0.2	LR	0.199	LR	3.109	LR	0.184	LR	2.592	LR	0.167	LR	2.420
			0.4	LR	0.332	LR	1.627	LR	0.280	LR	1.333	LR	0.281	LR	1.325
		0.4	0.2	LR	0.183	LR	2.859	LR	0.168	LR	2.366	LR	0.152	LR	2.203
			0.4	LR	0.314	LR	1.539	LR	0.266	LR	1.267	LR	0.266	LR	1.255
		0.6	0.2	LR	0.147	LR	2.297	LR	0.136	LR	1.915	LR	0.121	LR	1.754
			0.4	LR	0.275	LR	1.348	LR	0.238	LR	1.133	LR	0.238	LR	1.123
		0.8	0.2	LR	0.081	LR	1.266	LR	0.081	LR	1.141	LR	0.073	LR	1.058
			0.4	LR	0.213	LR	1.044	LR	0.204	LR	0.971	LR	0.206	LR	0.972
	2000	0.2	0.2	LR	0.193	LR	3.113	LR	0.186	LR	2.696	LR	0.173	LR	2.544
			0.4	LR	0.325	LR	1.633	LR	0.282	LR	1.376	LR	0.288	LR	1.391
		0.4	0.2	LR	0.176	LR	2.839	LR	0.170	LR	2.464	LR	0.158	LR	2.324
			0.4	LR	0.307	LR	1.543	LR	0.267	LR	1.302	LR	0.272	LR	1.314
		0.6	0.2	LR	0.141	LR	2.274	LR	0.137	LR	1.986	LR	0.125	LR	1.838
			0.4	LR	0.268	LR	1.347	LR	0.238	LR	1.161	LR	0.242	LR	1.169
		0.8	0.2	LR	0.077	LR	1.242	LR	0.081	LR	1.174	LR	0.074	LR	1.088
			0.4	LR	0.208	LR	1.045	LR	0.202	LR	0.985	LR	0.205	LR	0.990

Table 13: Best performing ML model in terms of AMSER and AMSE under Model III for CGADP.

$p$	$n$	$\vartheta$	$\sigma$	$\rho=0.2$				$\rho=0.4$				$\rho=0.6$			
				Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER	Best AMSE	AMSE	Best AMSER	AMSER
100	500	0.2	0.2	LR	0.242	Radial_SVM	2.394	LR	0.216	Radial_SVM	2.285	LR	0.195	LR	2.635
			0.4	LR	0.402	LR	1.576	LR	0.368	LR	1.421	LR	0.337	LR	1.322
		0.4	0.2	LR	0.227	Radial_SVM	2.328	LR	0.202	Radial_SVM	2.187	LR	0.181	LR	2.446
			0.4	LR	0.385	LR	1.510	LR	0.353	LR	1.363	LR	0.322	LR	1.263
		0.6	0.2	LR	0.193	Radial_SVM	2.058	LR	0.169	Radial_SVM	1.959	LR	0.149	LR	2.014
			0.4	LR	0.349	LR	1.369	LR	0.319	LR	1.232	LR	0.292	LR	1.145
		0.8	0.2	LR	0.118	Radial_SVM	1.526	LASSO	0.100	LR	1.365	LR	0.088	LR	1.189
			0.4	LASSO	0.273	LR	1.082	LASSO	0.252	LR	1.008	LASSO	0.238	LR	0.953
	1000	0.2	0.2	LR	0.209	LR	3.167	LR	0.191	LR	2.894	LR	0.177	LR	2.723
			0.4	LR	0.357	LR	1.559	LR	0.334	LR	1.421	LR	0.311	LR	1.346
		0.4	0.2	LR	0.193	LR	2.924	LR	0.176	LR	2.667	LR	0.162	LR	2.492
			0.4	LR	0.340	LR	1.485	LR	0.318	LR	1.353	LR	0.295	LR	1.277
		0.6	0.2	LR	0.160	LR	2.424	LR	0.144	LR	2.182	LR	0.130	LR	2.000
			0.4	LR	0.304	LR	1.328	LR	0.285	LR	1.213	LR	0.264	LR	1.143
		0.8	0.2	LR	0.092	LR	1.394	LR	0.083	LR	1.258	LR	0.073	LR	1.123
			0.4	LR	0.240	LR	1.048	LR	0.233	LR	0.991	LR	0.221	LR	0.957
	2000	0.2	0.2	LR	0.201	LR	3.190	LR	0.187	LR	2.968	LR	0.175	LR	2.823
			0.4	LR	0.348	LR	1.589	LR	0.331	LR	1.478	LR	0.310	LR	1.403
		0.4	0.2	LR	0.185	LR	2.937	LR	0.172	LR	2.730	LR	0.159	LR	2.565
			0.4	LR	0.330	LR	1.507	LR	0.314	LR	1.402	LR	0.293	LR	1.326
		0.6	0.2	LR	0.150	LR	2.381	LR	0.138	LR	2.190	LR	0.126	LR	2.032
			0.4	LR	0.293	LR	1.338	LR	0.279	LR	1.246	LR	0.260	LR	1.176
		0.8	0.2	LR	0.084	LR	1.333	LR	0.078	LR	1.238	LR	0.070	LR	1.129
			0.4	LR	0.229	LR	1.046	LR	0.227	LR	1.013	LR	0.216	LR	0.977

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