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# A Critique of Infant Mortality Estimates in India

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*Abstract*:

**Background:** Brass developed a procedure for converting proportions dead of children ever born reported by women in childbearing ages into estimates of the probability of dying before attaining certain exact childhood ages. The method has become very popular in less developed countries where direct mortality estimation is not possible due to incomplete death registration. However, the estimates of $q(x)$, the probability of dying before age $x$, obtained by Trussell's variant of Brass method are sometimes unrealistic, $q(x)$ being not monotonically increasing for increasing $x$.

**Method:** State level child mortality estimates obtained by Trussell's variant of Brass method from 1991 and 2001 Indian census data were made monotonically increasing by logit smoothing. Using two of the smoothed child mortality estimates, infant mortality estimate is obtained by fitting a two parameter Weibull survival function.

**Results:** It has been found that in many states and union territories infant mortality rates have increased between 1991 and 2001. Cross checking with the 1991 and 2001 census data on the increase/decrease of percentage of children died establishes the reliability of the estimates.

**Conclusion:** We have reason to suspect the trend of declining infant mortality as shown by the different agencies and researchers.

*Key words*: Child mortality, India, infant mortality, logit, state, Weibull.

## 1. Introduction

Infant and child mortality have traditionally been considered as the indicator of overall socio-economic wellbeing, child health and mortality conditions of a region (Chandrasekhar, 1972; Jain, 1985; Fayissa, 2001; Saha and Roy 2002). It is well known that the proportions of children ever born who have died are indicators of child mortality and can yield robust estimates of childhood mortality. Brass (1964; 1975) was the first to develop a procedure for converting proportions dead

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of children ever born reported by women in different age groups of childbearing period into estimates of the probability of dying before attaining certain exact childhood ages. He observed that the relation between the proportions of children dead by age group of mothers ($D(i)$, $i = 1$ for age group 15-19, $i = 2$ for age group 20-24, $\cdots$, $i = 7$ for age group 45-49) and the probability of dying before age $x$ [$q(x)$], is primarily influenced by the age pattern of fertility.

Brass established a set of correspondences between ages of mothers and ages of their children for whom cumulative mortality is best identified (see Table 1), and these correspondences have been widely used by all subsequent analysts. These correspondences, however, are not exact and depend on the reproductive histories of the particular group of women reporting their births (Preston *et al.*, 2003). Brass developed a set of multipliers (adjustment factors) to adjust for the particular reproductive histories of a group of women and to convert the observed values of the proportions of children dead into estimates of the probability of dying before age $x$. To increase flexibility of Brass' original method Sullivan (1972) developed a set of adjustment factors based on data generated from observed fertility schedules and the Coale-Demeny regional model life tables. Trussell (1975) also developed a set of multipliers based on a wider range of cases, using data generated from the model fertility schedules developed by Coale and Trussell (1974), which are now more commonly used.

Table 1: The correspondence relating to age group, age group index $i$ and age $x$ are:

| Age group: | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 |
|---|---|---|---|---|---|---|---|
| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $x$ | 1 | 2 | 3 | 5 | 10 | 15 | 20 |

Note: $x$ is regarded as function of the age group $i$, i.e., $x = x(i)$, but we have omitted the variable $i$ for convenience.

Brass method is, however, subject to certain types of errors. Such errors arise from misreporting of the number of children ever born and children dead; Inclusion of still births as live births and omission of living children who moved away from their mother's household; changes in fertility levels and from selective mortality among mothers. Fortunately, the bias from selective mortality is typically small (except in populations with high HIV prevalence) because female mortality is low in the child bearing interval (Preston *et al.*, 2003, Chapter 11). Another limitation of this method is that it suggests to disregard the estimate of $q(1)$, the probability of dying before age one, based on women aged 15-19, as this estimate is usually too high. This bias results from the fact that younger women have a high proportion of first births, which generally have above-average risks of mortality. Also the early child bearers tend to come from lower socioeconomic strata with higher mortality risks (Ewbank, 1982). Despite these limitations this method has many appealing characteristics and has been found to estimate child mortality levels and trends over roughly a 10 year period rather well (Hill, 1991).

Brass-type estimation has brought a revolution in child mortality estimates in less developed countries (Preston *et al.*, 2003, Chapter 11). However, it has been observed that in some data sets of children ever born ($CEB$) and children surviving ($CS$) to mothers of different age groups, the estimates of $q(x)$ obtained by Trussell's variant of Brass method are not monotonically increasing. For instance, the $CEB$, $CS$ data of Zimbabwe Demographic and Health Survey, 1994 yields the following estimates: $q(1) = 0.0605$, $q(2) = 0.0858$, $q(3) = 0.0760$, $q(5) = 0.0855$ and $q(10) = 0.0960$ (Preston *et al.*, 2003, Chapter 11, Page 232). Note that $q(2) > q(3)$ which does not conform to reality as the probability of dying before age 3 cannot be less than the probability of dying before age 2. This situation is frequently encountered while trying to estimate infant and child mortality for the states and the Union Territories (U/Ts) of India using 1991 and 2001 census data.

