

Matched Mass Imputation for Survey Data Integration

Supplementary Material

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This supplementary material contains additional simulation results. Specifically, we present the root mean square error ratio (RMSE) and absolute bias ratio (ABR) results in tabular format to complement the visualizations in Figures 1 and 2. While the figures provide an overview of the trends and patterns, these tables offer a more detailed numerical perspective. Together, the visual and tabular results enable a finer examination of key factors influencing estimator performance, including the relationship between the response and covariates, the level of bias in the nonprobability sample, and the relative size of the nonprobability to probability sample.

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Table 1: Performance of various estimators relative to $\hat{\mu}_\pi$ when $n_B = n_A$; to enhance readability, lowest values are bolded per row.

(a) RMSER.

Model	r	$\hat{\mu}_B$	$\hat{\mu}_{MIgam}$	$\hat{\mu}_{DRgam}$	$\hat{\mu}_{SMexact}$	$\hat{\mu}_{SMcem}$	$\hat{\mu}_{SMunn}$	$\hat{\mu}_{SMrnn}$	$\hat{\mu}_{MMIexact}$	$\hat{\mu}_{MMIcem}$	$\hat{\mu}_{MMIunn}$	$\hat{\mu}_{MMIrn}$
ξ_1	0.5	0.61	1.01	1.02	10.68	0.87	1.00	1.01	8.06E+07	1.03	1.30	1.28
	0.3	10.21	2.27	2.21	14.45	8.83	2.28	2.23	5.16E+07	2.27	1.42	1.42
	0.15	17.84	4.43	4.41	19.93	15.95	4.44	4.36	1.24E+08	4.40	1.87	1.91
	0.05	22.93	7.31	7.37	23.95	21.16	7.33	7.20	1.96E+08	7.35	3.15	3.18
	0.01	24.97	9.55	9.64	25.74	23.37	9.68	9.52	5.58E+07	9.81	6.21	6.33
ξ_2	0.5	0.57	1.04	1.07	10.23	0.85	1.04	1.04	2.90E+09	1.07	1.22	1.21
	0.3	10.11	1.66	1.76	13.80	8.54	1.66	1.66	2.70E+16	1.67	1.28	1.28
	0.15	17.68	2.79	2.88	19.59	15.45	2.79	2.79	5.00E+09	2.82	1.38	1.38
	0.05	22.72	4.40	4.55	23.81	20.57	4.42	4.41	6.77E+08	4.53	1.86	1.89
	0.01	24.74	5.67	5.90	25.70	22.76	5.85	5.84	5.94E+10	6.01	3.44	3.49
ξ_3	0.5	0.73	0.98	0.98	10.63	3.74	0.97	1.08	2.26E+09	85.14	1.60	1.38
	0.3	9.17	3.07	3.07	14.47	7.68	3.07	2.75	1.94E+12	111.56	2.93	2.53
	0.15	15.99	5.53	5.53	19.83	13.01	5.53	5.25	1.11E+09	307.42	4.38	3.92
	0.05	20.52	7.78	7.78	23.14	16.99	7.78	7.57	2.78E+10	220.05	6.08	5.72
	0.01	22.36	8.99	8.99	24.94	18.99	9.01	8.87	2.78E+10	1569.33	7.68	7.46
ξ_4	0.5	0.70	0.99	0.99	10.52	3.76	0.99	0.98	2.58E+09	35.48	2.30	2.29
	0.3	9.18	1.53	1.52	14.24	7.31	1.57	1.59	4.54E+07	467.79	1.96	1.96
	0.15	16.03	2.15	2.13	19.00	12.20	2.25	2.27	2.14E+08	1067.89	1.66	1.67
	0.05	20.58	3.20	3.19	22.66	16.50	3.44	3.44	2.90E+08	159.74	1.67	1.68
	0.01	22.41	4.36	4.35	24.24	18.62	5.02	5.01	2.29E+08	183.51	2.66	2.68
ξ_5	0.5	0.65	0.98	0.99	10.83	3.67	0.98	0.98	5.55E+09	41.56	1.37	1.37
	0.3	9.56	4.43	4.42	14.24	8.57	4.43	4.43	4.23E+08	1227.35	3.20	3.20
	0.15	16.74	8.38	8.37	19.51	14.24	8.38	8.38	1.85E+08	886.90	6.03	6.03
	0.05	21.53	12.04	12.03	23.26	18.66	12.04	12.04	9.45E+08	121.07	9.35	9.35
	0.01	23.45	13.97	13.97	25.24	20.65	14.00	14.00	4.45E+08	195.29	12.06	12.06

