Supplementary Material for Power Priors for Leveraging Historical Data: Looking Back and Looking Forward

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S.1 Additional Tables and Figures for MCMC Convergence Checks for Kociba and NTP Data

Table S.1: MCMC convergence check p-values for Kociba and NIP data

Prior	Geweke	Heidelberger-Welch (Stationary)	Test Outcome*
PP ($a_0 = 0.1$) β_0	0.1050	0.0715	Passed
PP ($a_0 = 0.1$) β_0	0.1864	0.0721	Passed
nPP β_0	0.4544	0.5168	Passed
nPP β_1	0.2040	0.1311	Passed
nPP a_0	0.1879	0.2911	Passed
pPP (Borrowing Intercept) β_0	0.3601	0.5505	Passed
pPP (Borrowing Intercept) β_1	0.2787	0.2027	Passed
pPP (Borrowing Intercept) β_1^*	0.4019	0.6410	Passed
pPP (Borrowing Slope) β_0	0.6766	0.7652	Passed
pPP (Borrowing Slope) β_1	0.5850	0.8937	Passed
pPP (Borrowing Slope) β_0^*	0.6780	0.7644	Passed

*based on SAS output

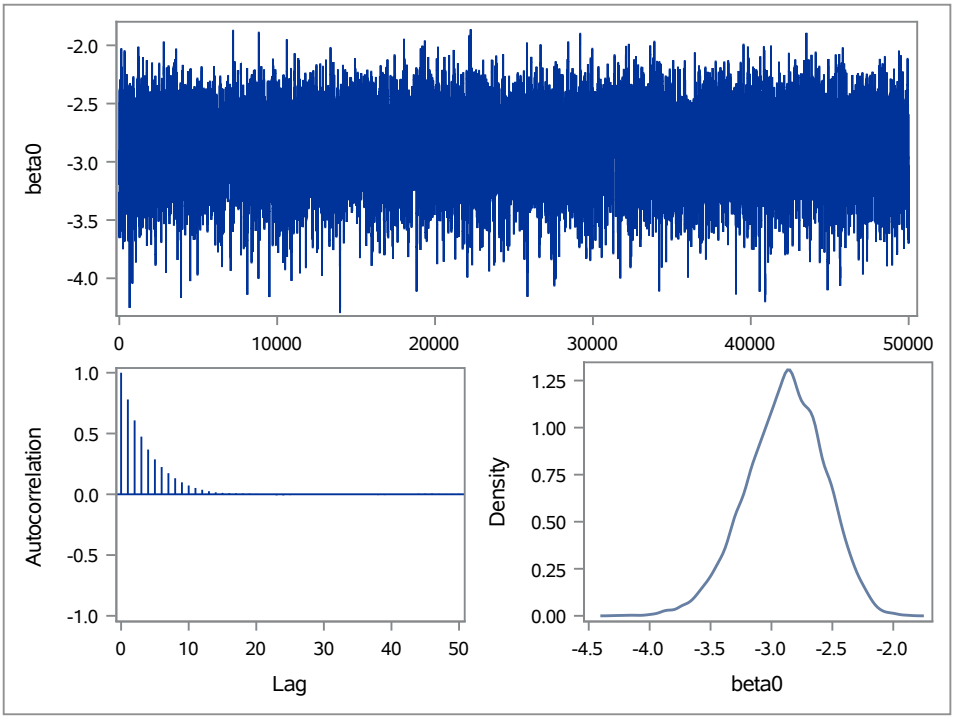

Figure S.1: Trace plot, autocorrelation function plot and marginal posterior density plot of β_0 (Intercept) with power prior ($a_0 = 0.1$).

