Supplemental Materials

S1. Gibbs sampling algorithm for the penalized spline prediction

We consider a Bayesian penalized spline with homoscedastic errors. Suppose $Y^* = X + \lambda Y$, we fit the hierarchical model $X|Y^* \sim N(W\beta + Z\gamma,\ \sigma^2)$, where $W = \begin{pmatrix} 1 & Y_1^* \\ \vdots & \vdots \\ 1 & Y_r^* \end{pmatrix}$, $Z = \begin{pmatrix} (Y_1{}^* - \kappa_1)_+ & \cdots & (Y_r^* - \kappa_k)_+ \\ \vdots & \ddots & \vdots \\ (Y_r^* - \kappa_1)_+ & \cdots & (Y_r^* - \kappa_k)_+ \end{pmatrix}$,

$\gamma_1, \ldots, \gamma_K \sim N(0,\ \tau^2)$, and r is the number of respondents. Here, we have K splines in the PSPP model represented by Z. We assign the following non-informative priors:

$$\beta \sim 1$$

$$\sigma^2 \sim InvGamma(10^{-5}, 10^{-5})$$

$$\tau^2 \sim InvGamma(10^{-5}, 10^{-5})$$

Estimates of the joint posterior distributions of the parameters, along with the posterior predictive distribution of X are obtained via Gibbs sampling algorithm. The method is summarized as follows:

1. Draw $\beta^{(d)}, \gamma^{(d)} | W, Z, \sigma^{2(d\text{-}1)}, \tau^{2(d\text{-}1)} \sim N((C'C + \frac{\sigma^{2(d-1)}}{\tau^{2(d-1)}}D)^{-1}C'X,\ \sigma^{2(d\text{-}1)}(C'C + \frac{\sigma^{2(d-1)}}{\tau^{2(d-1)}}D)^{-1})$, $C = [W\ Z]$, $D = \begin{pmatrix} 0_{2x2} & 0_{2xK} \\ 0_{Kx2} & 1_{KxK} \end{pmatrix}$
2. Draw $\tau^{2(d)} | W, Z, \sigma^{2(d\text{-}1)}, \beta^{(d)}, \gamma^{(d)} \sim InvGamma(10^{-5} + \frac{K}{2}, 10^{-5} + \frac{1}{2}\|\gamma\|^2)$
3. Draw $\sigma^{2(d)} | W, Z, \tau^{2(d)}, \beta^{(d)}, \gamma^{(d)} \sim InvGamma(10^{-5} + \frac{r}{2}, 10^{-5} + \frac{1}{2}(X - W\beta^{(d)} - Z\gamma^{(d)})'\ (X - W\beta^{(d)} - Z\gamma^{(d)}))$
4. Draw $X_{mis} | W, Z, \sigma^{2(d)}, \tau^{2(d)}, \beta^{(d)}, \gamma^{(d)} \sim N(W\beta^{(d)} + Z\gamma^{(d)}, \sigma^{2(d)})$

We repeat steps 1-4 for total of 2000 iterations, discarding the first 1000 as burn-in and taking every other 10 iterations, and apply the hotdeck procedure described in Section 2.1.

S2. Results from simulations

In scenario 1 we generate data under a pattern-mixture model, with *X* generated for the entire sample and *Y* generated only for respondents:

$$R \sim Bernoulli(0.5)$$

$$X, Y|R = 1 \sim N_2\left(\begin{matrix}0\\0\end{matrix}, \begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix}\right)$$

$$X|R = 0 \sim N(1, 1)$$

For all scenarios, we use sample sizes of n = 100 and n = 400, and set the mean nonresponse rate to be approximately 50%. Here, the true nonresponse mechanism depends on value of $X + \lambda_T Y$, where λ_T varies between 0, 1, and ∞. Given λ_T, the true mean of *Y* is then:

$$\mu_Y = \mu_Y^{(1)} + \pi^{(0)}\left(\frac{\sigma_{XY} + \lambda\sigma_{YY}}{\sigma_{XX} + \lambda\sigma_{XY}}\right)(\mu_X^{(0)} - \mu_X^{(1)})$$

For scenario 2, we assume bivariate data on *X* and *Y:*

$$X \sim Gamma(1, 1/4)$$

$$Y|X \sim N(10 + X, 1)$$

And a latent variable *U* such that *Y* is deleted if U > 0:

A. $U|X, Y \sim N(-1.5 + 0.5X, 4)$ ($\lambda_T = 0$)

B. $U|X, Y \sim N(-4 + 0.5(X + Y), 4)$ ($\lambda_T = 1$)

C. $U|X, Y \sim N(-6.5 + 0.5Y, 4)$ ($\lambda_T = \infty$)

Mechanisms A, B, and C simulate conditions where missingness depends on X ($\lambda_T = 0$), $X + Y$ ($\lambda_T = 1$), and Y ($\lambda_T = \infty$).