(b) ABR.

Model	r	$\hat{\mu}_B$	$\hat{\mu}_{MIgam}$	$\hat{\mu}_{DRgam}$	$\hat{\mu}_{SMexact}$	$\hat{\mu}_{SMcem}$	$\hat{\mu}_{SMunn}$	$\hat{\mu}_{SMrnn}$	$\hat{\mu}_{MMIexact}$	$\hat{\mu}_{MMIcem}$	$\hat{\mu}_{MMIunn}$	$\hat{\mu}_{MMIrn}$
ξ_1	0.5	0.65	2.78	2.51	10.73	0.06	2.63	0.34	3.06E+08	1.66	5.26	7.10
	0.3	1423.82	285.27	274.39	1424.40	1228.00	285.87	278.75	3.42E+08	282.77	80.82	84.71
	0.15	2489.87	603.44	599.61	2471.69	2224.52	604.74	592.19	3.53E+08	598.01	179.09	185.87
	0.05	3201.62	1012.09	1019.64	3159.91	2952.73	1014.83	995.76	7.98E+08	1017.12	381.99	390.40
	0.01	3486.45	1327.13	1340.46	3462.19	3261.63	1345.33	1323.98	2.34E+08	1363.85	833.76	851.59
ξ_2	0.5	5.99	8.26	95.70	44.07	15.16	8.62	8.30	3.18E+10	7.49	14.96	15.12
	0.3	3454.35	434.10	476.31	3365.03	2909.13	435.22	434.48	2.92E+17	436.87	75.36	75.31
	0.15	6045.57	884.40	917.02	5992.95	5280.40	886.40	885.24	5.39E+10	891.57	178.25	178.65
	0.05	7774.48	1463.33	1516.03	7744.67	7033.31	1469.70	1468.05	1.12E+09	1507.46	408.00	416.32
	0.01	8464.66	1910.31	1987.74	8487.51	7785.82	1970.69	1968.63	6.45E+11	2025.72	1037.79	1062.37
ξ_3	0.5	3.14	2.68	2.68	15.94	7.14	2.46	37.18	3.23E+09	414.88	93.22	46.07
	0.3	854.32	272.33	272.31	857.37	623.69	272.17	239.35	5.72E+12	573.66	248.90	205.02
	0.15	1492.89	509.87	509.85	1521.32	1164.92	509.78	482.51	5.52E+09	1740.98	395.10	349.64
	0.05	1917.36	722.94	722.97	1933.40	1552.59	722.95	702.54	8.19E+10	769.54	558.82	522.78
	0.01	2089.34	836.50	836.51	2131.07	1744.69	838.51	824.36	8.44E+10	5870.55	711.20	690.41
ξ_4	0.5	0.59	0.49	0.70	23.30	11.69	2.88	3.94	1.03E+10	146.04	233.45	233.04
	0.3	1098.67	133.27	131.27	1079.92	738.65	140.42	143.95	2.71E+08	1475.87	176.23	175.09
	0.15	1923.18	215.55	213.23	1916.83	1395.58	232.30	235.74	6.99E+08	4495.84	109.87	112.92
	0.05	2471.11	350.77	348.49	2446.71	1932.22	384.14	384.81	2.99E+08	1332.45	17.87	18.86
	0.01	2690.94	494.32	492.11	2680.76	2193.90	578.97	577.51	1.01E+09	1042.62	154.75	156.23
ξ_5	0.5	1.97	0.43	1.11	6.82	5.38	0.45	0.51	8.16E+09	85.94	1.39	1.22
	0.3	454.62	205.80	205.06	448.66	369.05	205.81	205.80	6.76E+08	1965.77	137.09	137.22
	0.15	797.14	396.91	396.30	798.07	656.17	396.92	396.91	3.49E+08	1325.97	279.95	279.90
	0.05	1026.07	572.29	572.01	1021.51	876.10	572.40	572.39	2.22E+09	660.65	441.03	441.36
	0.01	1117.28	664.71	664.57	1130.07	972.93	666.33	666.32	9.06E+08	799.94	572.14	571.82