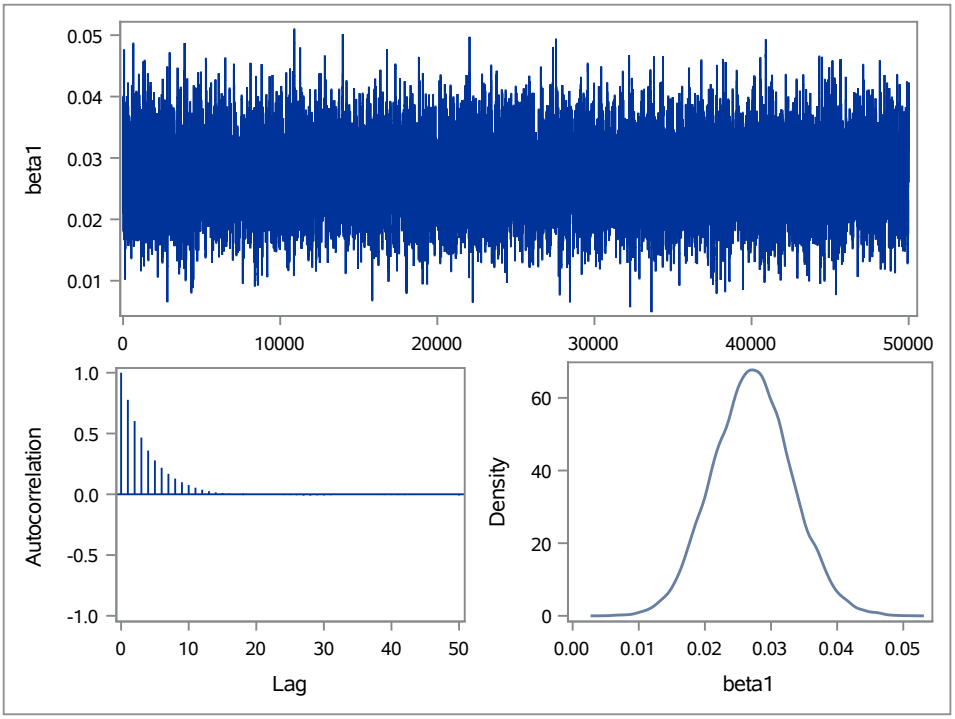

Figure S.2: Trace plot, autocorrelation function plot and marginal posterior density plot of β_1 (Slope) with power prior ($a_0 = 0.1$).

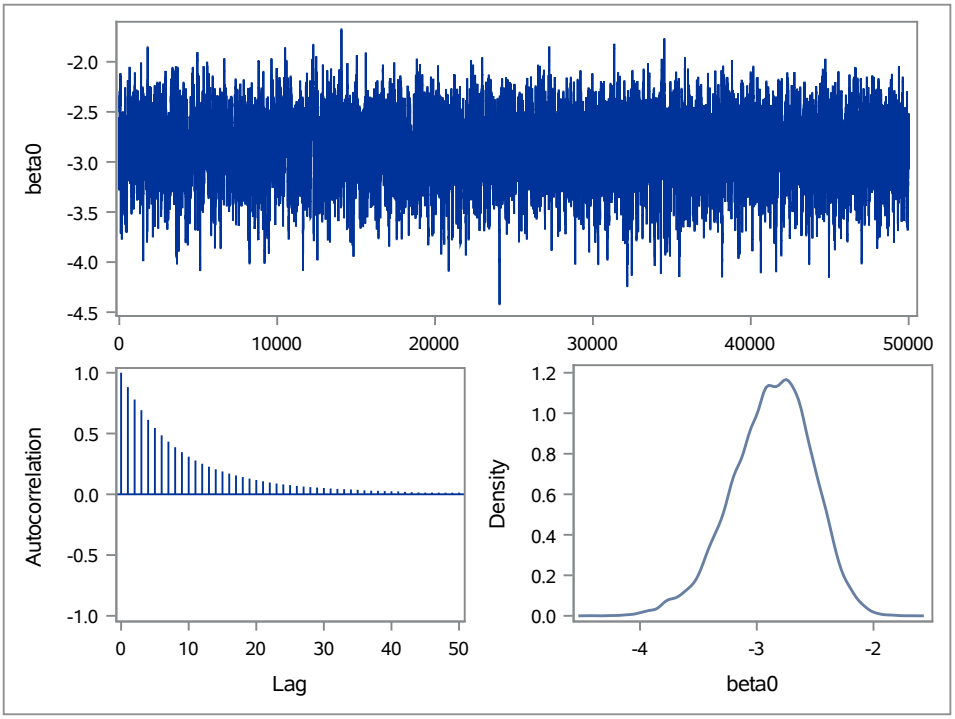

Figure S.3: Trace plot, autocorrelation function plot and marginal posterior density plot of β_0 (Intercept) with normalized power prior.

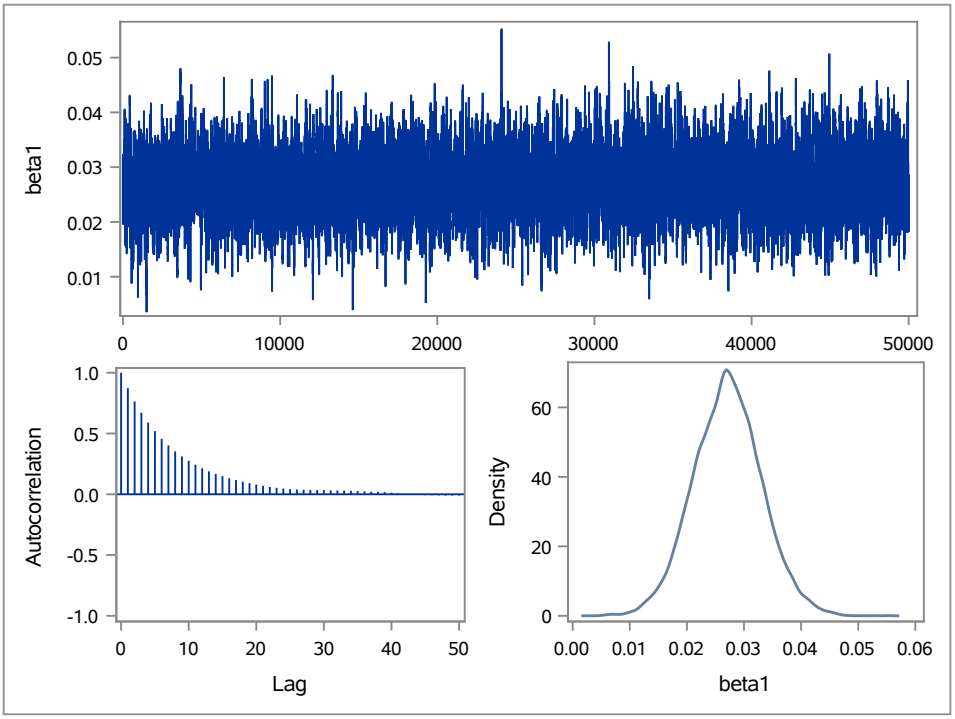

Figure S.4: Trace plot, autocorrelation function plot and marginal posterior density plot of β_1 (Slope) with normalized power prior.