Though the above method assumes constant fertility and childhood mortality in the recent past, there are methods to avoid the problems caused by declining fertility when data for true cohorts are available from censuses or surveys taken 5 or 10 years apart (United Nations, 1983, Chapter III). Even this does not remove the problem of not having monotonically increasing $q(x)$ for some states of India. For instance, consider the state of Maharashtra. The estimates based on 1991 and 2001 census data are: $q(3) = 0.0783$, $q(5) = 0.0841$ and $q(10) = 0.0812$. Here also, $q(5) > q(10)$, which violates reality.

Therefore, it is essential to look for some sort of smoothing in the results of $q(x)$ obtained by the above methods so that these estimates remain valid for all data sets. This can be done by logit smoothing technique using the logit of the Brass' general standard life table. Roy (1989) has derived reliable estimate of child survivorship function $[l(x) = 1 - q(x)]$ for India by logit smoothing. After getting the smoothed values of $l(2)$, $l(3)$, $l(5)$ (in case of data from one census) or the values of $l(3)$, $l(5)$ and $l(10)$ (in case of data from two censuses) one can estimate $l(1)$ [and consequently $q(1)$] by fitting a two parameter Weibull survival function using any two of the smoothed estimates (Choe, 1981).

In India, registration of deaths is not satisfactory, particularly at the vulnerable period of infancy and childhood. As a result, the death rate implied by the reported deaths is not reliable and usually under represents the true death rate prevalent in the population (Saha and Roy, 2002). The sample registration system (SRS) and the national family health surveys (NFHS) provide information on child mortality at the state level. State and district level estimates of child mortality were also made by Registrar General of India (1988; 1997) using 1981 and 1991 census data. A more detailed effort has been made by Rajan and Mohanachandran (1998), providing these estimates for the states and the districts based on 1991 census data. Recently, Rajan *et al.* (2008) estimated infant and child mortality using 2001 census data for the states and the districts adopting a different (modified) methodology. All these estimates showed a declining trend in infant mortality. However, it would be ideal to estimate the infant mortalities by a single method on different census data to ensure that the differences in the estimates of different periods are not due to the differences in methodologies. Further, it is intuitive that there is a correspondence between the decline in infant and child mortality and the decline in the percentage of children died over different periods. An analysis of the 1991 and 2001 census data on children ever born and children died as reported by mothers may help in revealing the status of the estimates by the Registrar General of India through SRS.

An effort has been made in this paper to estimate infant mortality using 1991 and 2001 census data for the states and the union territories of India. It is hoped that the scenario of infant mortality decline in the states and the union territories of India between 1991 and 2001 could be recaptured with the help of these estimates.

### 1.1 Objectives

The present study has the following objectives:

1. Estimation of infant mortality rates (IMR) at the state level using the 1991 census data and comparing the estimates with the ones obtained by SRS and Rajan and Mohanachandran (1998) to check for adequacy of our estimates.

2. Estimation of infant mortality rates (IMR) at the state level using the 2001 census data and comparing them with the estimates of 1991. Comparison of the decline in the IMR and the corresponding decline in the percentage of children died during this period.

## 2. Methods and Materials

For the convenience of the readers and to maintain continuity, we first briefly discuss the following:

### 2.1 Brass Method of Estimating Child Mortality Using Trussell's Multipliers

The average parity per woman is estimated by

$$P(i) = CEB(i)/W(i),$$

where $CEB(i)$ denotes the number of children ever born to women belonging to the age group $i$ and $W(i)$ denotes the total number of women belonging to the age group $i$ irrespective of their marital status. The relation between index $i$ and age group is presented in Table 1.

The proportion of children dead for each age group of mothers is estimated by

$$D(i) = \frac{CEB(i) - CS(i)}{CEB(i)} = \frac{CD(i)}{CEB(i)},$$

where $CS(i)$ denotes the number of surviving children reported by mothers belonging to the age group $i$ and $CD(i)$ denotes the number of dead children reported by mothers belonging to the age group $i$.

The multipliers $K(i)$'s are calculated according to the Trussell's variant of the original Brass method. The simplified equation is:

$$K(i) = a(i) + b(i)\frac{P(1)}{P(2)} + c(i)\frac{P(2)}{P(3)},$$

where $a(i)$, $b(i)$ and $c(i)$ are the coefficients for estimation of child mortality multipliers. The values of $a(i)$, $b(i)$ and $c(i)$ are provided in United Nations (1983) for the four different families (North, South, East and West) of model life tables developed in the Coale Demeny regional model life table system.