Table 2: Performance values of various estimators relative to $\hat{\mu}_\pi$ when $n_B = 10n_A$.

(a) RMSER.

Model	r	$\hat{\mu}_B$	$\hat{\mu}_{MIgam}$	$\hat{\mu}_{DRgam}$	$\hat{\mu}_{SMexact}$	$\hat{\mu}_{SMcem}$	$\hat{\mu}_{SMunn}$	$\hat{\mu}_{SMrnn}$	$\hat{\mu}_{MMIexact}$	$\hat{\mu}_{MMIcem}$	$\hat{\mu}_{MMIunn}$	$\hat{\mu}_{MMIrrnn}$
ξ_1	0.5	0.19	0.90	0.90	3.34	0.57	0.90	0.91	2.24	0.91	1.13	1.14
	0.3	10.19	2.22	2.22	10.69	7.75	2.22	2.18	3.03	2.22	1.24	1.23
	0.15	17.84	4.38	4.40	17.95	14.52	4.38	4.29	5.07	4.35	1.58	1.58
	0.05	22.93	7.20	7.21	23.09	19.79	7.20	7.06	7.91	7.28	2.33	2.38
	0.01	24.97	9.44	9.41	25.08	22.08	9.44	9.27	10.04	9.85	3.67	3.71
ξ_2	0.5	0.18	0.93	0.93	3.33	0.59	0.93	0.93	1.72	0.94	1.10	1.10
	0.3	10.09	1.55	1.57	10.45	7.29	1.55	1.55	2.14	1.56	1.08	1.07
	0.15	17.65	2.74	2.76	17.83	13.81	2.74	2.73	3.24	2.78	1.15	1.16
	0.05	22.69	4.42	4.45	22.85	18.95	4.42	4.41	4.84	4.64	1.28	1.27
	0.01	24.71	5.63	5.68	24.77	21.15	5.63	5.63	5.85	6.18	1.65	1.69
ξ_3	0.5	0.21	0.93	0.93	3.18	1.45	0.93	1.08	2.52	1.17	1.62	1.27
	0.3	9.12	3.06	3.06	9.74	5.25	3.06	2.71	3.91	3.11	2.95	2.45
	0.15	15.94	5.50	5.50	16.21	9.95	5.50	5.21	5.97	5.26	4.17	3.70
	0.05	20.49	7.78	7.78	20.79	14.26	7.78	7.57	8.05	7.28	5.50	5.09
	0.01	22.31	8.90	8.90	22.35	16.43	8.90	8.73	9.23	8.50	6.43	6.09
ξ_4	0.5	0.21	0.84	0.84	3.21	1.42	0.84	0.84	2.80	1.13	2.31	2.32
	0.3	9.15	1.45	1.45	9.87	4.14	1.45	1.47	3.18	1.86	2.21	2.17
	0.15	16.01	2.09	2.08	16.45	7.49	2.10	2.14	3.89	3.01	2.11	2.02
	0.05	20.58	3.19	3.18	20.76	11.43	3.22	3.26	5.18	4.73	1.73	1.71
	0.01	22.41	4.43	4.42	22.84	13.63	4.55	4.54	5.88	6.60	1.43	1.45
ξ_5	0.5	0.20	0.75	0.75	3.29	1.36	0.75	0.75	3.61	1.16	1.08	1.09
	0.3	9.59	4.43	4.43	10.16	6.24	4.43	4.43	5.49	4.71	3.10	3.09
	0.15	16.77	8.35	8.35	16.96	11.27	8.35	8.35	9.09	8.92	5.67	5.67
	0.05	21.57	12.00	12.00	21.77	15.54	12.00	12.00	12.45	12.84	8.47	8.48
	0.01	23.48	13.95	13.95	23.65	17.60	13.95	13.95	14.26	14.97	10.41	10.43

(b) ABR.