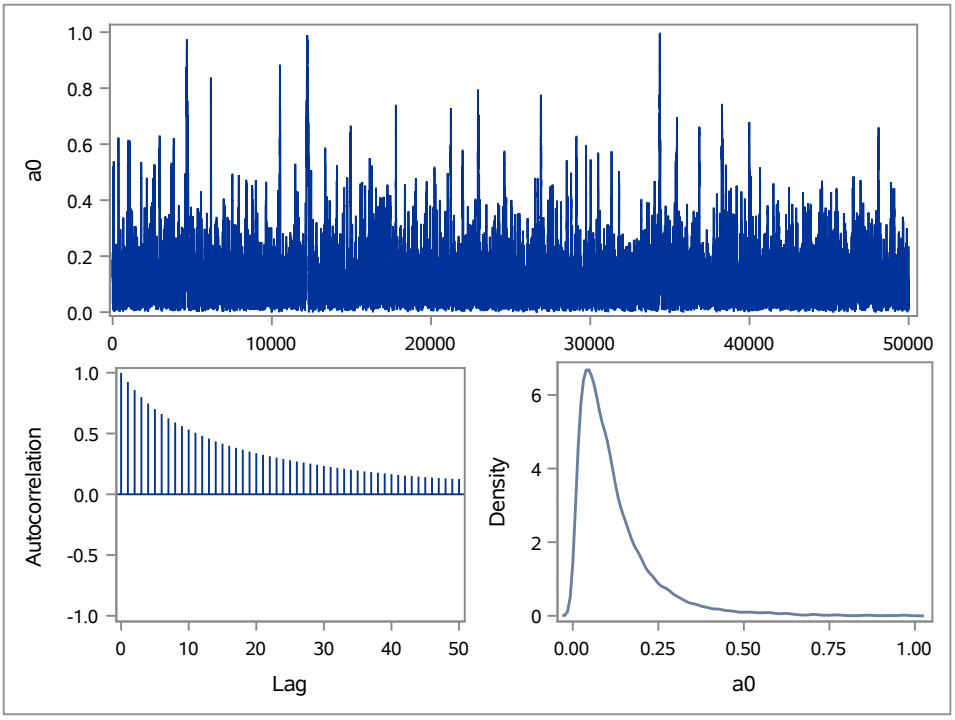

Figure S.5: Trace plot, autocorrelation function plot and marginal posterior density plot of a_0 with normalized power prior.

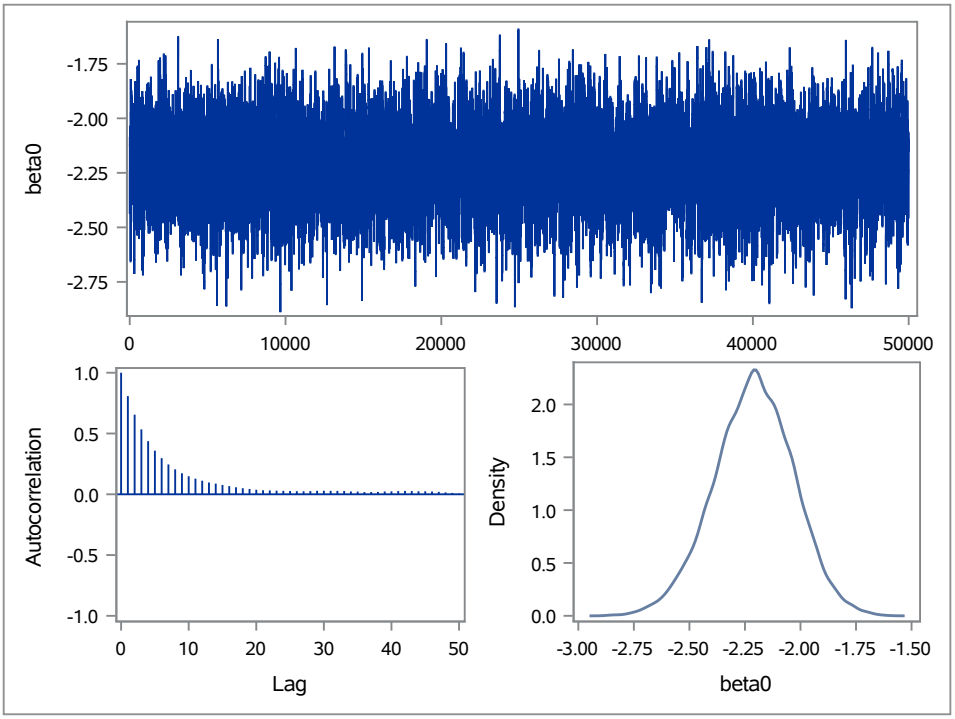

Figure S.6: Trace plot, autocorrelation function plot and marginal posterior density plot of β_0 (Intercept) with partial borrowing power prior (Borrow Intercept).

