SUPPLEMENTARY MATERIAL

A Description of optimization algorithms: This table includes descriptions of each of the optimization algorithms tested in this article. (LaTeX tables)

B Performance tables: These tables provide additional information about the performance of the different optimization algorithms. (LaTeX tables)

R-package for EZtune: R-package EZtune that can implement autotuning of SVMs, GBMs, and adaboost using the Hooke-Jeeves algorithm and genetic algorithm. The package also contains Lichen and Mullein datasets used in the examples in the article. The package is currently available on CRAN and updates are available at https://github.com/jillbo1000/EZtune. (GNU zipped tar file)

Code and data for creating grids and performing optimization tests: The code and data used to create the error and time response surfaces and the code for testing the optimization algorithms is available at https://github.com/jillbo1000/autotune.

A Description of optimization algorithms

Table A.1: List of optimization algorithms used to search tuning parameter spaces with a brief description of each method.

Algorithm	Type	Description
Ant Lion Mirjalili (2015a)	Metaheuristic	Based on the hunting mechanisms of antlions
BOBYQA Powell (2009)	Derivative free	Derivative free optimization by quadratic approximation
Dragonfly Mirjalili (2016a)	Metaheuristic	Based on static and dynamic swarming behaviors of dragonflies
Firefly Yang (2009)	Metaheuristic	Based on fireflies use of light to attract other fireflies
Genetic algorithm Goldberg (1999)	Metaheuristic	Uses the principles of natural selection in successive generations to find an optimal solution
Grasshopper Saremi et al. (2017)	Metaheuristic	Mimics the behavior of grasshopper swarms
Grey wolf Mirjalili et al. (2014)	Metaheuristic	Mimics leadership hierarchy and hunting methods of grey wolves
Hooke-Jeeves Hooke and Jeeves (1961)	Derivative free	Pattern search that does a local search to find a direction where performance improves and then moves in that direction making larger moves as long as improvement continues
Improved harmony search Mahdavi et al. (2007)	Metaheuristic	Mimics the improvisational process of musicians
L-BFGS Byrd et al. (1995)	Quasi-Newton	Second order method that estimates the Hessian using only recent gradients
Moth flame Mirjalili (2015b)	Metaheuristic	Based on the navigation method of moths called transverse orientation
Nelder-Mead Kelley (1999)	Derivative free	Direct search algorithm that generates a simplex from sample points, x, and uses values of $f(x)$ at the vertices to search for an optimal solution
Nonlinear conjugate gradient Dai and Yuan (2001)	Gradient	The residual is replaced by a gradient and combined with a line search method
Particle swarm	Metaheuristic	Based on the evolutionary

Continued on next page

Table A.1 – *Continued from previous page*

Algorithm	Type	Description
Shi and Eberhart (1998)		mechanisms that allows organisms to adjust their flying based on its own flying experience and the experiences of its companions
Sine cosine Mirjalili (2016b)	Metaheuristic	Creates multiple initial random possible solutions and requires them to fluctuate towards the optimal solution using a mathematical model based on sine and cosine functions
Spectral projected gradient Birgin et al. (2000)	Gradient	Uses the spectrum of the underlying Hessian to determine the step lengths for gradient descent
Whale Mirjalili and Lewis (2016)	Metaheuristic	Mimics the bubble-net hunting strategy of humpback whales

B Performance tables

Table B.1: Average mean squared errors from cross validation model verification and computation times in seconds for support vector regression with EZtune. The best mean squared errors from the grid search are included in the table for reference. Table entries are (cross validated MSE, computation time in seconds).