Model	r	$\hat{\mu}_B$	$\hat{\mu}_{MIgam}$	$\hat{\mu}_{DRgam}$	$\hat{\mu}_{SMexact}$	$\hat{\mu}_{SMcem}$	$\hat{\mu}_{SMunn}$	$\hat{\mu}_{SMrnn}$	$\hat{\mu}_{MMIexact}$	$\hat{\mu}_{MMIcem}$	$\hat{\mu}_{MMIunn}$	$\hat{\mu}_{MMIrrnn}$
ξ_1	0.5	0.57	2.61	2.62	27.78	4.32	2.60	4.79	6.30	2.76	3.95	5.61
	0.3	1423.31	284.08	283.32	1432.76	1079.34	284.12	276.64	290.26	282.96	67.22	66.25
	0.15	2490.66	600.39	602.60	2478.81	2026.29	600.46	587.71	609.48	595.79	150.46	156.27
	0.05	3201.83	1000.20	1000.69	3210.04	2762.85	1000.27	980.37	1024.68	1011.14	285.46	293.72
	0.01	3486.72	1315.21	1310.68	3490.95	3082.49	1315.48	1290.79	1334.97	1372.05	484.56	491.78
ξ_2	0.5	2.63	19.25	0.33	14.41	14.37	19.24	19.58	17.42	18.96	41.32	40.94
	0.3	3451.06	429.58	438.67	3428.61	2487.11	429.61	428.85	411.33	432.18	43.14	35.97
	0.15	6041.54	884.10	890.46	6033.21	4724.70	884.14	882.92	883.65	900.20	114.39	112.01
	0.05	7765.80	1484.61	1494.76	7781.38	6482.10	1484.69	1482.92	1488.96	1562.47	241.39	247.04
	0.01	8455.79	1906.76	1921.95	8448.76	7237.73	1907.66	1905.43	1831.41	2099.56	392.29	409.04
ξ_3	0.5	0.53	3.97	3.99	1.57	15.57	3.97	47.68	7.47	5.55	105.24	46.75
	0.3	852.52	272.77	272.75	855.37	472.75	272.77	237.57	282.73	271.81	255.29	202.24
	0.15	1490.53	507.87	507.73	1484.70	921.14	507.87	479.91	504.65	481.73	377.05	329.75
	0.05	1916.33	723.79	723.86	1918.83	1328.28	723.79	703.33	716.70	674.79	505.74	466.07
	0.01	2086.48	828.62	828.80	2070.07	1531.41	828.65	812.31	834.43	789.52	593.61	560.56
ξ_4	0.5	1.27	0.03	0.10	26.01	3.06	0.01	0.08	5.14	0.97	246.19	247.36
	0.3	1098.87	133.69	133.10	1113.90	468.73	134.00	137.73	149.97	176.87	227.67	223.17
	0.15	1924.23	221.81	220.92	1936.83	885.78	223.19	230.04	222.01	336.77	209.12	197.19
	0.05	2473.46	365.09	363.78	2468.30	1365.89	369.65	373.79	351.48	553.23	149.10	145.65
	0.01	2693.31	520.26	518.75	2722.25	1631.48	534.59	533.97	365.47	781.07	47.92	50.46
ξ_5	0.5	0.49	0.26	0.16	9.77	5.03	0.26	0.24	6.54	1.99	11.24	11.39
	0.3	457.06	208.16	207.97	460.08	290.44	208.17	208.16	208.17	217.86	138.95	139.08
	0.15	799.42	396.64	396.43	795.77	534.28	396.64	396.63	397.83	422.19	265.94	265.80
	0.05	1028.26	571.44	571.25	1029.28	739.11	571.45	571.44	568.65	610.91	401.71	401.99
	0.01	1119.59	664.52	664.32	1120.44	837.67	664.55	664.54	658.70	712.43	494.52	495.53

Table 3: Performance values of various estimators relative to $\hat{\mu}_\pi$ when $n_B = 20n_A$.