For scenario 3, we assume data on a set of fully observed covariates $Z_1 - Z_3$ and a *Y* subject to missingness with the following distributions:

$$Z_1 \sim N(0, 1)$$

$$Z_2 \sim N(0, 1)$$

$$Z_3 \sim N(0, 1)$$

$$Y | Z_1, Z_2, Z_3 \sim N(15 + Z_1 + 2Z_2 + Z_3, 1)$$

We generate nonresponse under the following logistic models:

A. $Logit[Pr(R = 0)] = 0.5(Z_1 + 2Z_2 + Z_3)$ $(\lambda_T = 0)$

B. $Logit[Pr(R = 0)] = -3.5 + 0.25(0.98Z_1 + 1.95Z_2 + 0.98Z_3 + Y)$ $(\lambda_T = 1)$

C. $Logit[Pr(R = 0)] = -7.5 + 0.5Y$ $(\lambda_T = \infty)$

D. $Logit[Pr(R = 0)] = 2Z_2$

E. $Logit[Pr(R = 0)] = -7.5 + 0.5(2Z_2 + Y)$

For mechanisms A-C, there is a correct value of λ_A for which the assumption that missingness depends on $Y^* = X + \lambda_A Y$ is correct. Mechanisms D and E, however, cannot be captured any value of λ_A.

In scenario 4, we vary the distributions of Z:

$$Z_1 \sim N(0, 1)$$

$$Z_2 \sim GAMMA(1, 1)$$

$$Z_3 \sim BERNOULLI(0.5)$$

$$Y | Z_1, Z_2, Z_3 \sim N(10 + Z_1 + 4Z_2 + Z_3, 1)$$

As in scenario 3, we generate nonresponse under the following selection models:

A. $Logit[Pr(R = 0)] = -2 + 0.5(Z_1 + 4Z_2 + Z_3)$ $(\lambda_T = 0)$

B. $Logit[Pr(R = 0)] = -4.5 + 0.25(0.98Z_1 + 3.9Z_2 + 0.98Z_3 + Y)$ $(\lambda_T = 1)$

C. $Logit[Pr(R = 0)] = -7 + 0.5Y$ $(\lambda_T = \infty)$

D. $Logit[Pr(R = 0)] = -1 + Z_2$

E. $Logit[Pr(R = 0)] = -4 + 0.25(2Z_2 + Y)$

where in D and E the assumption that nonresponse depends on $Y^* = X + \lambda_A Y$ is violated for all values of λ_A.

In scenario 5, we let the mean of Y be dependent on a quadratic Z^2:

$$Z_1 \sim N(0, 1)$$

$$Z_2 \sim N(0, 1)$$

$$Y|Z_1, Z_2 \sim N(10 + Z_1 + Z_2 + 2Z_2^2, 1)$$

and generate nonresponse under the selection models:

A. $Logit[Pr(R = 0)] = -1 + 0.5(Z_1 + Z_2 + 2Z_2^2)$ $(\lambda_T = 0)$

B. $Logit[Pr(R = 0)] = -3 + 0.25(0.97Z_1 + 0.97Z_2 + 1.95Z_2^2 + Y)$ $(\lambda_T = 1)$

C. $Logit[Pr(R = 0)] = -6 + 0.5Y$ $(\lambda_T = \infty)$

D. $Logit[Pr(R = 0)] = 4Z_2$

E. $Logit[Pr(R = 0)] = -5.5 + 0.5(4Z_2 + Y)$

As before mechanisms D and E do not correspond to any value of λ_T.

Lastly, in scenario 6 we introduce an interaction term in the mean of Y:

$$Z_1 \sim N(0, 1)$$

$$Z_2 \sim N(0, 1)$$

$$Y|Z_1, Z_2 \sim N(20 + Z_1 + Z_2 + 2Z_1Z_2, 1)$$

And delete Y according to the following models:

A. $Logit[Pr(R = 0)] = Z_1 + Z_2 + 2Z_1Z_2$ $(\lambda_T = 0)$

B. $Logit[Pr(R = 0)] = -20 + 0.98Z_1 + 0.98Z_2 + 1.96Z_1Z_2 + Y$ $(\lambda_T = 1)$

C. $Logit[Pr(R = 0)] = -20 + Y$ $(\lambda_T = \infty)$

D. $Logit[Pr(R = 0)] = 5Z_2$

E. $Logit[Pr(R = 0)] = -10 + 0.5(5Z_2 + Y)$

In each scenario and nonresponse mechanism, we first obtain the proxy X from a correctly specified regression of Y on $Z_1, \ldots, Z_p$ using the respondent sample, then apply S-PPMA and PPMA to X and Y to estimate the mean of Y over values of $\lambda_A = 0$, 1, and ∞, where λ_A is the assumed value of λ for the missing data mechanism. For PPMA, we draw $\hat{\mu}_Y$ from its posterior distribution as described in Little (1994), from which we obtain 95% credibility intervals and coverage (PPMA-BAYES). For comparison we include maximum likelihood estimates for PPMA with variance estimated from 200 bootstrap samples (PPMA-ML). For S-PPMA, we perform the hotdeck procedure described in the article to generate 100 imputed datasets, and obtain $\hat{\mu}_Y$ and its variance via (6) and (7). Finally, for PPMA-ML and S-PPMA, we estimate the 95% confidence interval as

$$95\%\ \text{CI} = (\hat{\mu}_Y - t_{n\text{-}1,0.975}\sqrt{Var(\hat{\mu}_Y)},\ \hat{\mu}_Y + t_{n\text{-}1,0.975}\sqrt{Var(\hat{\mu}_Y)})$$

where $t_{n\text{-}1,0.975}$ is the 97.5th percentile of the t-distribution with n-1 degrees of freedom, and $Var(\hat{\mu}_Y)$ is the estimated variance of the mean.

The following tables summarize performances of each method in terms of root mean square error (RMSE), 95% confidence interval width (CIW), and non-coverage rate out of 1000 replications. Estimates under $\lambda_A = 0$, 1, and ∞ are produced for each λ_T, the true underlying value of λ.

Table 1a. Results for scenario 1 under $\lambda_T = 0$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-250	290	397	645
	0	S-BNPM	1	161	625	61
		BNPM-ML	2	159	595	70
		BNPM-BAYES	0	159	607	64
	1	S-BNPM	269	334	678	323
		BNPM-ML	262	324	712	267
		BNPM-BAYES	260	322	722	335
	∞	S-BNPM	808	905	1706	691
		BNPM-ML	852	995	42061	265
		BNPM-BAYES	835	938	3390	863
400		CC	-249	259	196	984
	0	S-BNPM	-1	73	299	45
		BNPM-ML	0	72	289	52
		BNPM-BAYES	0	72	290	53
	1	S-BNPM	251	266	326	862
		BNPM-ML	248	262	337	853
		BNPM-BAYES	248	261	336	871
	∞	S-BNPM	750	769	620	1000
		BNPM-ML	755	773	707	1000
		BNPM-BAYES	755	773	701	1000

Table 1b. Results for scenario 1 under $\lambda_T = 1$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-500	519	395	982
	0	S-BNPM	-250	291	628	356
		BNPM-ML	-250	290	596	400
		BNPM-BAYES	-252	292	607	370
	1	S-BNPM	20	183	682	54
		BNPM-ML	15	178	721	43
		BNPM-BAYES	13	178	728	52
	∞	S-BNPM	568	692	1671	238
		BNPM-ML	643	1401	46654	30
		BNPM-BAYES	592	723	3391	493
400		CC	-501	506	196	1000
	0	S-BNPM	-250	262	301	885
		BNPM-ML	-250	261	291	897
		BNPM-BAYES	-250	262	293	892
	1	S-BNPM	5	93	329	66
		BNPM-ML	3	91	339	51
		BNPM-BAYES	3	91	340	55
	∞	S-BNPM	513	545	634	946
		BNPM-ML	521	552	728	930
		BNPM-BAYES	521	552	720	977

Table 1c. Results for scenario 1 under $\lambda_T = \infty$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1001	1011	394	1000
	0	S-BNPM	-746	762	619	991
		BNPM-ML	-747	763	592	996
		BNPM-BAYES	-749	765	600	995
	1	S-BNPM	-486	519	671	771
		BNPM-ML	-494	524	705	753
		BNPM-BAYES	-496	526	712	691
	∞	S-BNPM	55	405	1623	70
		BNPM-ML	81	444	25807	35
		BNPM-BAYES	74	415	3014	34
400		CC	-996	998	197	1000
	0	S-BNPM	-745	748	301	1000
		BNPM-ML	-745	748	292	1000
		BNPM-BAYES	-745	749	292	1000
	1	S-BNPM	-492	500	328	998
		BNPM-ML	-494	502	339	999
		BNPM-BAYES	-495	502	339	998
	∞	S-BNPM	6	174	621	76
		BNPM-ML	12	174	710	45
		BNPM-BAYES	12	174	702	59