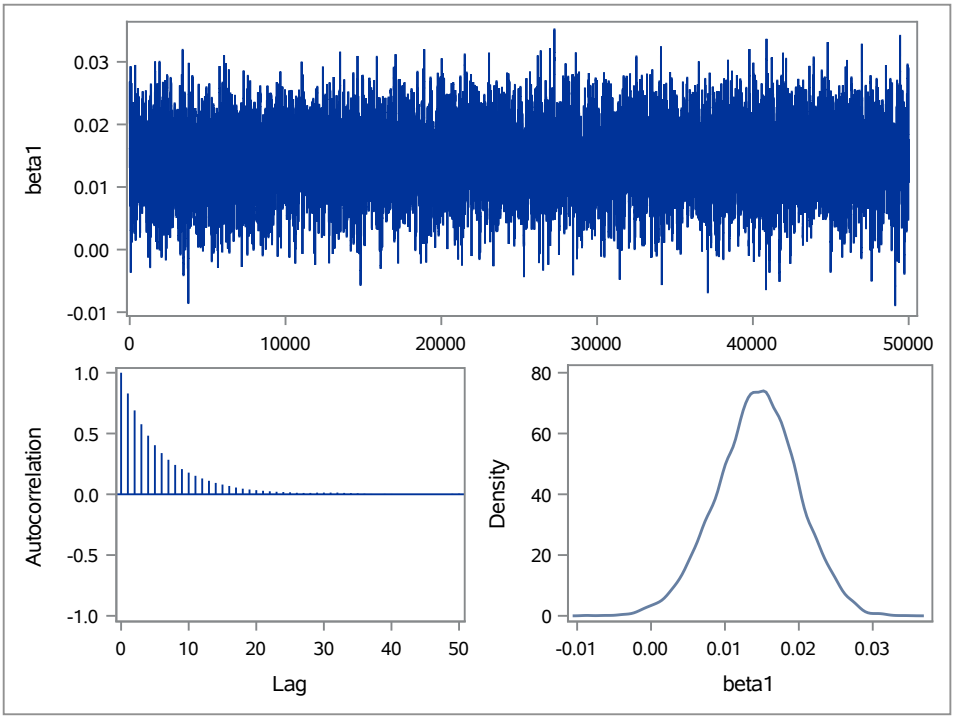

Figure S.7: Trace plot, autocorrelation function plot and marginal posterior density plot of β_1 (Slope) with partial borrowing power prior (Borrow Intercept).

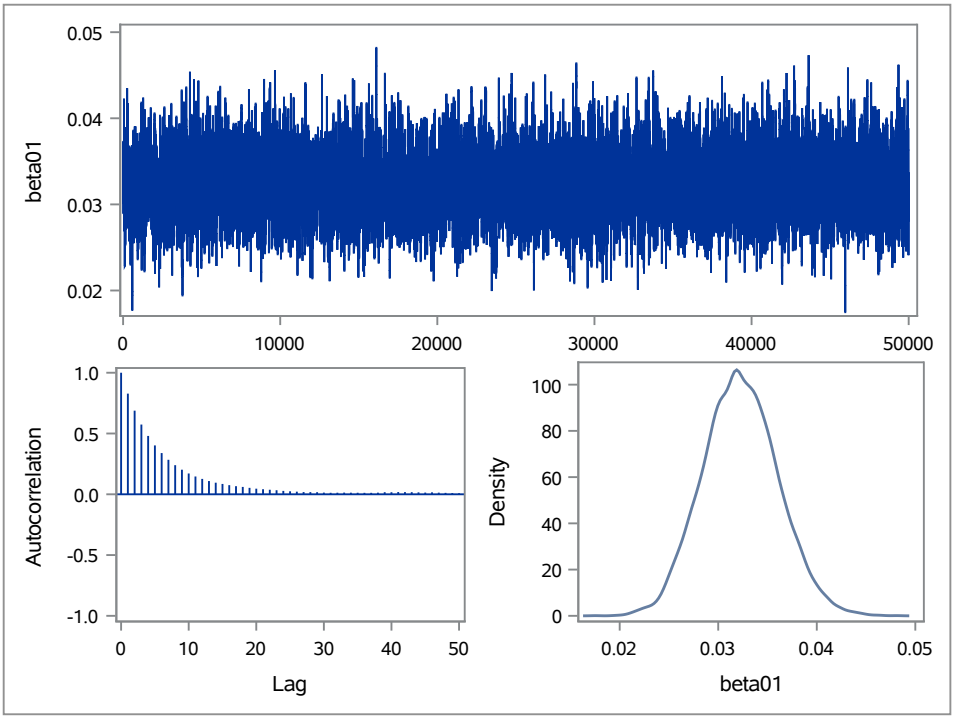

Figure S.8: Trace plot, autocorrelation function plot and marginal posterior density plot of β_0^* (Intercept) with partial borrowing power prior (Borrow Intercept).