(a) RMSER.

Model	r	$\hat{\mu}_B$	$\hat{\mu}_{MIgam}$	$\hat{\mu}_{DRgam}$	$\hat{\mu}_{SMexact}$	$\hat{\mu}_{SMcem}$	$\hat{\mu}_{SMunn}$	$\hat{\mu}_{SMrnn}$	$\hat{\mu}_{MMIexact}$	$\hat{\mu}_{MMIcem}$	$\hat{\mu}_{MMIunn}$	$\hat{\mu}_{MMIrn}$
ξ_1	0.5	0.13	0.88	0.88	2.20	0.53	0.88	0.89	1.41	0.88	1.10	1.10
	0.3	10.20	2.26	2.26	10.43	7.64	2.26	2.21	2.56	2.25	1.21	1.24
	0.15	17.84	4.37	4.39	17.88	14.40	4.37	4.28	4.64	4.34	1.55	1.59
	0.05	22.93	7.16	7.16	23.03	19.66	7.16	7.02	7.54	7.24	2.26	2.31
	0.01	24.97	9.45	9.43	25.08	21.95	9.45	9.28	9.79	9.86	3.43	3.51
ξ_2	0.5	0.12	0.92	0.92	2.20	0.61	0.92	0.92	1.26	0.92	1.12	1.09
	0.3	10.09	1.56	1.57	10.28	7.17	1.56	1.56	1.76	1.57	1.09	1.07
	0.15	17.65	2.73	2.73	17.72	13.65	2.73	2.72	2.90	2.77	1.14	1.13
	0.05	22.69	4.37	4.38	22.78	18.79	4.37	4.36	4.49	4.60	1.26	1.26
	0.01	24.72	5.61	5.62	24.66	20.98	5.61	5.60	5.77	6.18	1.43	1.46
ξ_3	0.5	0.14	0.87	0.87	2.16	1.18	0.87	0.99	1.46	1.02	1.61	1.27
	0.3	9.10	3.02	3.02	9.30	5.05	3.02	2.66	3.21	3.04	2.89	2.39
	0.15	15.94	5.46	5.46	16.13	9.66	5.46	5.17	5.55	5.18	4.12	3.63
	0.05	20.49	7.72	7.73	20.56	14.06	7.72	7.50	7.88	7.24	5.40	4.94
	0.01	22.30	8.99	8.99	22.51	16.27	8.99	8.82	9.06	8.57	6.33	6.00
ξ_4	0.5	0.14	0.86	0.86	2.24	1.15	0.86	0.86	1.53	1.02	2.31	2.28
	0.3	9.14	1.45	1.45	9.37	3.78	1.45	1.48	1.92	1.77	2.18	2.14
	0.15	16.00	2.01	2.00	16.06	6.93	2.01	2.07	2.47	2.92	2.22	2.14
	0.05	20.57	3.13	3.12	20.69	10.89	3.15	3.19	3.66	4.74	1.87	1.87
	0.01	22.41	4.51	4.49	22.54	13.08	4.57	4.57	4.62	6.66	1.40	1.42
ξ_5	0.5	0.13	0.75	0.75	2.30	1.06	0.75	0.75	1.97	0.95	1.06	1.07
	0.3	9.58	4.43	4.42	9.89	6.00	4.43	4.43	4.78	4.64	3.11	3.09
	0.15	16.78	8.34	8.33	16.98	10.92	8.34	8.34	8.60	8.83	5.65	5.66
	0.05	21.57	11.98	11.98	21.68	15.14	11.98	11.98	12.19	12.81	8.37	8.37
	0.01	23.49	13.92	13.92	23.52	17.21	13.92	13.92	13.97	14.94	10.11	10.12

(b) ABR.