Finally,

$$q(x) = K(i) * D(i).$$

The proportions of children surviving to the date of the survey are the net result of the mortality conditions in the past rather than the mortality conditions prevalent on the date the survey was done. However, since mortality too is not constant and changes over different time periods, it is important to identify the time period to which Brass-type estimates most closely pertain. Following on the work of Griffith Feeney (1976), Coale and Trussell (1977) developed formulas for the estimation of the reference period, $t(x)$ (number of years prior to the survey), to which the values of $q(x)$ refer. These equations have same format as those for the estimation of the adjustment factors $K(i)$ (Preston *et al.*, 2003, Chapter 11). The equation to estimate $t(x)$ is

$$t(x) = \alpha(i) + \beta(i)\frac{P(1)}{P(2)} + \gamma(i)\frac{P(2)}{P(3)},$$

where $\alpha(i)$, $\beta(i)$ and $\gamma(i)$ are the coefficients for estimation of $t(x)$. The values of $\alpha(i)$, $\beta(i)$ and $\gamma(i)$ are also provided in United Nations (1983) for the four different families of model life tables developed in the Coale Demeny regional model life table system.

**Choice of the Model Life Table**

The necessary coefficients to estimate the multipliers differ for each of the four different families of the life tables in the Coale-Demeny regional model life table system (United Nations, 1983, Chapter III). The Coale-Demeny regional model life tables were derived in 1966 from 192 life tables by sex recorded for actual populations. A preliminary analysis of the tables revealed that four different mortality patterns (also termed as four different families) could be distinguished among them. These patterns were labeled "North", "South", "East" and "West" because of the predominance of the European countries belonging to the various regions in each category. The North model is based largely on the Nordic countries, and is characterized by comparatively low infant mortality, relatively high child mortality and low old age mortality beyond age 50. The South model is based on life tables from the countries of Southern Europe, and has a mortality pattern characterized by high mortality under age 5, low mortality from about age 40 to age 60, and high mortality over age 65. The East model comes mainly from the Eastern European countries, and is characterized by high mortality rate in infancy and at older ages (over age 50). The West model is based on the residual tables not used in the other regional sets (i.e., countries of Western Europe and most of the non-European populations). It is characterized by a pattern intermediate between North and the East patterns (United Nations, 1983, Chapter I).

Hence the choice of the family is extremely important. In one hand, the West model is derived from the largest number and broadest variety of cases and is believed to represent the most general mortality pattern. On the other hand, the South model has a mortality pattern which is, perhaps, closer to the South Asian pattern (to which India belongs) of the United Nations Model life Tables for developing countries characterized by high mortality below age 15, low mortality between age 15 to age 55 and high mortality above age 55. To be doubly ensured, we have used both the West and South model coefficients and found that the infant mortality estimates of the states obtained by using the West model coefficients are slightly less or equal to the ones obtained by using the South model coefficients (the difference being at most two infant deaths per thousand live births). Considering the high level of infant mortality in India, this difference may be overlooked. However, we have taken the estimates based on South model coefficients for comparison as Rajan and Mahanachandran (1998) also used this model for estimating the infant and child mortality rates.

### 2.2 Modeling the Age Pattern of Infant and Child Mortality by Weibull Survival Distribution

According to the Weibull model, the survival probability at age $x$ is of the form

$$l(x) = \exp(-\lambda x^{\gamma}); \quad \lambda > 0,\ \gamma > 0,\ x \geq 0. \qquad \text{(Choe, 1981)} \tag{1}$$

The distribution can be identified alternatively by the instantaneous force of mortality or the hazard rate, as

$$\mu(x) = -\frac{l'(x)}{l(x)} = \lambda\gamma x^{\gamma-1}. \tag{2}$$

When $0 < \gamma < 1$, $\mu(x)$ is monotonically decreasing function, suggesting application to mortality at young ages. From (1),

$$\ln(\ln(1/l(x))) = \ln\lambda + \gamma\ln(x), \tag{3}$$

$$q(1) = 1 - l(1) = 1 - \exp(-\lambda). \tag{4}$$

### 2.3 Description of the Method

We have first estimated the $l(x)$ values for $x = 1, 2, 3$ and 5 by Brass method using Trussell's multipliers (South model) from census data on $CEB$ and $CS$. In many states these values are found not to be monotonically decreasing (as $q(x)$ values are not monotonically increasing). We have applied logit smoothing to make the $l(x)$ values monotonically decreasing. The logit transformation of $l(x)$ for $x = 2, 3, 5$, ignoring $l(1)$ (as $q(1)$ is suggested to be disregarded), are obtained by

$$y(x) = 0.5\ln\left(\frac{1 - l(x)}{l(x)}\right).$$

The smoothed values of $y(x)$ are then obtained as

$$\hat{y}(x) = y_s(x) + \frac{1}{3}\sum_{\substack{x=2,3,5 \\ (x\neq 4)}}[y(x) - y_s(x)]. \qquad \text{(Roy, 1989)}$$

For $x = 2, 3$ and 5 where $y_s(x)$ is the logit of the Brass General Standard Life Table (These values are provided in United Nations, 1983, Chapter III, Page 77).