Optimizer	Type	Abalone	BostonHousing	CO2	Crime	AmesHousing	Union	Wage
Hooke-Jeeves	Resub	(7.386, 918s)	(81.7, 19s)	(72.42, 1s)	(1114, 1s)	(5.42e+09, 346s)	(264.8, 2s)	(2510, 0s)
Hooke-Jeeves	CV = 10	(4.409, 976s)	(7.941, 156s)	(16.92, 8s)	(659.5, 1s)	(8.61e+08, 1554s)	(76.46, 1s)	(343.8, 1s)
Hooke-Jeeves	CV = 3	(4.424, 353s)	(8.991, 34s)	(16.87, 2s)	(754.1, 1s)	(8.64e+08, 647s)	(75.41, 1s)	(422.1, 1s)
Hooke-Jeeves	Fast = TRUE	(4.46, 7s)	(9.886, 3s)	(23.5, 1s)	(848.6, 1s)	(9.26e+08, 25s)	(77.69, 1s)	(617.4, 1s)
Hooke-Jeeves	Fast = 0.25	(4.45, 31s)	(9.797, 2s)	(22.66, 1s)	(1053, 1s)	(9.41e+08, 40s)	(74.05, 1s)	(675.2, 1s)
Hooke-Jeeves	Fast = 0.5	(4.446, 85s)	(9.482, 5s)	(22.79, 1s)	(827.7, 1s)	(9.51e+08, 79s)	(75.61, 1s)	(526.9, 1s)
Hooke-Jeeves	Fast = 0.75	(4.459, 132s)	(9.754, 9s)	(21.94, 1s)	(793.8, 1s)	(9.40e+08, 161s)	(76.87, 1s)	(705.9, 1s)
Hooke-Jeeves	Fast = 0.9	(4.451, 174s)	(9.762, 13s)	(18.79, 1s)	(853.6, 1s)	(9.26e+08, 221s)	(79.69, 1s)	(689.1, 1s)
Hooke-Jeeves	Fast = 100	(4.462, 6s)	(10.1, 2s)	NA	NA	(9.60e+08, 18s)	NA	NA
Hooke-Jeeves	Fast = 200	(4.464, 8s)	(9.588, 4s)	NA	NA	(9.64e+08, 25s)	NA	NA
Hooke-Jeeves	Fast = 300	(4.478, 11s)	(9.194, 9s)	NA	NA	(9.94e+08, 36s)	NA	NA
Hooke-Jeeves	Fast = 400	(4.454, 14s)	(9.805, 8s)	NA	NA	(9.35e+08, 48s)	NA	NA
Genetic Algorithm	Resub	(7.667, 10494s)	(39.06, 27s)	(77.76, 4s)	(1319, 3s)	(4.58e+09, 352s)	(441.4, 3s)	(2396, 2s)
Genetic Algorithm	CV = 10	(4.459, 34760s)	(7.999, 283s)	(13.32, 26s)	(611, 7s)	(8.57e+08, 1620s)	(73.09, 8s)	(323.9, 3s)
Genetic Algorithm	CV = 3	(4.514, 8307s)	(8.393, 83s)	(12.82, 8s)	(634.7, 5s)	(9.11e+08, 548s)	(76.99, 4s)	(330.5, 3s)
Genetic Algorithm	Fast = TRUE	(4.62, 40s)	(10.1, 13s)	(14.1, 4s)	(727.5, 4s)	(8.95e+08, 44s)	(83.97, 3s)	(717.1, 3s)
Genetic Algorithm	Fast = 0.25	(4.574, 252s)	(10.42, 9s)	(13.58, 4s)	(724.9, 4s)	(8.86e+08, 72s)	(116.3, 3s)	(696.8, 3s)
Genetic Algorithm	Fast = 0.5	(4.586, 973s)	(9.817, 16s)	(12.9, 5s)	(817.4, 4s)	(8.80e+08, 144s)	(77.97, 3s)	(676.3, 3s)
Genetic Algorithm	Fast = 0.75	(4.581, 2251s)	(9.963, 26s)	(16.38, 5s)	(847, 5s)	(9.54e+08, 194s)	(78.87, 3s)	(784.9, 3s)
Genetic Algorithm	Fast = 0.9	(4.67, 2659s)	(10.16, 36s)	(18.77, 5s)	(900, 4s)	(9.11e+08, 271s)	(90.47, 3s)	(626.1, 4s)
Genetic Algorithm	Fast = 100	(4.548, 32s)	(9.378, 8s)	NA	NA	(9.59e+08, 35s)	NA	NA
Genetic Algorithm	Fast = 200	(4.659, 41s)	(10.11, 11s)	NA	NA	(8.57e+08, 43s)	NA	NA
Genetic Algorithm	Fast = 300	(4.581, 53s)	(9.699, 21s)	NA	NA	(9.15e+08, 53s)	NA	NA
Genetic Algorithm	Fast = 400	(4.566, 71s)	(8.84, 29s)	NA	NA	(8.80e+08, 77s)	NA	NA
Best Grid		4.389	7.183	10.57	487.2	7.37e+08	62.7	4.71e-03

Table B.2: Average mean squared errors from cross validation model verification and computation times in seconds for gradient boosting regression with EZtune. The best mean squared errors from the grid search are included in the table for reference. Table entries are (cross validated MSE, computation time in seconds).