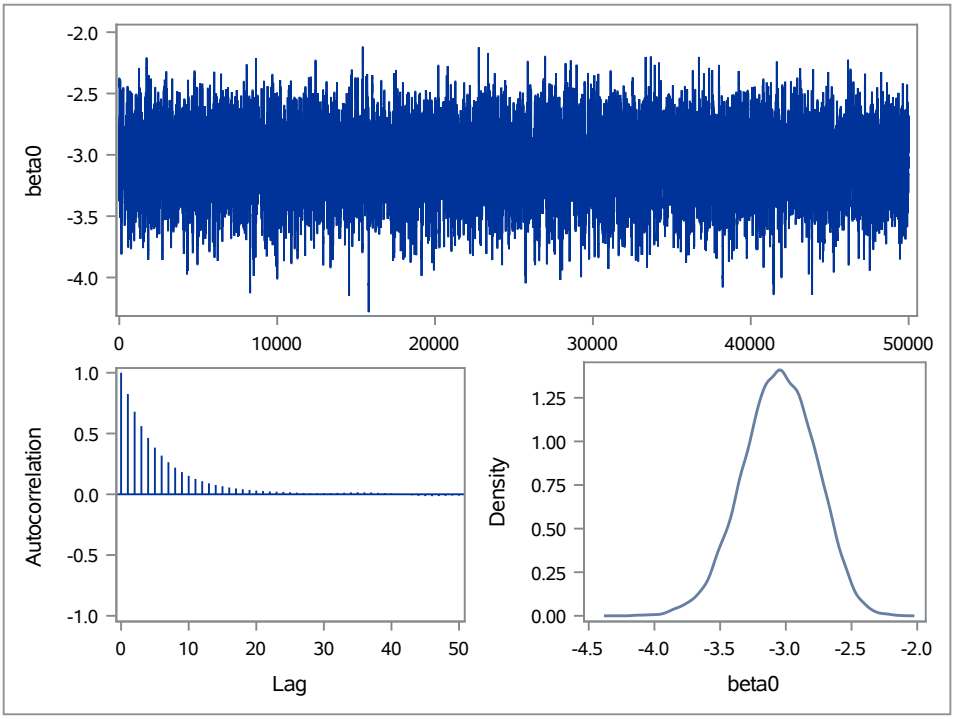

Figure S.9: Trace plot, autocorrelation function plot and marginal posterior density plot of β_0 (Intercept) with partial borrowing power prior (Borrow Slope)

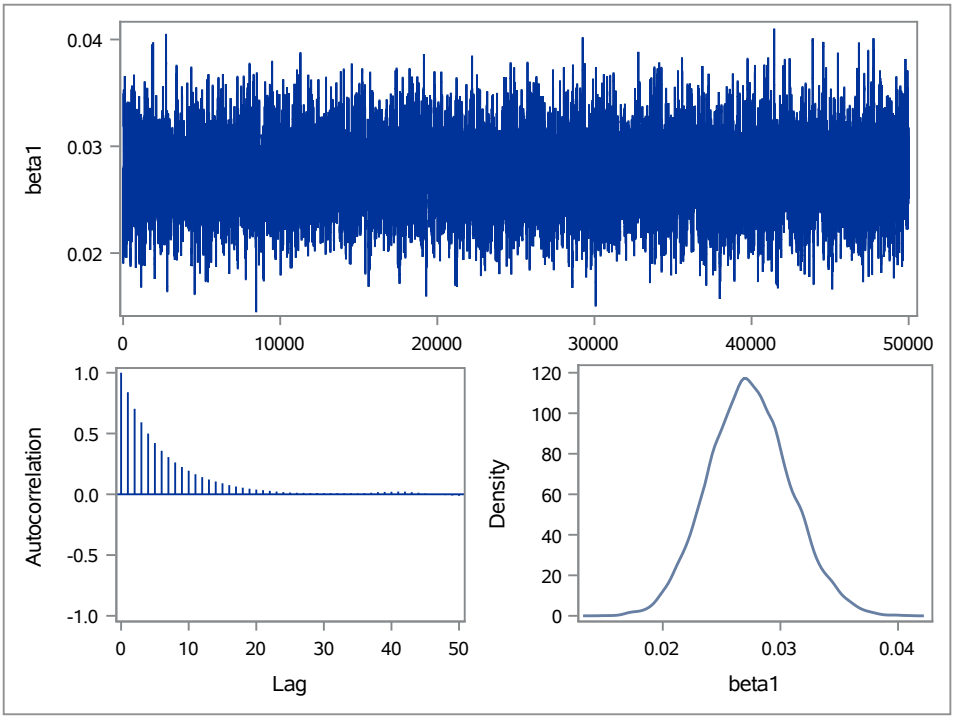

Figure S.10: Trace plot, autocorrelation function plot and marginal posterior density plot of β_1 (Slope) with partial borrowing power prior (Borrow Slope).