From these smoothed logit values $[\hat{y}(x)]$, the smoothed $l(x)$ values for $x = 2, 3$ and 5 are obtained from

$$\hat{l}(x) = \frac{1}{1 + \exp[2\hat{y}(x)]}.$$

It is to be noted that, since the Weibull model has only two parameters, a pair of $l(x)$ values are sufficient to estimate them (Choe, 1981). In our case, we have three smoothed $l(x)$ values, and a decision has to be made regarding which pair is to be used in estimating the parameters. Our experimentation with south model life table survival probabilities by taking three combinations $l(2)$, $l(3)$; $l(2)$, $l(5)$ and $l(3)$, $l(5)$ shows that the combination of $l(2)$ and $l(3)$ gives the best estimate of $l(1)$. Moreover, $l(2)$ and $l(3)$ are supposed to be more reliable estimates of child survival as they are derived from information of mothers in the relatively young age groups of 20-24 and 25-29 respectively (Shryock and Seigel, 1976, Chapter 24).

Using $\hat{l}(2)$ and $\hat{l}(3)$ in (3), $\gamma$ is estimated as

$$\hat{\gamma} = \frac{\ln[\ln\{1/\hat{l}(2)\}] - \ln[\ln\{1/\hat{l}(3)\}]}{\ln(2) - \ln(3)}.$$

Then using $\hat{l}(2)$ in (3), $\lambda$ is estimated as

$$\hat{\lambda} = \exp(\ln(\ln(1/\hat{l}(2))) - \hat{\gamma}\ln(2)).$$

The final estimates of $l(x)$ and consequently $q(x)$ are obtained by using $\hat{\lambda}$ and $\hat{\gamma}$ in (1).

An example of the computational steps is given in the Appendix with the $CEB$, $CS$ data of 2001 census for the state of West Bengal.

**The data to be used in this study are:**

1. 1991 and 2001 census data on children ever born and children surviving of both sexes combined classified by five years age group of women for the states and union territories of India. We have considered the women aged 15-34 instead of the entire childbearing period 15-49 as we are interested in estimating $q(x)$ only up to age 5.

2. 1991 and 2001 census data on total female population in the age group 15-34 classified by five years age interval.

## 3. Results and Discussion

Table 2 presents the IMR estimates of the states and U/Ts based on 1991 and 2001 census data along with the estimates of Rajan and Mahanachandran (1998) (based on 1991 census); Rajan *et al.* (2008) (based on 2001 census); SRS estimates (1990; 2000) and the estimates based on the method of two censuses (1991; 2001). Interestingly, in many states, our estimates based on 2001 census are greater than the ones based on 1991 census whereas SRS and Rajan *et al.* (2008) found a decline in IMR in all the states. We have also estimated the IMR by the two census method to avoid the problems caused by declining fertility and found that these estimates are not much different to the estimates based on 2001 census.

Table 2: The proposed IMR ($q(1)$) estimates of the states and U/Ts along with the estimates of Rajan and Mahanachandran (based on 1991 census); Rajan *et al.* (based on 2001 census), SRS (1990; 2000) estimates and the estimates based on the method of two censuses (1991; 2001)