Optimizer	Type	Abalone	BostonHousing	CO2	Crime	AmesHousing	Union	Wage
Hooke-Jeeves	Resub	(5.659, 1832s)	(8.036, 468s)	(8.409, 13s)	(570.7, 13s)	(7.63e+08, 4154s)	(131.4, 5s)	(2201, 4s)
Hooke-Jeeves	CV = 10	(4.588, 11366s)	(7.821, 2657s)	(5.281, 103s)	(580.2, 102s)	(7.15e+08, 22685s)	(94.73, 37s)	(1455, 39s)
Hooke-Jeeves	CV = 3	(4.579, 2515s)	(7.664, 668s)	(6.567, 25s)	(550.4, 20s)	(6.92e+08, 5539s)	(96.38, 8s)	NA
Hooke-Jeeves	Fast = TRUE	(4.961, 163s)	(8.47, 144s)	(7.174, 6s)	(572.8, 5s)	(7.18e+08, 527s)	(94.61, 2s)	NA
Hooke-Jeeves	Fast = 0.25	(4.721, 423s)	(7.889, 90s)	(5.505, 1s)	(569.8, 1s)	(6.67e+08, 794s)	(107.1, 0s)	NA
Hooke-Jeeves	Fast = 0.5	(4.573, 706s)	(7.878, 167s)	(7.017, 6s)	(645.6, 5s)	(6.74e+08, 1577s)	(98.26, 2s)	NA
Hooke-Jeeves	Fast = 0.75	(4.583, 881s)	(8.206, 253s)	(5.355, 10s)	(628.4, 8s)	(7.09e+08, 1907s)	(105.4, 3s)	NA
Hooke-Jeeves	Fast = 0.9	(4.657, 1193s)	(8.051, 322s)	(6.248, 12s)	(607.4, 11s)	(7.06e+08, 2534s)	(99.76, 4s)	(1744, 3s)
Hooke-Jeeves	Fast = 100	(4.928, 104s)	(7.923, 63s)	NA	NA	(7.03e+08, 261s)	NA	NA
Hooke-Jeeves	Fast = 200	(4.947, 161s)	(8.131, 144s)	NA	NA	(7.00e+08, 519s)	NA	NA
Hooke-Jeeves	Fast = 300	(5.015, 182s)	(7.728, 209s)	NA	NA	(7.26e+08, 736s)	NA	NA
Hooke-Jeeves	Fast = 400	(4.97, 199s)	(8.176, 253s)	NA	NA	(7.06e+08, 981s)	NA	NA
Genetic Algorithm	Resub	(5.778, 4067s)	(8.082, 611s)	(9.286, 34s)	(594.7, 20s)	(8.03e+08, 5941s)	(133.2, 15s)	NA
Genetic Algorithm	CV = 10	(4.571, 12460s)	(7.987, 5843s)	(6.343, 179s)	(570.2, 194s)	(7.00e+08, 58628s)	(99.73, 48s)	(1751, 36s)
Genetic Algorithm	CV = 3	(4.581, 3549s)	(7.778, 1398s)	(6.517, 53s)	(556, 42s)	(6.58e+08, 12907s)	(111.3, 14s)	NA
Genetic Algorithm	Fast = TRUE	(4.726, 209s)	(7.965, 259s)	(6.882, 13s)	(580.7, 10s)	(7.19e+08, 927s)	(117.5, 6s)	NA
Genetic Algorithm	Fast = 0.25	(4.588, 567s)	(8.097, 144s)	(7.975, 2s)	NA	(6.72e+08, 1649s)	(124.9, 2s)	NA
Genetic Algorithm	Fast = 0.5	(4.598, 1098s)	(7.808, 419s)	(7.059, 15s)	(544.5, 12s)	(7.26e+08, 2780s)	(121.6, 4s)	NA
Genetic Algorithm	Fast = 0.75	(4.641, 1489s)	(7.853, 447s)	(6.758, 20s)	(573.4, 18s)	(7.36e+08, 5229s)	(126.1, 9s)	NA
Genetic Algorithm	Fast = 0.9	(4.688, 1814s)	(8.379, 538s)	(7.084, 25s)	(580.7, 22s)	(7.40e+08, 4433s)	(119.2, 10s)	(2118, 7s)
Genetic Algorithm	Fast = 100	(4.656, 169s)	(8.207, 113s)	NA	NA	(7.10e+08, 469s)	NA	NA
Genetic Algorithm	Fast = 200	(4.69, 221s)	(7.968, 326s)	NA	NA	(6.98e+08, 1052s)	NA	NA
Genetic Algorithm	Fast = 300	(4.738, 273s)	(8.039, 484s)	NA	NA	(7.24e+08, 1524s)	NA	NA
Genetic Algorithm	Fast = 400	(4.681, 303s)	(7.834, 603s)	NA	NA	(6.86e+08, 1580s)	NA	NA
Best Grid		4.442	6.8	4.568	391.7	6.04e+08	82.86	0.0262