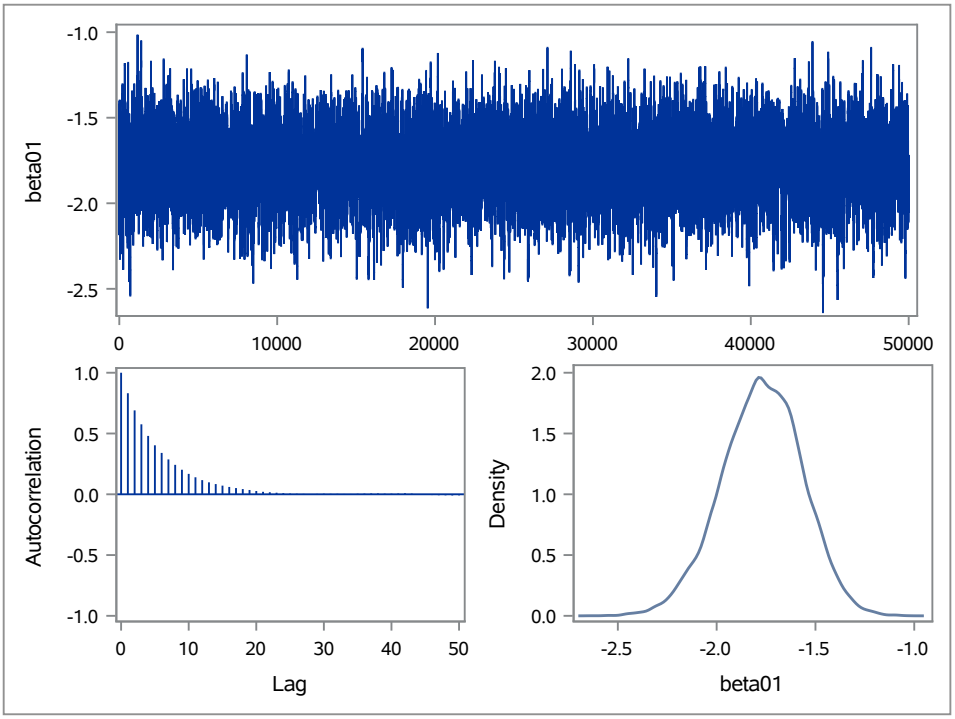

Figure S.11: Trace plot, autocorrelation function plot and marginal posterior density plot of β_1^* (Slope) with partial borrowing power prior (Borrow Slope).

S.2 Additional Tables and Figures for MCMC Convergence Checks for ADNI Data

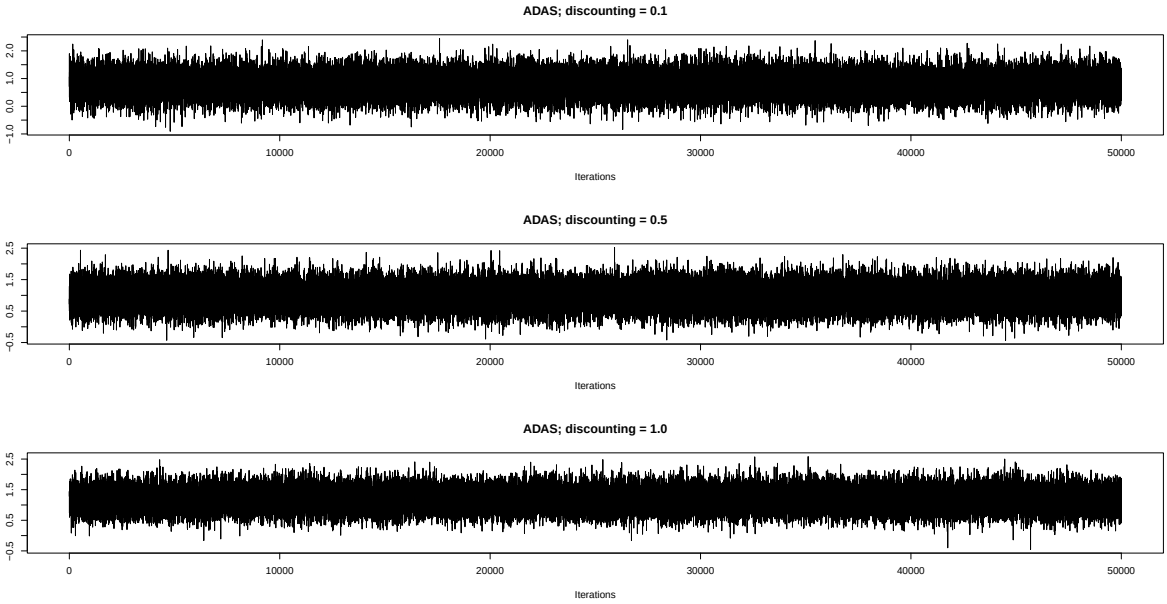

Figure S.12: Trace plots for Gibbs samples of the MCI exposure effect under ADAS response and the partial borrowing power prior with $a_0 \in \{0.1, 0.5, 1\}$. 50k samples after 10k burn-in period.