| State/(U/ T) | Proposed (Based on 1991census, South model) | SRS (1990) | R&M (Based on 1991census) | Proposed (Based on 2001census, South model) | SRS (2000) | Rajan *et al.* (Based on 2001census) | Based on 1991, 2001 censuses |
|---|---|---|---|---|---|---|---|
| Andhra Pradesh | 0.044 | 0.070 | 0.049 | 0.050 | 0.065 | 0.043 | 0.055 |
| Assam | 0.078 | 0.076 | 0.085 | 0.075 | 0.075 | 0.070 | 0.080 |
| Bihar | 0.063 | 0.075 | 0.070 | 0.072 | 0.062 | 0.057 | 0.078 |
| Gujarat | 0.057 | 0.072 | 0.069 | 0.056 | 0.062 | 0.059 | 0.060 |
| Haryana | 0.049 | 0.069 | 0.055 | 0.070 | 0.067 | 0.040 | 0.079 |
| Himachal Pradesh | 0.064 | 0.069 | 0.075 | 0.063 | 0.060 | 0.045 | 0.067 |
| Karnataka | 0.054 | 0.070 | 0.060 | 0.053 | 0.057 | 0.054 | 0.057 |
| Kerala | 0.031 | 0.017 | 0.037 | 0.037 | 0.014 | 0.018 | 0.042 |
| Madhya Pradesh | 0.103 | 0.111 | 0.107 | 0.098 | 0.087 | 0.094 | 0.103 |
| Maharashtra | 0.049 | 0.058 | 0.058 | 0.047 | 0.048 | 0.049 | 0.050 |
| Orissa | 0.101 | 0.122 | 0.108 | 0.089 | 0.095 | 0.090 | 0.092 |
| Punjab | 0.044 | 0.061 | 0.054 | 0.060 | 0.052 | 0.043 | 0.066 |
| Rajasthan | 0.076 | 0.084 | 0.081 | 0.078 | 0.079 | 0.079 | 0.084 |
| Tamil Nadu | 0.046 | 0.059 | 0.053 | 0.058 | 0.051 | 0.044 | 0.064 |
| Uttar Pradesh | 0.084 | 0.099 | 0.089 | 0.089 | 0.083 | 0.084 | 0.096 |
| West Bengal | 0.060 | 0.063 | 0.067 | 0.068 | 0.051 | 0.059 | 0.074 |
| Arunachal Pradesh | 0.087 | NA | 0.083 | 0.096 | 0.044 | 0.061 | 0.104 |
| Chattishgarh | NA | NA | NA | 0.088 | 0.079 | 0.068 | NA |
| Goa | 0.033 | NA | 0.034 | 0.033 | 0.023 | 0.028 | 0.036 |
| Jharkhand | NA | NA | NA | 0.065 | 0.070 | 0.050 | NA |
| Jammu& Kashmir | NA | NA | NA | 0.075 | 0.050 | 0.047 | NA |
| Manipur | 0.028 | NA | 0.036 | 0.053 | 0.023 | 0.031 | 0.059 |
| Meghalaya | 0.069 | NA | 0.076 | 0.085 | 0.058 | 0.058 | 0.093 |
| Mizoram | 0.049 | NA | 0.058 | 0.058 | 0.021 | 0.041 | 0.063 |
| Nagaland | 0.045 | NA | 0.055 | 0.077 | NA | 0.039 | 0.083 |
| Sikkim | 0.054 | NA | 0.057 | 0.064 | 0.049 | 0.043 | 0.071 |
| Tripura | 0.072 | NA | 0.078 | 0.066 | 0.041 | 0.064 | 0.070 |
| Uttaranchal | NA | NA | NA | 0.065 | 0.050 | 0.063 | NA |
| #A & N Islands | 0.045 | NA | 0.049 | 0.056 | 0.023 | 0.041 | 0.058 |
| #Chandigarh | 0.033 | NA | 0.046 | 0.068 | 0.028 | 0.044 | 0.077 |
| #D & N Haveli | 0.059 | NA | 0.063 | 0.058 | 0.058 | 0.046 | 0.063 |
| #Daman & Diu | 0.049 | NA | 0.060 | 0.042 | 0.048 | 0.032 | 0.045 |
| #Delhi | 0.040 | NA | 0.049 | 0.054 | 0.032 | 0.040 | 0.060 |
| #Lakshadweep | 0.085 | NA | 0.080 | 0.064 | 0.027 | 0.060 | 0.066 |
| #Pondicherry | 0.044 | NA | 0.047 | 0.042 | 0.023 | 0.028 | 0.046 |

# indicates U/T, NA: Not available.

A serious question arises as we have found in as many as 15 states and 3 U/T's (States: Andhra Pradesh, Bihar, Haryana, Kerala, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, West Bengal, Arunachal Pradesh, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim; U/Ts: Andaman & Nicober, Chandigarh and Delhi) the IMR estimates increased in between 1991 and 2001. Among these states and U/T's Kerala, Punjab, Delhi and Chandigarh are considered to be the most developed ones. The IMR of Chandigarh has almost doubled with an increase of 35 infant deaths per thousand live births. Punjab, Delhi and Kerala experienced respectively an increase of 16, 14 and 6 infant deaths per thousand live births during 1991-2001. Among the other states, Haryana, Tamil Nadu, Manipur and Nagaland experienced significant increase in IMR with an increase of 21, 12, 25 and 32 infant deaths per thousand live births respectively during 1991-2001. On the other hand less developed states like Assam, Madhya Pradesh and Orissa showed a decline in IMR during 1991-2001.

Infant and child mortality estimation generally suffers from misclassification of women in different age groups and underreporting of child deaths. Considering a relatively young group of mothers (15-34) for our estimation purpose possibly eliminates the age misclassification problem. Regarding underreporting of child deaths, a possibility may be that the level of underreporting in 1991 is significantly higher than in 2001 in the states where IMR increased during 1991-2001.

Analysis of such results necessitates a further review of 1991 and 2001 census data regarding the women in the 15-34 age group (the age group responsible for producing estimates of $q(x)$, $x = 1, 2, 3, 5$ by Brass method) and their child bearing and child surviving histories. Table 3 presents the percentage increase in the number of women, the percentage increase in $CEB$ and $CS$, percentage of children died to children ever born in 1991 and 2001 censuses and the decrease/increase in the percentage of children died of women in the age group 15-34. It has been observed that a decrease in the percentage of children died to