Model	r	$\hat{\mu}_B$	$\hat{\mu}_{MIgam}$	$\hat{\mu}_{DRgam}$	$\hat{\mu}_{SMexact}$	$\hat{\mu}_{SMcem}$	$\hat{\mu}_{SMunn}$	$\hat{\mu}_{SMrnn}$	$\hat{\mu}_{MMIexact}$	$\hat{\mu}_{MMIcem}$	$\hat{\mu}_{MMIunn}$	$\hat{\mu}_{MMIrn}$
ξ_1	0.5	0.98	1.79	1.81	11.16	2.23	1.79	4.57	5.20	2.08	2.42	3.84
	0.3	1423.79	291.26	292.22	1426.68	1063.97	291.27	284.17	287.72	289.57	73.07	77.44
	0.15	2491.46	597.58	600.80	2483.36	2010.04	597.61	584.99	606.48	593.52	142.25	149.47
	0.05	3201.79	993.52	994.48	3208.18	2745.60	993.55	973.97	1021.24	1005.83	271.96	278.43
	0.01	3487.03	1316.07	1312.78	3496.66	3065.38	1316.11	1291.37	1347.42	1374.37	450.68	461.67
ξ_2	0.5	0.28	11.21	4.83	47.88	5.37	11.21	11.55	23.87	12.29	33.17	35.54
	0.3	3452.31	436.91	440.17	3448.39	2447.41	436.91	436.16	429.64	441.64	55.09	45.32
	0.15	6041.18	880.54	883.09	6027.57	4669.59	880.55	879.35	873.77	897.82	114.81	110.88
	0.05	7766.29	1465.35	1469.13	7776.88	6430.46	1465.37	1463.57	1455.30	1547.48	219.09	214.62
	0.01	8459.70	1900.53	1905.56	8424.41	7180.06	1900.80	1898.57	1910.27	2102.34	329.31	335.95
ξ_3	0.5	0.67	2.28	2.62	5.03	1.21	2.28	41.51	4.87	6.29	109.40	54.83
	0.3	851.31	270.49	270.53	841.07	460.65	270.49	235.22	270.00	269.22	251.21	199.70
	0.15	1490.73	504.66	504.63	1492.39	897.75	504.66	476.20	501.59	475.69	372.14	324.23
	0.05	1915.92	718.32	718.46	1911.34	1310.54	718.32	697.29	725.94	670.96	495.51	451.32
	0.01	2086.11	837.28	837.53	2095.53	1517.93	837.28	821.04	838.97	796.61	584.88	552.79
ξ_4	0.5	0.21	0.87	0.88	16.32	5.26	0.88	0.64	1.58	2.21	245.81	242.36
	0.3	1098.95	137.06	136.74	1092.75	433.07	137.13	140.92	132.33	177.08	225.89	220.36
	0.15	1922.69	214.99	214.20	1912.03	823.03	215.46	223.00	193.86	332.43	229.07	217.89
	0.05	2471.86	357.11	355.69	2473.57	1302.78	359.12	364.43	326.14	557.77	174.23	171.85
	0.01	2692.74	529.70	527.91	2696.14	1567.06	537.89	538.02	451.03	792.54	74.56	78.71
ξ_5	0.5	0.17	0.43	0.41	4.34	1.64	0.43	0.41	2.61	1.91	12.64	12.31
	0.3	456.88	208.03	207.93	458.84	281.26	208.03	208.02	209.71	216.48	139.77	139.10
	0.15	799.93	396.05	395.90	803.18	518.70	396.05	396.04	399.26	419.31	265.32	266.05
	0.05	1028.58	570.31	570.13	1030.08	720.68	570.31	570.31	573.90	609.58	396.42	396.84
	0.01	1119.75	663.13	662.90	1118.14	819.83	663.13	663.12	661.02	711.37	480.01	480.58