Table B.3: Average classification errors from cross validation model verification and computation times in seconds for support vector classification with EZtune. The best classification errors from the grid search are included in the table for reference. Table entries are (cross validated error rate, computation time in seconds).

Optimizer	Type	BreastCancer	Ionosphere	Lichen	Mullein	Pima	Sonar
Hooke-Jeeves	Resub	(0.0455, 2s)	(0.0521, 2s)	(0.1821, 11s)	(0.0581, 6998s)	(0.3031, 7s)	(0.1327, 12s)
Hooke-Jeeves	CV = 10	(0.0312, 5s)	(0.0544, 7s)	(0.1446, 44s)	(0.0574, 70130s)	(0.2363, 26s)	(0.1197, 48s)
Hooke-Jeeves	CV = 3	(0.0338, 2s)	(0.0587, 3s)	(0.1538, 20s)	(0.0575, 18827s)	(0.2384, 10s)	(0.1274, 26s)
Hooke-Jeeves	Fast = TRUE	(0.0372, 1s)	(0.0587, 2s)	(0.1599, 4s)	(0.1748, 70s)	(0.2384, 1s)	(0.1264, 15s)
Hooke-Jeeves	Fast = 0.25	(0.0379, 1s)	(0.0581, 2s)	(0.1693, 3s)	(0.0783, 539s)	(0.2393, 1s)	(0.1274, 13s)
Hooke-Jeeves	Fast = 0.5	(0.0401, 1s)	(0.057, 2s)	(0.164, 5s)	(0.0581, 2745s)	(0.2336, 3s)	(0.1264, 15s)
Hooke-Jeeves	Fast = 0.75	(0.0379, 1s)	(0.055, 3s)	(0.1705, 8s)	(0.0576, 5128s)	(0.2534, 4s)	(0.1298, 17s)
Hooke-Jeeves	Fast = 0.9	(0.0397, 1s)	(0.0544, 2s)	(0.1681, 11s)	(0.0681, 7179s)	(0.2436, 5s)	(0.1308, 19s)
Hooke-Jeeves	Fast = 100	(0.0391, 1s)	(0.0524, 2s)	(0.1623, 3s)	(0.1645, 57s)	(0.2374, 1s)	(0.125, 15s)
Hooke-Jeeves	Fast = 200	(0.0351, 1s)	(0.0553, 2s)	(0.174, 4s)	(0.169, 64s)	(0.243, 1s)	(0.1298, 16s)
Hooke-Jeeves	Fast = 300	(0.0365, 1s)	(0.0524, 2s)	(0.16, 5s)	(0.1627, 70s)	(0.2417, 2s)	NA
Hooke-Jeeves	Fast = 400	(0.0344, 1s)	NA	(0.1582, 6s)	(0.1563, 80s)	(0.2435, 2s)	NA
Genetic Algorithm	Resub	(0.053, 8s)	(0.1598, 10s)	(0.2098, 45s)	(0.0578, 13064s)	(0.3234, 25s)	(0.3111, 58s)
Genetic Algorithm	CV = 10	(0.0321, 26s)	(0.0481, 28s)	(0.1496, 227s)	(0.0583, 175904s)	(0.2302, 248s)	(0.1212, 176s)
Genetic Algorithm	CV = 3	(0.0327, 11s)	(0.055, 15s)	(0.1494, 84s)	(0.0587, 58531s)	(0.2355, 72s)	(0.1212, 105s)
Genetic Algorithm	Fast = TRUE	(0.0347, 5s)	(0.0575, 9s)	(0.1601, 14s)	(0.1825, 137s)	(0.2322, 8s)	(0.1269, 80s)
Genetic Algorithm	Fast = 0.25	(0.0362, 4s)	(0.0584, 8s)	(0.1526, 15s)	(0.075, 1338s)	(0.2359, 6s)	(0.1298, 67s)
Genetic Algorithm	Fast = 0.5	(0.0329, 5s)	(0.0581, 9s)	(0.1487, 24s)	(0.0608, 5163s)	(0.2371, 15s)	(0.126, 76s)
Genetic Algorithm	Fast = 0.75	(0.0378, 6s)	(0.0601, 9s)	(0.1606, 38s)	(0.0587, 13604s)	(0.2384, 25s)	(0.1173, 76s)
Genetic Algorithm	Fast = 0.9	(0.0394, 6s)	(0.0655, 8s)	(0.1568, 41s)	(0.059, 13408s)	(0.244, 38s)	(0.1212, 75s)
Genetic Algorithm	Fast = 100	(0.0397, 4s)	(0.0604, 8s)	(0.1662, 11s)	(0.1898, 117s)	(0.2348, 5s)	(0.1269, 74s)
Genetic Algorithm	Fast = 200	(0.0354, 5s)	(0.0621, 10s)	(0.1539, 15s)	(0.1813, 126s)	(0.2357, 8s)	(0.1231, 64s)
Genetic Algorithm	Fast = 300	(0.0359, 4s)	(0.0644, 9s)	(0.1551, 17s)	(0.1786, 142s)	(0.2353, 12s)	NA
Genetic Algorithm	Fast = 400	(0.0329, 6s)	NA	(0.1515, 22s)	(0.1709, 202s)	(0.232, 15s)	NA
Best Grid		0.0234	0.0427	0.131	0.0682	0.2174	0.101