Table 3: The percentage increase in the number of women, the percentage increase in $CEB$ and $CS$, percentage of children died to children ever born in 1991 and 2001 census and the decrease/increase in the percentage of children died to women in the age group 15-35

| State/Union Territory | % increase during 1991-2001 in | | | % of children died to $CEB$ in | | Decrease in % died | Decrease in $q(1)$ |
|---|---|---|---|---|---|---|---|
| | Women | $CEB$ | $CS$ | 1991 | 2001 | (1991-2001) | (1991-2001) |
| Andhra Pradesh | 18.33 | 2.16 | 1.41 | 6.12 | 6.81 | -0.69 | -0.006 |
| Assam | 18.70 | 0.95 | 1.58 | 10.16 | 9.59 | 0.56 | 0.003 |
| Bihar[1] | 26.23 | 28.24 | 27.26 | 8.50 | 9.20 | -0.70 | -0.009 |
| Gujarat | 21.92 | 13.35 | 13.69 | 7.31 | 7.04 | 0.28 | 0.001 |
| Haryana | 30.73 | 26.52 | 23.03 | 6.59 | 9.17 | -2.57 | -0.021 |
| Himachal Pradesh | 19.62 | 2.58 | 3.18 | 8.35 | 7.81 | 0.54 | 0.001 |
| Karnataka | 20.09 | 1.66 | 2.06 | 7.35 | 6.99 | 0.36 | 0.001 |
| Kerala | 3.81 | -2.88 | -3.40 | 4.12 | 4.64 | -0.52 | -0.006 |
| Madhya Pradesh[2] | 22.24 | 18.16 | 19.73 | 13.68 | 12.53 | 1.15 | 0.005 |
| Maharashtra | 23.73 | 8.78 | 9.46 | 6.67 | 6.09 | 0.58 | 0.002 |
| Orissa | 18.54 | 6.30 | 8.49 | 12.88 | 11.08 | 1.80 | 0.012 |
| Punjab | 20.40 | 9.87 | 7.90 | 5.70 | 7.38 | -1.69 | -0.016 |
| Rajasthan | 28.17 | 28.37 | 28.15 | 10.17 | 10.33 | -0.16 | -0.002 |
| Tamil Nadu | 12.74 | 3.48 | 2.10 | 6.22 | 7.46 | -1.25 | -0.012 |
| Uttar Pradesh[3] | 26.93 | 30.13 | 29.97 | 11.21 | 11.31 | -0.11 | -0.005 |
| West Bengal | 18.99 | 11.09 | 9.87 | 8.22 | 9.22 | -1.01 | -0.008 |
| Arunachal Pradesh | 31.04 | 36.87 | 36.06 | 11.92 | 12.44 | -0.53 | -0.009 |
| Goa | 13.09 | 4.65 | 5.07 | 4.30 | 3.91 | 0.39 | 0.000 |
| Manipur | 21.67 | 1.72 | -1.35 | 3.50 | 6.41 | -2.91 | -0.025 |
| Meghalaya | 31.97 | 19.12 | 16.88 | 9.03 | 10.74 | -1.70 | -0.016 |
| Mizoram | 34.00 | 26.82 | 25.43 | 6.27 | 7.30 | -1.03 | -0.009 |
| Nagaland | 77.53 | 68.86 | 62.59 | 5.50 | 9.02 | -3.51 | -0.032 |
| Sikkim | 39.89 | 26.17 | 24.79 | 7.30 | 8.31 | -1.02 | -0.010 |
| Tripura | 19.55 | -4.46 | -3.65 | 9.44 | 8.67 | 0.77 | 0.006 |
| #Andaman & Nicober | 35.09 | 5.96 | 4.62 | 6.13 | 7.31 | -1.19 | -0.011 |
| #Chandigarh | 38.99 | 29.06 | 23.69 | 4.22 | 8.21 | -3.99 | -0.035 |
| #Dadra & Nagar Haveli | 49.13 | 48.03 | 48.53 | 8.09 | 7.78 | 0.31 | 0.001 |
| #Daman & Diu | 46.05 | 35.83 | 37.43 | 6.51 | 5.41 | 1.10 | 0.007 |
| #Delhi | 45.40 | 29.70 | 27.65 | 5.24 | 6.74 | -1.50 | -0.014 |
| #Lakshwadeep | 24.33 | 18.64 | 21.52 | 10.90 | 8.73 | 2.16 | 0.021 |
| #Pondecheri | 22.87 | 14.21 | 14.70 | 5.96 | 5.55 | 0.40 | 0.002 |

[1]including Jharkhand, [2]including Chhatishgarh, [3]including Uttaranchal. Negative sign indicates increase.

children ever born results in a corresponding decrease in IMR and vice versa. These observations clearly indicates that the increase/decrease in the IMR's are due to the changes that take place in the female population in the age group 15-34 and their *CEB*, *CS* characteristics during 1991-2001. From the decrease in the percentage of children died and the decrease in IMR, a table (Table 4) is