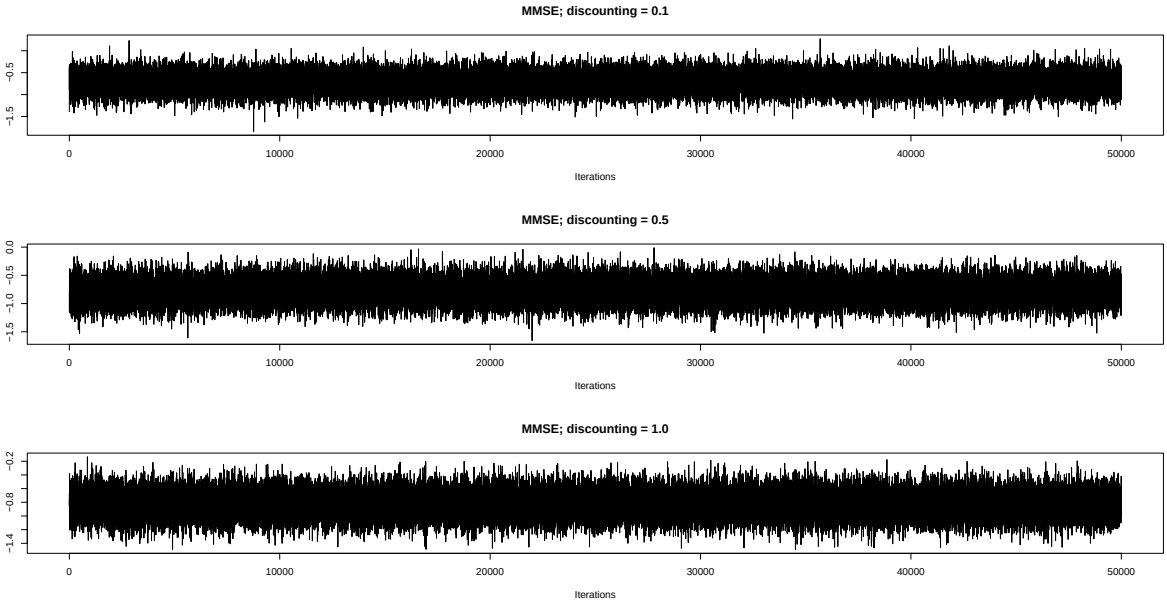

Figure S.13: Trace plots for Gibbs samples of the MCI exposure effect under MMSE response and the partial borrowing power prior with $a_0 \in \{0.1, 0.5, 1\}$. 50k samples after 10k burn-in period.

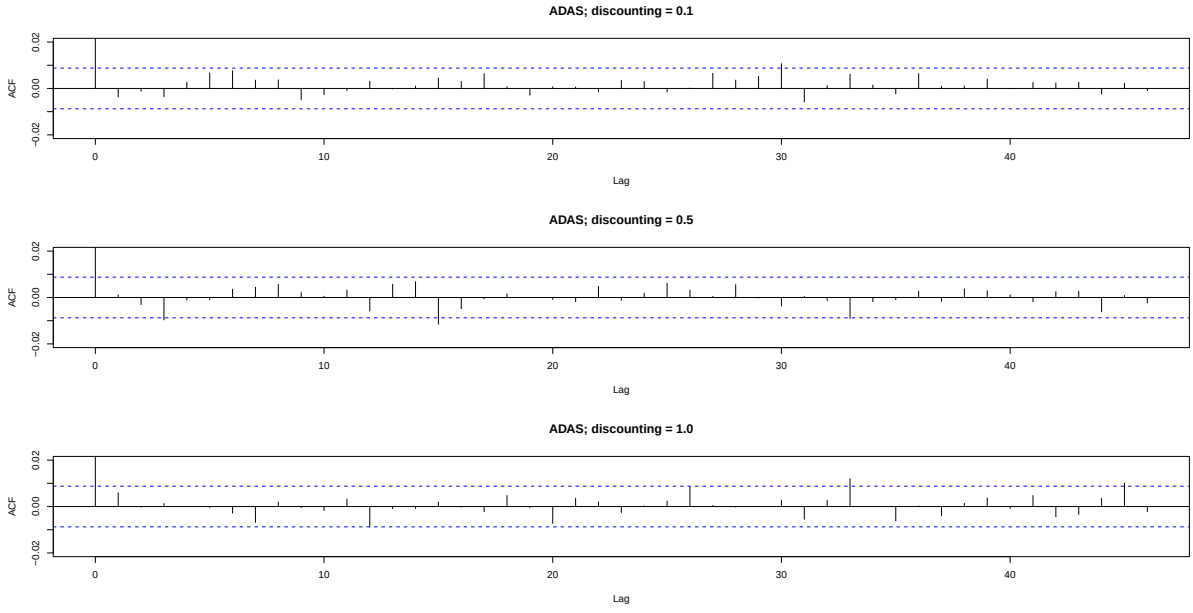

Figure S.14: ACF plots for Gibbs samples of the MCI exposure effect under ADAS response and the partial borrowing power prior with $a_0 \in \{0.1, 0.5, 1\}$. 50k samples after 10k burn-in period.