Table B.4: Average classification errors from cross validation model verification and computation times in seconds for gradient boosting classification with EZtune. The best classification errors from the grid search are included in the table for reference. Table entries are (cross validated error rate, computation time in seconds).

Optimizer	Type	BreastCancer	Ionosphere	Lichen	Mullein	Pima	Sonar
Hooke-Jeeves	Resub	(0.0303, 30s)	(0.0681, 58s)	(0.1606, 124s)	(0.0779, 6388s)	(0.269, 47s)	(0.1341, 347s)
Hooke-Jeeves	CV = 10	(0.0313, 331s)	(0.0698, 667s)	(0.163, 1529s)	(0.0787, 54699s)	(0.2587, 372s)	(0.1279, 4171s)
Hooke-Jeeves	CV = 3	(0.0318, 79s)	(0.067, 157s)	(0.1629, 460s)	(0.0786, 11786s)	(0.2577, 107s)	(0.1308, 1081s)
Hooke-Jeeves	Fast = TRUE	(0.0319, 15s)	(0.0712, 47s)	(0.1592, 48s)	(0.1399, 115s)	(0.2665, 17s)	(0.1457, 263s)
Hooke-Jeeves	Fast = 0.25	(0.0297, 14s)	(0.0687, 23s)	(0.1617, 54s)	(0.0939, 1070s)	(0.2642, 15s)	(0.1428, 107s)
Hooke-Jeeves	Fast = 0.5	(0.0293, 21s)	(0.0675, 42s)	(0.1581, 114s)	(0.089, 2589s)	(0.2585, 23s)	(0.1288, 275s)
Hooke-Jeeves	Fast = 0.75	(0.0331, 27s)	(0.0718, 68s)	(0.163, 145s)	(0.0836, 4395s)	(0.2643, 34s)	(0.1428, 381s)
Hooke-Jeeves	Fast = 0.9	(0.0319, 27s)	(0.0672, 77s)	(0.1611, 152s)	(0.101, 3720s)	(0.2646, 47s)	(0.126, 420s)
Hooke-Jeeves	Fast = 100	(0.0309, 9s)	(0.0675, 27s)	(0.1594, 30s)	(0.1207, 99s)	(0.2651, 11s)	(0.1327, 247s)
Hooke-Jeeves	Fast = 200	(0.0312, 15s)	(0.0704, 46s)	(0.1623, 50s)	(0.1217, 137s)	(0.2604, 16s)	(0.1428, 401s)
Hooke-Jeeves	Fast = 300	(0.0318, 18s)	(0.0655, 65s)	(0.1618, 82s)	(0.1131, 166s)	(0.2613, 22s)	NA
Hooke-Jeeves	Fast = 400	(0.0306, 27s)	NA	(0.164, 106s)	(0.1062, 187s)	(0.2642, 28s)	NA