Table 2a. Results for scenario 2 under $\lambda_T = 0$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-2401	2412	932	1000
		True	-37	503	2028	56
	0	S-BNPM	-35	553	2372	46
		BNPM-ML	-37	503	2006	65
		BNPM-BAYES	-63	504	2015	51
	1	S-BNPM	368	761	2572	39
		BNPM-ML	716	963	2546	121
		BNPM-BAYES	693	943	2497	197
	∞	S-BNPM	986	1370	3262	89
		BNPM-ML	1490	1744	3805	243
		BNPM-BAYES	1476	1730	3360	527
400		CC	-2399	2402	463	1000
		True	-1	245	989	47
	0	S-BNPM	1	276	1384	12
		BNPM-ML	-1	245	985	48
		BNPM-BAYES	-8	245	984	49
	1	S-BNPM	503	809	2446	8
		BNPM-ML	719	782	1206	640
		BNPM-BAYES	714	777	1202	695
	∞	S-BNPM	864	1066	2704	52
		BNPM-ML	1443	1499	1586	986
		BNPM-BAYES	1440	1496	1525	993

Table 2b. Results for scenario 2 under $\lambda_T = 1$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-2082	2101	1107	1000
		True	27	472	2216	23
	0	S-BNPM	-123	499	1927	81
		BNPM-ML	-122	475	1788	91
		BNPM-BAYES	-141	478	1798	73
	1	S-BNPM	72	537	1994	61
		BNPM-ML	241	569	2031	47
		BNPM-BAYES	225	559	2035	62
	∞	S-BNPM	304	677	2196	53
		BNPM-ML	618	869	2498	78
		BNPM-BAYES	607	859	2389	151
400		CC	-2107	2112	545	1000
		True	3	236	1061	28
	0	S-BNPM	-147	279	985	101
		BNPM-ML	-142	269	876	123
		BNPM-BAYES	-147	272	878	120
	1	S-BNPM	52	292	1140	40
		BNPM-ML	206	328	977	115
		BNPM-BAYES	202	326	988	130
	∞	S-BNPM	183	367	1184	44
		BNPM-ML	564	637	1146	458
		BNPM-BAYES	561	634	1133	520

Table 2c. Results for scenario 2 under $\lambda_T = \infty$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1917	1940	1155	999
		True	40	463	2051	31
	0	S-BNPM	-181	488	1792	99
		BNPM-ML	-179	478	1702	107
		BNPM-BAYES	-194	481	1725	95
	1	S-BNPM	-47	477	1826	80
		BNPM-ML	71	487	1859	56
		BNPM-BAYES	59	482	1898	54
	∞	S-BNPM	94	527	1945	61
		BNPM-ML	332	641	2149	48
		BNPM-BAYES	321	632	2129	74
400		CC	-1911	1917	573	1000
		True	10	220	984	25
	0	S-BNPM	-201	296	899	153
		BNPM-ML	-195	288	837	173
		BNPM-BAYES	-198	291	837	165
	1	S-BNPM	-73	265	1013	80
		BNPM-ML	38	229	901	50
		BNPM-BAYES	34	229	916	45
	∞	S-BNPM	55	304	1124	44
		BNPM-ML	280	375	1002	150
		BNPM-BAYES	278	374	1012	171

Table 3a. Results for scenario 3 under $\lambda_T = 0$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1146	1191	939	981
	0	S-BNPM	5	291	1185	50
		BNPM-ML	6	288	1165	53
		BNPM-BAYES	2	288	1171	53
	1	S-BNPM	131	326	1200	63
		BNPM-ML	126	320	1203	60
		BNPM-BAYES	123	319	1201	63
	∞	S-BNPM	253	401	1245	104
		BNPM-ML	246	394	1250	97
		BNPM-BAYES	246	393	1255	117
400		CC	-1157	1168	467	1000
	0	S-BNPM	3	148	584	54
		BNPM-ML	3	147	576	52
		BNPM-BAYES	2	146	574	51
	1	S-BNPM	131	202	599	132
		BNPM-ML	126	196	593	124
		BNPM-BAYES	126	196	588	131
	∞	S-BNPM	256	300	617	345
		BNPM-ML	250	295	614	326
		BNPM-BAYES	250	295	612	345