Table 4: The approximate decrease in children died per thousand and decrease in IMR per thousand

| State/Union Territory | Decrease in the number of children died (per thousand) | Decrease infant mortality (per thousand) |
|---|---|---|
| Andhra Pradesh | -6.9 | -6 |
| Assam | 5.6 | 3 |
| Bihar[1] | -7.0 | -9 |
| Gujarat | 2.8 | 1 |
| Haryana | -25.7 | -21 |
| Himachal Pradesh | 5.4 | 1 |
| Karnataka | 3.6 | 1 |
| Kerala | -5.2 | -6 |
| Madhya Pradesh[2] | 11.5 | 5 |
| Maharashtra | 5.8 | 2 |
| Orissa | 18.0 | 12 |
| Punjab | -16.9 | -16 |
| Rajasthan | -1.6 | -2 |
| Tamil Nadu | -12.5 | -12 |
| Uttar Pradesh[3] | -1.1 | -5 |
| West Bengal | -10.1 | -8 |
| Arunachal Pradesh | -5.3 | -9 |
| Goa | 3.9 | 0 |
| Manipur | -29.1 | -25 |
| Meghalaya | -17.0 | -16 |
| Mizoram | -10.3 | -9 |
| Nagaland | -35.1 | -32 |
| Sikkim | -10.2 | -10 |
| Tripura | 7.7 | 6 |
| #Andaman & Nicober | -11.9 | -11 |
| #Chandigarh | -39.9 | -35 |
| #Dadra & Nagar Haveli | 3.1 | 1 |
| #Daman & Diu | 11.0 | 7 |
| #Delhi | -15.0 | -14 |
| #Lakshwadeep | 21.6 | 21 |
| #Pondecheri | 4.0 | 2 |

[1]including Jharkhand, [2]including Chhatishgarh, [3]including Uttaranchal. Negative sign indicates increase.

constructed showing the approximate decrease in children died per thousand and decrease in infant mortality per thousand. This data confirms the direct correspondence between the decrease in the percentage of children died and decrease in IMR with a Spearman's correlation coefficient of 0.99. Thus, the 35 point increase in IMR of Chandigarh may be attributed to the almost 40 point increase in the number of children died. Similarly, the increase of 16, 14 and 6 point IMR during 1991-2001 in Punjab, Delhi and Kerala respectively may be attributed to the respective increase of 17, 15 and 5 point in the number of children died and so on. This, perhaps, establishes the reliability of our estimates, assuming the same level of underreporting of child deaths in 1991 and 2001 censuses, which in turn, lead to suspect the trend of declining infant mortality as suggested by others.

**Appendix: Estimates of $q(x)$ for $x = 1, 2, 3$ and 5 for the state of West Bengal (both sexes combined, 2001 census data, south model multipliers)**

| Age Group | $i$ | $W(i)$ | $CEB(i)$ | $CS(i)$ | $P(i)$ | $D(i)$ | $K(i)$ | $x$ | $q(x)$ | $l(x)$ | $t(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 15-19 | 1 | 3564573 | 598807 | 538330 | 0.1680 | 0.1010 | 1.0947 | 1 | 0.1106 | 0.8894 | 0.85 |
| 20-24 | 2 | 3495373 | 3868115 | 3522999 | 1.1066 | 0.0892 | 1.0278 | 2 | 0.0917 | 0.9083 | 2.26 |
| 25-29 | 3 | 3564430 | 7318556 | 6654778 | 2.0532 | 0.0907 | 0.9817 | 3 | 0.0890 | 0.9110 | 4.48 |
| 30-34 | 4 | 3023596 | 8119231 | 7352420 | 2.6853 | 0.0944 | 0.9886 | 5 | 0.0934 | 0.9066 | 7.19 |

$$P(1) = \frac{CEB(1)}{W(1)} = \frac{598807}{3564573} = 0.1680, \text{ etc.}$$

$$D(1) = \frac{CEB(1) - CS(1)}{CEB(1)} = \frac{598807 - 538330}{598807} = 0.1010, \text{ etc.}$$

$$K(1) = a(1) + b(1)\frac{P(1)}{P(2)} + c(1)\frac{P(2)}{P(3)}$$

$$= 1.0819 - 3.0005\frac{P(1)}{P(2)} + 0.8689\frac{P(2)}{P(3)} = 1.0947, \text{ etc.}$$

The Values of $a(1)$, $b(1)$ and $c(1)$ etc. are provided in United Nations, 1983 (Chapter III, Page 77).

$$q(2) = K(2) * D(2) = 1.0278 * 0.0892 = 0.0917, \text{ etc. [Disregarding } q(1)].$$

Note that $l(2)$, $l(3)$ and $l(5)$ are not monotonically decreasing.

$$\begin{aligned} t(1) &= \alpha(1) + \beta(1)\frac{P(1)}{P(2)} + \gamma(1)\frac{P(2)}{P(3)} \\ &= 1.0900 + 5.4443\frac{P(1)}{P(2)} - 1.9721\frac{P(2)}{P(3)} = 0.85, \text{ etc.} \end{aligned}$$

The reference dates $[t(x)]$, to which the $q(x)$ values refer, imply that the estimates of $q(1)$, $q(2)$, $q(3)$ and $q(5)$ obtained above refer to mortality conditions prevalent approximately one year, two and a quarter year, four and a half year and seven years before the census respectively.