Genetic Algorithm	Resub	(0.0328, 224s)	(0.0724, 438s)	(0.1585, 1100s)	(0.0703, 17910s)	(0.2747, 266s)	(0.1524, 466s)
Genetic Algorithm	CV = 10	(0.0294, 2192s)	(0.0644, 4026s)	(0.154, 11211s)	(0.0705, 159183s)	(0.2423, 1609s)	(0.1337, 4398s)
Genetic Algorithm	CV = 3	(0.0324, 609s)	(0.0692, 820s)	(0.1574, 2654s)	(0.0712, 62064s)	(0.2401, 458s)	(0.1361, 809s)
Genetic Algorithm	Fast = TRUE	(0.0312, 104s)	(0.0695, 241s)	(0.1549, 231s)	(0.1113, 784s)	(0.2495, 80s)	(0.1394, 1152s)
Genetic Algorithm	Fast = 0.25	(0.0322, 84s)	(0.0675, 115s)	(0.156, 279s)	(0.0746, 6952s)	(0.243, 71s)	(0.1577, 373s)
Genetic Algorithm	Fast = 0.5	(0.0312, 149s)	(0.0692, 221s)	(0.1575, 712s)	(0.071, 14104s)	(0.2448, 129s)	(0.1457, 941s)
Genetic Algorithm	Fast = 0.75	(0.0322, 196s)	(0.0738, 379s)	(0.1625, 985s)	(0.0729, 36854s)	(0.2458, 190s)	(0.1481, 1900s)
Genetic Algorithm	Fast = 0.9	(0.0316, 215s)	(0.0681, 364s)	(0.1585, 1050s)	(0.0709, 34875s)	(0.2544, 222s)	(0.1495, 1875s)
Genetic Algorithm	Fast = 100	(0.0299, 58s)	(0.0652, 128s)	(0.1565, 142s)	(0.0957, 462s)	(0.2454, 41s)	(0.1375, 1155s)
Genetic Algorithm	Fast = 200	(0.0309, 103s)	(0.0709, 291s)	(0.159, 344s)	(0.1126, 735s)	(0.2402, 72s)	(0.137, 2034s)
Genetic Algorithm	Fast = 300	(0.031, 159s)	(0.0687, 387s)	(0.1558, 529s)	(0.089, 1010s)	(0.2406, 95s)	NA
Genetic Algorithm	Fast = 400	(0.0313, 210s)	NA	(0.1558, 674s)	(0.1026, 1158s)	(0.2443, 112s)	NA
Best Grid		0.022	0.0484	0.1333	0.0733	0.2214	0.0962

Table B.5: Average classification errors from cross validation model verification and computation times in seconds for adaboost with EZtune. The best classification errors from the grid search are included in the table for reference. Table entries are (cross validated error rate, computation time in seconds).