Table 3b. Results for scenario 3 under $\lambda_T = 1$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1349	1393	914	995
	0	S-BNPM	-117	319	1197	65
		BNPM-ML	-116	317	1180	70
		BNPM-BAYES	-122	319	1178	70
	1	S-BNPM	26	309	1216	42
		BNPM-ML	18	303	1225	46
		BNPM-BAYES	14	302	1210	47
	∞	S-BNPM	163	366	1267	64
		BNPM-ML	151	353	1280	57
		BNPM-BAYES	151	353	1276	72
400		CC	-1351	1361	458	1000
	0	S-BNPM	-139	198	587	140
		BNPM-ML	-138	197	576	152
		BNPM-BAYES	-140	198	576	145
	1	S-BNPM	-2	147	601	41
		BNPM-ML	-5	145	594	42
		BNPM-BAYES	-5	145	590	46
	∞	S-BNPM	133	204	619	113
		BNPM-ML	129	201	617	116
		BNPM-BAYES	129	201	617	120

Table 3c. Results for scenario 3 under $\lambda_T = \infty$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1314	1357	914	995
	0	S-BNPM	-233	372	1139	133
		BNPM-ML	-231	370	1125	136
		BNPM-BAYES	-235	373	1129	132
	1	S-BNPM	-115	319	1148	87
		BNPM-ML	-118	316	1160	78
		BNPM-BAYES	-119	316	1156	81
	∞	S-BNPM	0	307	1193	51
		BNPM-ML	-5	304	1204	50
		BNPM-BAYES	-5	304	1207	48
400		CC	-1312	1322	452	1000
	0	S-BNPM	-240	277	560	387
		BNPM-ML	-238	275	552	392
		BNPM-BAYES	-239	276	553	395
	1	S-BNPM	-118	185	572	133
		BNPM-ML	-119	185	568	135
		BNPM-BAYES	-120	185	567	129
	∞	S-BNPM	1	149	590	47
		BNPM-ML	-1	148	589	44
		BNPM-BAYES	-1	147	588	42

Table 3d. Results for scenario 3 under mechanism D.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1221	1265	930	988
	0	S-BNPM	7	309	1201	58
		BNPM-ML	6	307	1219	55
		BNPM-BAYES	3	306	1178	65
	1	S-BNPM	145	354	1220	68
		BNPM-ML	137	344	1269	53
		BNPM-BAYES	134	343	1214	68
	∞	S-BNPM	277	436	1267	123
		BNPM-ML	267	425	1327	89
		BNPM-BAYES	267	425	1273	134
400		CC	-1204	1216	461	1000
	0	S-BNPM	9	150	589	48
		BNPM-ML	8	149	597	44
		BNPM-BAYES	8	149	577	49
	1	S-BNPM	148	214	607	159
		BNPM-ML	141	208	619	137
		BNPM-BAYES	141	208	593	167
	∞	S-BNPM	280	325	626	408
		BNPM-ML	274	318	646	364
		BNPM-BAYES	274	318	617	421

Table 3e. Results for scenario 3 under mechanism E.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1581	1610	840	1000
	0	S-BNPM	-239	375	1196	127
		BNPM-ML	-230	365	1160	127
		BNPM-BAYES	-235	368	1153	126
	1	S-BNPM	-41	318	1238	54
		BNPM-ML	-48	304	1231	55
		BNPM-BAYES	-51	303	1206	60
	∞	S-BNPM	149	375	1315	60
		BNPM-ML	134	355	1315	55
		BNPM-BAYES	134	355	1300	68
400		CC	-1576	1583	417	1000
	0	S-BNPM	-235	277	592	342
		BNPM-ML	-226	267	566	355
		BNPM-BAYES	-227	268	565	353
	1	S-BNPM	-40	161	625	56
		BNPM-ML	-42	155	598	57
		BNPM-BAYES	-42	155	589	54
	∞	S-BNPM	152	226	658	137
		BNPM-ML	142	214	636	133
		BNPM-BAYES	141	214	631	148

Table 4a. Results for scenario 4 under $\lambda_T = 0$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-2146	2169	891	1000
	0	S-BNPM	5	479	1968	45
		BNPM-ML	2	467	1872	55
		BNPM-BAYES	-14	465	1845	53
	1	S-BNPM	220	556	2089	43
		BNPM-ML	271	568	2052	51
		BNPM-BAYES	256	559	2009	58
	∞	S-BNPM	445	720	2289	53
		BNPM-ML	540	775	2240	92
		BNPM-BAYES	528	765	2223	134
400		CC	-2155	2161	442	1000
	0	S-BNPM	3	231	999	36
		BNPM-ML	1	227	916	51
		BNPM-BAYES	-3	227	907	54
	1	S-BNPM	186	318	1123	51
		BNPM-ML	265	358	1000	146
		BNPM-BAYES	261	356	988	167
	∞	S-BNPM	358	463	1233	137
		BNPM-ML	528	592	1087	458
		BNPM-BAYES	526	589	1079	503