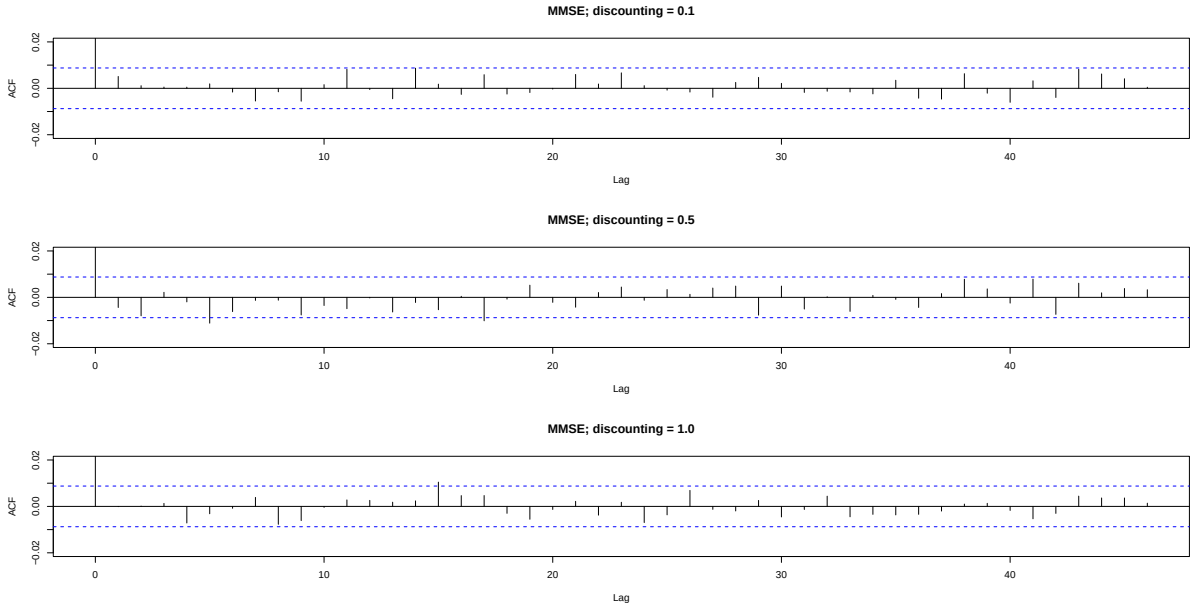

Figure S.15: ACF plots for Gibbs samples of the MCI exposure effect under MMSE response and the partial borrowing power prior with $a_0 \in \{0.1, 0.5, 1\}$. 50k samples after 10k burn-in period.

S.3 Analysis of Simulated Dataset

In this section, we carry out an analysis of a simulated dataset, `sim_data.csv`, which mimics the ADNI data. The dataset and `R` code can be found at `https://github.com/MinLinSTAT/PPreview`. Below are the posterior estimates of γ for this simulated dataset.

Table S.2: Posterior Estimates of γ in for the Simulated Data

Prior	a_0	Estimate	SD	95% HPD
No Borrow	0.0	0.629	0.380	(−0.115, 1.374)
PP	0.1	0.751	0.367	(0.032, 1.470)
	0.5	1.091	0.325	(0.454, 1.727)
	1.0	1.339	0.288	(0.775, 1.903)
pPP	0.1	0.709	0.364	(−0.023, 1.398)
	0.5	0.960	0.323	(0.327, 1.596)
	1.0	1.181	0.287	(0.611, 1.737)
iptwPP	given by (26)	1.102	0.305	(0.505, 1.700)
$\bar{\text{p}}$PP	(0.1, 0.0)	0.751	0.359	(0.047, 1.455)
	(0.1, 0.5)	0.751	0.386	(−0.006, 1.509)
	(0.1, 1.0)	0.751	0.400	(−0.032, 1.535)
	(0.5, 0.0)	1.091	0.303	(0.496, 1.685)
	(0.5, 1.0)	1.091	0.335	(0.433, 1.748)
	(1.0, 0.0)	1.339	0.261	(0.827, 1.851)
	(1.0, 0.5)	1.339	0.279	(0.792, 1.886)
p$\bar{\text{p}}$PP	(0.1, 0.1)	0.792	0.371	(0.065, 1.520)
	(0.1, 0.5)	0.792	0.391	(0.027, 1.558)
	(0.1, 1.0)	0.792	0.404	(0.001, 1.584)
	(0.5, 0.1)	1.210	0.313	(0.596, 1.824)
	(0.5, 0.5)	1.210	0.330	(0.564, 1.856)
	(0.5, 1.0)	1.210	0.341	(0.542, 1.877)
	(1.0, 0.1)	1.483	0.268	(0.958, 2.008)
	(1.0, 0.5)	1.483	0.282	(0.930, 2.036)
	(1.0, 1.0)	1.483	0.292	(0.911, 2.055)