Optimizer	Type	BreastCancer	Ionosphere	Lichen	Mullein	Pima	Sonar
Hooke-Jeeves	Resub	(0.0347, 131s)	(0.0738, 207s)	(0.1689, 381s)	(0.1254, 12601s)	(0.2786, 1216s)	(0.1668, 234s)
Hooke-Jeeves	CV = 10	(0.0351, 1568s)	(0.0812, 2145s)	(0.1615, 3472s)	(0.1265, 123910s)	(0.2452, 8702s)	(0.1659, 3099s)
Hooke-Jeeves	CV = 3	(0.0344, 455s)	(0.0781, 662s)	(0.1615, 1086s)	(0.1255, 27970s)	(0.2409, 2596s)	(0.1553, 851s)
Hooke-Jeeves	Fast = TRUE	(0.0357, 114s)	(0.0755, 221s)	(0.165, 299s)	(0.1795, 528s)	(0.2518, 709s)	(0.1663, 322s)
Hooke-Jeeves	Fast = 0.25	(0.034, 114s)	(0.0789, 210s)	(0.1708, 298s)	(0.1298, 2964s)	(0.249, 672s)	(0.1582, 265s)
Hooke-Jeeves	Fast = 0.5	(0.036, 148s)	(0.0764, 198s)	(0.1638, 322s)	(0.1272, 6286s)	(0.2574, 778s)	(0.1524, 315s)
Hooke-Jeeves	Fast = 0.75	(0.0334, 163s)	(0.0789, 218s)	(0.1599, 386s)	(0.1292, 7831s)	(0.2479, 909s)	(0.1562, 302s)
Hooke-Jeeves	Fast = 0.9	(0.0354, 145s)	(0.0772, 245s)	(0.1658, 370s)	(0.1327, 9046s)	(0.2496, 940s)	(0.1471, 312s)
Hooke-Jeeves	Fast = 100	(0.0366, 108s)	(0.0778, 174s)	(0.1667, 227s)	(0.1874, 412s)	(0.2543, 595s)	(0.1591, 316s)
Hooke-Jeeves	Fast = 200	(0.034, 124s)	(0.0766, 239s)	(0.1682, 220s)	(0.1864, 445s)	(0.2491, 767s)	(0.1639, 228s)
Hooke-Jeeves	Fast = 300	(0.0351, 128s)	(0.0795, 232s)	(0.1667, 310s)	(0.1942, 427s)	(0.2418, 705s)	NA
Hooke-Jeeves	Fast = 400	(0.0348, 150s)	NA	(0.1687, 297s)	(0.1802, 523s)	(0.247, 703s)	NA
Genetic Algorithm	Resub	(0.0306, 523s)	(0.0718, 847s)	(0.1582, 1999s)	(0.1137, 51756s)	(0.276, 4191s)	(0.1404, 1070s)
Genetic Algorithm	CV = 10	(0.0327, 5165s)	(0.0704, 12083s)	(0.1635, 22474s)	(0.1282, 224020s)	NA	(0.1341, 14681s)
Genetic Algorithm	CV = 3	(0.0318, 1772s)	(0.0687, 2917s)	(0.1573, 5970s)	(0.1202, 77899s)	(0.2607, 12742s)	(0.1365, 4128s)
Genetic Algorithm	Fast = TRUE	(0.031, 568s)	(0.0724, 793s)	(0.1619, 1228s)	(0.1412, 1837s)	(0.2698, 3064s)	(0.1351, 1358s)
Genetic Algorithm	Fast = 0.25	(0.0328, 528s)	(0.0698, 738s)	(0.1587, 1305s)	(0.1265, 11429s)	(0.2642, 3588s)	(0.1413, 1120s)
Genetic Algorithm	Fast = 0.5	(0.0321, 641s)	(0.0724, 848s)	(0.1543, 1395s)	(0.1281, 21992s)	(0.2618, 3508s)	(0.1457, 1210s)
Genetic Algorithm	Fast = 0.75	(0.03, 676s)	(0.0709, 1070s)	(0.1611, 1770s)	(0.125, 35719s)	(0.2674, 4699s)	(0.1317, 1332s)
Genetic Algorithm	Fast = 0.9	(0.0316, 473s)	(0.0709, 794s)	(0.1571, 2247s)	(0.1277, 47479s)	(0.2661, 5161s)	(0.1308, 1283s)
Genetic Algorithm	Fast = 100	(0.0335, 428s)	(0.0729, 944s)	(0.1608, 933s)	(0.1383, 1337s)	(0.2745, 2580s)	(0.1418, 1099s)
Genetic Algorithm	Fast = 200	(0.0309, 466s)	(0.0721, 825s)	(0.157, 1165s)	(0.1359, 1729s)	(0.2682, 3134s)	(0.1413, 1137s)
Genetic Algorithm	Fast = 300	(0.0313, 821s)	(0.0746, 859s)	(0.1568, 1502s)	(0.1271, 2433s)	(0.2755, 3895s)	NA
Genetic Algorithm	Fast = 400	(0.0324, 807s)	NA	(0.1599, 1576s)	(0.1294, 2619s)	(0.2669, 4392s)	NA
Best Grid		0.019	0.0484	0.125	0.0814	0.2109	0.0865

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