Table 4b. Results for scenario 4 under $\lambda_T = 1$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-2208	2229	883	1000
	0	S-BNPM	-98	503	1938	62
		BNPM-ML	-93	494	1847	66
		BNPM-BAYES	-110	495	1823	67
	1	S-BNPM	116	546	2051	37
		BNPM-ML	166	548	2022	47
		BNPM-BAYES	153	542	1974	66
	∞	S-BNPM	334	679	2236	50
		BNPM-ML	426	717	2204	70
		BNPM-BAYES	415	709	2181	119
400		CC	-2201	2206	437	1000
	0	S-BNPM	-136	270	967	96
		BNPM-ML	-137	263	888	116
		BNPM-BAYES	-141	265	882	111
	1	S-BNPM	38	264	1079	37
		BNPM-ML	121	271	971	55
		BNPM-BAYES	117	269	959	69
	∞	S-BNPM	198	352	1185	61
		BNPM-ML	379	466	1057	267
		BNPM-BAYES	376	463	1050	309

Table 4c. Results for scenario 4 under $\lambda_T = \infty$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-2279	2301	868	1000
	0	S-BNPM	-281	537	1869	115
		BNPM-ML	-275	524	1780	125
		BNPM-BAYES	-291	532	1760	106
	1	S-BNPM	-79	503	1983	60
		BNPM-ML	-24	481	1950	55
		BNPM-BAYES	-38	478	1913	55
	∞	S-BNPM	136	576	2181	51
		BNPM-ML	226	584	2127	47
		BNPM-BAYES	215	579	2115	66
400		CC	-2280	2285	431	1000
	0	S-BNPM	-255	336	950	186
		BNPM-ML	-257	333	875	221
		BNPM-BAYES	-261	336	868	204
	1	S-BNPM	-85	253	1058	59
		BNPM-ML	-5	226	955	45
		BNPM-BAYES	-9	226	942	42
	∞	S-BNPM	76	280	1167	32
		BNPM-ML	247	351	1039	109
		BNPM-BAYES	244	349	1031	132

Table 4d. Results for scenario 4 under mechanism D.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1392	1453	1124	950
	0	S-BNPM	11	462	1788	59
		BNPM-ML	11	460	1771	60
		BNPM-BAYES	2	458	1777	57
	1	S-BNPM	98	485	1823	53
		BNPM-ML	115	486	1846	52
		BNPM-BAYES	107	483	1848	54
	∞	S-BNPM	174	523	1893	50
		BNPM-ML	219	537	1926	52
		BNPM-BAYES	211	532	1934	59
400		CC	-1406	1420	559	1000
	0	S-BNPM	-3	229	887	52
		BNPM-ML	-2	228	876	58
		BNPM-BAYES	-4	228	873	55
	1	S-BNPM	65	240	907	51
		BNPM-ML	98	253	911	61
		BNPM-BAYES	96	251	907	68
	∞	S-BNPM	120	266	934	67
		BNPM-ML	198	311	949	105
		BNPM-BAYES	197	310	945	117

Table 4e. Results for scenario 4 under mechanism E.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1712	1748	1041	997
	0	S-BNPM	-123	455	1770	72
		BNPM-ML	-122	451	1731	73
		BNPM-BAYES	-131	452	1746	66
	1	S-BNPM	-18	451	1810	57
		BNPM-ML	13	450	1824	51
		BNPM-BAYES	4	448	1838	43
	∞	S-BNPM	83	478	1893	41
		BNPM-ML	148	496	1921	39
		BNPM-BAYES	142	493	1939	48
400		CC	-1714	1723	519	1000
	0	S-BNPM	-113	244	884	87
		BNPM-ML	-114	243	865	89
		BNPM-BAYES	-116	244	862	89
	1	S-BNPM	-23	224	918	46
		BNPM-ML	19	222	910	46
		BNPM-BAYES	17	222	905	48
	∞	S-BNPM	51	240	957	43
		BNPM-ML	151	276	957	69
		BNPM-BAYES	149	275	952	79

Table 5a. Results for scenario 5 under $\lambda_T = 0$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1304	1327	730	998
	0	S-BNPM	-13	374	1519	57
		BNPM-ML	-15	368	1502	50
		BNPM-BAYES	-28	367	1441	54
	1	S-BNPM	184	440	1645	39
		BNPM-ML	255	480	1742	22
		BNPM-BAYES	245	473	1646	61
	∞	S-BNPM	388	614	1886	42
		BNPM-ML	525	709	1996	86
		BNPM-BAYES	516	701	1913	186
400		CC	-1287	1294	368	1000
	0	S-BNPM	-2	188	763	46
		BNPM-ML	-1	185	718	47
		BNPM-BAYES	-4	185	712	52
	1	S-BNPM	153	259	864	58
		BNPM-ML	255	328	821	191
		BNPM-BAYES	252	326	805	231
	∞	S-BNPM	274	364	953	125
		BNPM-ML	511	565	930	597
		BNPM-BAYES	509	564	920	630