Next we apply logit smoothing to get monotonically decreasing $l(x)$ for $x = 2, 3, 5$. From two of the smoothed estimates, viz., $l(2)$ and $l(3)$, improved estimates of $l(x)$ [and consequently $q(x)$] for $x = 1, 2, 3$ and 5 are obtained by fitting the Weibull survival function as shown below.

| Age Group | $i$ | $x$ | $l(x)$ | $y(x)$ | $y_s(x)$ | $y(x) - y_s(x)$ | $\hat{y}(x)$ | $\hat{l}(x)$ | Final estimates $l(x)$ | Final estimates $q(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 15-19 | 1 | 1 | - | - | - | - | - | - | 0.9319 | 0.0681 |
| 20-24 | 2 | 2 | 0.9083 | -1.1465 | -0.7152 | -0.4313 | -1.2065 | 0.9178 | 0.9178 | 0.0822 |
| 25-29 | 3 | 3 | 0.9110 | -1.1627 | -0.6552 | -0.5075 | -1.1465 | 0.9083 | 0.9083 | 0.0917 |
| 30-34 | 4 | 5 | 0.9066 | -1.1366 | -0.6015 | -0.5351 | -1.0928 | 0.8990 | 0.8948 | 0.1052 |
| Total | | | | | | -1.4739 | | | | |

$y_s(x)$ values are provide in United Nations, 1983 (Chapter I, page 19)

$$\hat{y}(x) = y_s(x) + \frac{1}{3}\sum_{\substack{x=2,3,5\\(x\neq 4)}}[y(x) - y_s(x)], \quad \text{for } x = 2, 3, 5,$$

$$\begin{aligned} \hat{y}(2) &= y_s(2) + \frac{1}{3}\sum_{\substack{x=2,3,5\\(x\neq 4)}}[(y(x) - y_s(x)] \\ &= y_s(2) + \frac{1}{3}[\{y(2) - y_s(2)\} + \{y(3) - y_s(3)\} + \{y(5) - y_s(5)\}] \\ &= -0.7152 - \frac{1}{3}[-1.4739] = -0.7152 - 0.4913 = -1.2065, \end{aligned}$$

$$\hat{y}(3) = y_s(3) + \frac{1}{3}\sum_{\substack{x=2,3,5\\(x\neq 4)}}[(y(x) - y_s(x)]$$

$$= y_s(3) + \frac{1}{3}[\{y(2) - y_s(2)\} + \{y(3) - y_s(3)\} + \{y(5) - y_s(5)\}]$$
$$= -0.6552 - \frac{1}{3}[-1.4739] = -0.6552 - 0.4913 = -1.1465,$$
$$\hat{y}(5) = y_s(5) + \frac{1}{3} \sum_{\substack{x = 2,3,5 \\ (x \neq 4)}} [(y(x) - y_s(x)]$$
$$= y_s(5) + \frac{1}{3}[\{y(2) - y_s(2)\} + \{y(3) - y_s(3)\} + \{y(5) - y_s(5)\}]$$
$$= -0.6015 - \frac{1}{3}[-1.4739] = -0.6015 - 0.4913 = -1.0928,$$
$$\hat{l}(2) = \frac{1}{1 + \exp[2\hat{y}(2)]} = \frac{1}{1 + \exp[2(-1.2065)]} = 0.9178,$$
$$\hat{l}(3) = \frac{1}{1 + \exp[2\hat{y}(3)]} = \frac{1}{1 + \exp[2(-1.1465)]} = 0.9083,$$
$$\hat{l}(5) = \frac{1}{1 + \exp[2\hat{y}(5)]} = \frac{1}{1 + \exp[2(-1.0928)]} = 0.8990,$$
$$\hat{\gamma} = \frac{\ln[\ln\{1/\hat{l}(2)\}] - \ln[\ln\{1/\hat{l}(3)\}]}{\ln(2) - \ln(3)}$$
$$= \frac{\ln[\ln\{1/0.9178\} - \ln[\ln\{1/0.9083\}]}{\ln(2) - \ln(3)} = 0.2829,$$
$$\hat{\lambda} = \exp(\ln(\ln 1/\hat{l}(2))) - \hat{\gamma}\ln(2)$$
$$= \exp(\ln(\ln(1/0.9178))) - 0.2829\ln(2) = 0.0705.$$

Finally,

$$l(x) = \exp(-\hat{\lambda}x^{\hat{\gamma}}) = \exp(-0.0705\ x^{0.2829}), \quad \text{for } x = 1, 2, 3 \text{ and } 5,$$

and,

$$q(x) = 1 - l(x), \quad \text{for } x = 1, 2, 3 \text{ and } 5.$$

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