Table 5b. Results for scenario 5 under $\lambda_T = 1$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1521	1543	701	1000
	0	S-BNPM	-143	427	1616	101
		BNPM-ML	-149	416	1639	89
		BNPM-BAYES	-164	419	1492	105
	1	S-BNPM	106	460	1800	45
		BNPM-ML	166	469	1950	27
		BNPM-BAYES	154	461	1716	57
	∞	S-BNPM	361	652	2143	31
		BNPM-ML	481	721	2279	35
		BNPM-BAYES	473	714	2065	116
400		CC	-1510	1516	350	1000
	0	S-BNPM	-145	245	805	130
		BNPM-ML	-145	241	745	155
		BNPM-BAYES	-149	244	725	154
	1	S-BNPM	53	232	953	32
		BNPM-ML	163	274	869	85
		BNPM-BAYES	160	271	832	120
	∞	S-BNPM	216	342	1072	64
		BNPM-ML	472	543	1004	427
		BNPM-BAYES	470	541	976	492

Table 5c. Results for scenario 5 under $\lambda_T = \infty$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1444	1465	713	1000
	0	S-BNPM	-226	426	1470	129
		BNPM-ML	-226	417	1451	118
		BNPM-BAYES	-238	422	1398	121
	1	S-BNPM	-34	398	1603	60
		BNPM-ML	29	390	1683	38
		BNPM-BAYES	19	387	1594	42
	∞	S-BNPM	154	493	1833	29
		BNPM-ML	285	541	1930	24
		BNPM-BAYES	277	536	1858	71
400		CC	-1443	1449	356	1000
	0	S-BNPM	-233	295	731	261
		BNPM-ML	-233	292	688	277
		BNPM-BAYES	-236	295	679	277
	1	S-BNPM	-82	212	833	75
		BNPM-ML	17	194	789	50
		BNPM-BAYES	15	194	771	49
	∞	S-BNPM	43	229	929	32
		BNPM-ML	268	350	897	163
		BNPM-BAYES	265	348	885	215

Table 5d. Results for scenario 5 under mechanism D.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-728	822	1033	691
	0	S-BNPM	8	529	1413	196
		BNPM-ML	8	528	2141	54
		BNPM-BAYES	3	526	1409	194
	1	S-BNPM	74	555	1426	195
		BNPM-ML	83	561	2307	42
		BNPM-BAYES	78	558	1469	197
	∞	S-BNPM	118	582	1464	196
		BNPM-ML	158	610	2479	41
		BNPM-BAYES	154	606	1539	202
400		CC	-731	756	520	980
	0	S-BNPM	1	237	690	147
		BNPM-ML	1	237	938	53
		BNPM-BAYES	0	236	688	154
	1	S-BNPM	63	252	698	161
		BNPM-ML	65	256	994	42
		BNPM-BAYES	65	256	709	165
	∞	S-BNPM	93	269	701	181
		BNPM-ML	129	291	1052	49
		BNPM-BAYES	129	291	736	199

Table 5e. Results for scenario 5 under mechanism E.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1217	1261	887	971
	0	S-BNPM	-279	518	1357	233
		BNPM-ML	-279	517	1728	138
		BNPM-BAYES	-287	519	1347	231
	1	S-BNPM	-174	482	1389	188
		BNPM-ML	-156	485	1901	82
		BNPM-BAYES	-161	484	1436	167
	∞	S-BNPM	-99	484	1451	158
		BNPM-ML	-32	500	2081	59
		BNPM-BAYES	-36	499	1551	126
400		CC	-1207	1217	442	1000
	0	S-BNPM	-283	349	661	439
		BNPM-ML	-283	350	789	322
		BNPM-BAYES	-284	351	654	434
	1	S-BNPM	-193	285	676	282
		BNPM-ML	-172	277	852	142
		BNPM-BAYES	-174	278	692	235
	∞	S-BNPM	-143	261	690	214
		BNPM-ML	-62	243	920	69
		BNPM-BAYES	-63	243	737	146

Table 6a. Results for scenario 6 under $\lambda_T = 0$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1220	1250	790	999
	0	S-BNPM	2	311	1251	54
		BNPM-ML	2	306	1209	58
		BNPM-BAYES	-4	304	1185	62
	1	S-BNPM	358	547	1517	72
		BNPM-ML	210	405	1328	65
		BNPM-BAYES	206	401	1281	97
	∞	S-BNPM	688	871	1923	164
		BNPM-ML	418	584	1460	160
		BNPM-BAYES	416	581	1437	241
400		CC	-1217	1225	395	1000
	0	S-BNPM	5	155	644	34
		BNPM-ML	4	150	593	45
		BNPM-BAYES	2	150	584	54
	1	S-BNPM	330	394	855	279
		BNPM-ML	207	268	644	238
		BNPM-BAYES	206	268	627	278
	∞	S-BNPM	671	742	1226	610
		BNPM-ML	409	456	701	602
		BNPM-BAYES	409	455	690	634

Table 6b. Results for scenario 6 under $\lambda_T = 1$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1056	1096	844	989
	0	S-BNPM	-137	310	1150	86
		BNPM-ML	-134	307	1133	85
		BNPM-BAYES	-139	308	1136	77
	1	S-BNPM	20	303	1217	41
		BNPM-ML	-6	298	1200	47
		BNPM-BAYES	-9	297	1192	43
	∞	S-BNPM	179	397	1375	42
		BNPM-ML	121	350	1278	49
		BNPM-BAYES	120	349	1279	70
400		CC	-1048	1059	423	1000
	0	S-BNPM	-128	191	574	154
		BNPM-ML	-127	189	560	151
		BNPM-BAYES	-128	189	559	154
	1	S-BNPM	7	156	613	46
		BNPM-ML	-2	153	588	53
		BNPM-BAYES	-3	153	584	61
	∞	S-BNPM	131	223	683	82
		BNPM-ML	124	210	622	123
		BNPM-BAYES	123	209	621	135

Table 6c. Results for scenario 6 under $\lambda_T = \infty$.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1118	1156	835	992
	0	S-BNPM	-219	356	1140	145
		BNPM-ML	-215	353	1121	150
		BNPM-BAYES	-220	354	1124	139
	1	S-BNPM	-65	315	1202	69
		BNPM-ML	-90	314	1188	77
		BNPM-BAYES	-93	314	1182	78
	∞	S-BNPM	95	378	1369	44
		BNPM-ML	35	335	1268	54
		BNPM-BAYES	33	334	1270	56
400		CC	-1134	1145	418	1000
	0	S-BNPM	-238	279	565	404
		BNPM-ML	-235	276	551	423
		BNPM-BAYES	-237	277	550	407
	1	S-BNPM	-109	188	600	125
		BNPM-ML	-115	192	579	147
		BNPM-BAYES	-116	192	575	146
	∞	S-BNPM	8	171	658	49
		BNPM-ML	6	167	612	55
		BNPM-BAYES	5	167	610	56

Table 6d. Results for scenario 6 under mechanism D.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-745	789	726	937
	0	S-BNPM	-11	332	1219	61
		BNPM-ML	-12	328	1352	40
		BNPM-BAYES	-19	327	1173	81
	1	S-BNPM	221	484	1444	81
		BNPM-ML	149	408	1566	45
		BNPM-BAYES	144	405	1320	112
	∞	S-BNPM	452	713	1812	135
		BNPM-ML	310	545	1799	86
		BNPM-BAYES	306	542	1516	181
400		CC	-746	758	362	1000
	0	S-BNPM	3	164	604	77
		BNPM-ML	2	161	644	59
		BNPM-BAYES	0	161	571	91
	1	S-BNPM	153	254	708	131
		BNPM-ML	162	247	733	120
		BNPM-BAYES	160	246	635	195
	∞	S-BNPM	310	410	841	265
		BNPM-ML	321	389	832	305
		BNPM-BAYES	320	388	715	448

Table 6e. Results for scenario 6 under mechanism E.

n	λ_A	Model	Bias (x1000)	RMSE (x1000)	CIW (x1000)	Non-Coverage
100		CC	-1043	1075	750	996
	0	S-BNPM	-220	379	1178	157
		BNPM-ML	-211	371	1240	119
		BNPM-BAYES	-217	372	1138	148
	1	S-BNPM	41	424	1396	70
		BNPM-ML	-43	361	1406	64
		BNPM-BAYES	-48	359	1253	86
	∞	S-BNPM	313	632	1773	57
		BNPM-ML	125	446	1591	51
		BNPM-BAYES	121	443	1422	104
400		CC	-1040	1048	377	1000
	0	S-BNPM	-220	266	583	345
		BNPM-ML	-210	257	595	303
		BNPM-BAYES	-212	258	555	334
	1	S-BNPM	-49	184	672	78
		BNPM-ML	-59	177	658	73
		BNPM-BAYES	-60	178	600	101
	∞	S-BNPM	127	280	806	68
		BNPM-ML	92	213	729	79
		BNPM-BAYES	91	213	663	120